Survey on Clutter Spatial Intensity Estimation Methods for Target Tracking

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This paper is a comprehensive and up-to-date survey on clutter spatial intensity estimation methods proposed over a span of two decades for radar target tracking. Here, the methods are grouped into three categories. The first category of methods is applicable to measurements in a track validation gate, while the second category of methods compute the clutter spatial intensity of any measurement in measurement space. Finally, the third category uses the concept of clutter generators, which act as the source of clutter measurements. Then, probability hypothesis density filter equations are used to derive the clutter spatial intensity of measurements. The above classification of different methods is based on the techniques used and on the assumptions made while computing the spatial intensity of clutter. This paper emphasizes the underlying ideas and assumptions of each of the methods so that the reader could understand not only how each method works but also their pros and cons. Also, an effort is made to bring out the interrelationship between different methods, wherever possible.

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I. INTRODUCTION

The surveillance systems like radar generate detections (measurements) from the received signals based on a defined detection threshold. These detections may be from targets of interest or from random objects in surveillance space. If a target is present among the detections, then it is detected with a detection probability ($P_D < 1$). The detections from random objects are termed clutter or false alarms. These detections are fed to multitarget trackers (MTTs) without any prior information about the source of measurements. The primary task of any MTT algorithm is to solve the measurement-origin uncertainty, i.e., to identify target-originated measurements and clutter measurements. The above task is called “measurement-to-track association” in the target tracking world.

Usually, in multitarget tracking, the clutter is modeled as a nonhomogeneous Poisson process (NHPP) in measurement space [1]. The distribution of an NHPP process is fully described by its spatial intensity function [6], [7]. Hence, the clutter process can be explained by some clutter spatial intensity function, otherwise called a jargon clutter density in target tracking. Many practical MTTs require the clutter spatial intensity to compute the measurement likelihood ratio (i.e., ratio of the probability that a measurement is generated by a target to the probability that it is clutter) for the purpose of measurement-to-track association and new track initialization [8]–[10], [12]. An overoptimistic estimate for clutter spatial intensity leads to lower data association probabilities of measurement-to-tracks, thereby degrading the performance of MTT. Whereas, a pessimistic estimate of clutter spatial intensity may lead to faster confirmation of false tracks. Hence, it is important to have an optimal estimate of clutter spatial intensity at the tracker.

This paper is a consolidated survey on clutter spatial intensity estimation methods proposed over a span of two decades for radar target tracking. The research work on clutter spatial intensity estimators is scattered over many literatures with no consolidation of the work available till date. This paper is the first comprehensive and up-to-date survey on clutter spatial intensity estimators [3]–[5] to the best of the authors’ knowledge. The contributions of the authors are as follows. We broadly group the different spatial intensity estimators into three categories. The first category of methods [1], [12], [14] computes the clutter spatial intensity of those measurements that are only in the track validation gate. The second category of methods [11], [18], [20]–[24], [38], [46], [49], [50] evaluates spatial intensity at any detection point in the surveillance region using some nonparametric density estimation methods. Finally, the third category of methods [27]–[29], [31]–[33], [36], [39], [40], [48] uses a notion of clutter generators independent of the actual targets, and they act as the source of clutter measurements. Thereafter, a probability hypothesis density

(PHD) filter-based recursions are derived for evaluating the intensity of clutter generators. This intensity is used to estimate the required clutter spatial intensity at any detection point. Here, this survey emphasizes the underlying ideas and assumptions of each of the methods so that the reader could understand not only how each method works but also their pros and cons. Moreover, this paper also brings out the interrelationship between different methods, wherever possible. However, the issues related to the implementation of methods are not discussed in this paper.

The rest of the paper is organized as follows: A mathematical definition for clutter spatial intensity is presented in Section II. The clutter density estimation methods based on a track validation gate are explained in Section III. Thereafter, Section IV describes the clutter spatial intensity estimators for any detection in measurement space. The clutter density estimators based on a clutter generator assumption are explained in Section V. Finally, an interrelation between the methods is discussed in Section VI.

II. DEFINITION OF CLUTTER SPATIAL INTENSITY

Assume that the clutter measurements follow an NHPP with spatial intensity function $c(\mathbf{z})$ varying with the d-dimensional location in space $\mathbf{z}$, and if $c(\mathbf{z})$ is continuous and locally integrable in small infinitesimal volume dV, then [41]

$$\lambda = \int_{V}^{V+dV} c(\mathbf{z})\mathbf{dz}, \tag{1}$$

where λ is the mean number of clutter points in the volume. Thus, clutter spatial intensity can be defined as the average number of clutter measurements in dV. From (1), it can be seen that the number of clutter measurements in surveillance space is not uniformly distributed but dependent on spatial coordinate $\mathbf{z}$. The clutter spatial intensity estimators presented in the subsequent sections either try to estimate or find some approximations for this clutter spatial intensity.

III. CLUTTER SPATIAL INTENSITY ESTIMATORS BASED ON TRACK VALIDATION GATE

It is common in the target tracking world to define a validation gate in order to select only the probable measurements corresponding to tracks. A simple ellipsoidal gate [2] is used as the validation gate in the case of single target trackers (STTs). Whereas, a union of validation gates (effective validation gate) of those tracks sharing measurements is used in the case of MTT. The type of estimators discussed in this section only evaluate the clutter spatial intensity of measurements that are in the validation gate/effective validation gate of track(s). Such estimators assume the spatial variation of clutter to be uniformly distributed in the volume of the validation gate/effective validation gate. The clutter spatial intensity estimate in such estimators is calculated as

$$\hat{\lambda}(\mathbf{z}) = \frac{\hat{m}_k}{V}, \tag{2}$$

where $\hat{m}_k$ is the expected number of clutter measurements in the validation gate, V is the volume of the validation gate, and $\hat{\lambda}(\mathbf{z})$ is the estimate for clutter spatial intensity. The effective track validation gate volume discussed in [13] can be used for clutter intensity estimation when the measurements are shared by different tracks. In [1], MTT uses sample spatial intensity (2) obtained from the set of measurements in the effective validation gate as the clutter spatial intensity. In this case, $\hat{m}_k = m_k$, where m_k is the total number of measurements in the validation gate. However, the method in [1] gives a biased estimate as it does not distinguish between target originated measurements and clutter measurements in the validation gate.

However, the method in [14] uses "track perceivability"[15] and that in [12] uses "track existence probability"[16] in order to give an unbiased estimate for the clutter spatial intensity of measurements in the effective track validation gate. The conditional mean estimator of spatial intensity for false alarms proposed in [14] is defined, for a single target, on the assumption that at any time there is one or no target measurement present in the validation gate. Therein, the mean number of false measurements is computed by excluding the target-originated measurements stochastically. The mean number of false alarms $\hat{m}_k$ can be evaluated as [14]

$$\hat{m}_k = m_k - \frac{r_k \frac{m_k}{V}}{\hat{\lambda}_k + r_k \frac{m_k}{V}}. \tag{3}$$

In the above, r_k, a measure of target-originated measurements, is defined as

$$r_k = \frac{P_D P_G P\left(\chi_k \mid Z^{k-1}\right)}{1 - P_D P_G P\left(\chi_k \mid Z^{k-1}\right)}. \tag{4}$$

Here, $P\left(\chi_k \mid Z^{k-1}\right)$ denotes the predicted track quality parameter, i.e., the track exists at time k given the set of previous measurements, and P_D and P_G denoting the track-detection probability and the probability that the target measurement is in its validation gate, respectively. Once $\hat{m}_k$ is computed, the spatial intensity of clutter can be approximated as given in (2). The conditional estimate of (3) is dependent on an initial estimate $\hat{\lambda}_k$, which can be computed using a maximum-likelihood (ML) estimator [14] as

$$\hat{\lambda}_k = \frac{m_k}{2V}\left(1 - r_k + \sqrt{(1 - r_k)^2 + \frac{4m_k - 1}{m_k} r_k}\right). \tag{5}$$

In [12], the predicted track existence probability of tracks is used to stochastically identify target-originated measurements under the assumption of at most one detection per target. In a single-target case, the mean number of false alarms therein can be shown to be

$$\hat{m}_k = m_k - P_D P_G P\left(\chi_k \mid Z^{k-1}\right). \tag{6}$$

Thereafter, (2) can be used to evaluate clutter spatial intensity. However, for a multitarget case, the extension is slightly exhaustive [12].

Validation gates in MTT are created using innovation matrices of tracks [2]. Hence, a measurement shared by different tracks will have different false alarm spatial intensities when used in updating different tracks. So, the effective track validation gate used in the above methods cannot be a suitable choice to estimate the spatial clutter intensity. Furthermore, the estimators based on a track validation gate only evaluate the clutter spatial intensity of those measurements inside the track validation gate. However, the false-alarm intensity of measurements outside the gates of tracks is also required as shown in [17] to compute the cost of newly initialized tracks.

IV. CLUTTER SPATIAL INTENSITY ESTIMATORS IN MEASUREMENT SPACE

Spatial intensity estimators of this category try to evaluate the false-alarm intensity of any detection point in measurement space or in a surveillance region. They utilize statistical methods like nonparametric density estimation [19] or expectation maximization (EM) toward this purpose. The classic clutter map estimator in [18] and [11] divides the whole measurement space into cells, and the false-alarm spatial intensity of measurements belonging to a cell is assumed constant. The false-alarm intensity in cell i is given by

$$\hat{\lambda}(\mathbf{z}) = \frac{\text{number of false alarms in cell } i}{\text{volume of cell} \times \text{number of scans}}. \tag{7}$$

Note that (7) returns a smoothed clutter intensity over a number of scans. Here, the spatial intensity estimate depends on the size of the cell, which also determines the bias of the estimate. The clutter map estimator can be compared to a multivariate histogram method [19]. The histogram method is discontinuous and unsuitable for data of two or more dimensions [19] and also gives a block nature to the estimated spatial intensity function. Even though the notion of dividing the measurement space into cells captures nonhomogeneity in some sense, the spatial intensity estimate evaluated by a clutter map method is overoptimistic, as shown in [18].

Two other methods, namely spatial clutter map estimator [18], [20] and temporal clutter map estimator [18], also require the surveillance region to be divided into cells manually and assume a stationary Poisson clutter process in each cell. Both methods define a mathematical distance to compute the inverse of clutter spatial intensity, called sparsity, of the measurements falling in a cell. The former relies on the nearest neighbor measurement distance, which is equal to the mathematical distance from the center of the cell to the nearest neighbor measurement, and the latter uses interarrival time between two consecutive measurements falling in the cell as the mathematical distance. The estimates obtained are averaged over time in order to smoothen the spatial intensity. However, the exact definition of mathematical distance between the center of the cell and the nearest measurement is not given therein. In order to use the nearest neighbor distance or the interarrival time between measurements in evaluating the clutter spatial intensity, it is assumed that the spatial intensity is homogeneous and isotropic in and around the cell. But, the inverse of the defined mathematical distance does not hold in the case of NHPP [41]. It was shown in [18] that a spatial clutter map estimator is effective when compared to a temporal clutter map estimator in a dynamic clutter.

The need to divide the measurement space into cells is eliminated in a spatial clutter measurement density estimator (SCMDE) [21] unlike the previous methods. It proposes to calculate the sparsity at any detection point by evaluating the volume of hypersphere with center at the detection point and radius equal to the l_2-norm distance to nth nearest detection point. The sparsity therein is the inverse of clutter spatial intensity at that point. The sparsity $\hat{\gamma}(\mathbf{z})$ in SCMDE is defined as

$$\hat{\gamma}(\mathbf{z}) = \frac{V\left(r^n(\mathbf{z})\right)}{n}, \tag{8}$$

where $V\left(r^n(\mathbf{z})\right)$ is the volume of the d-dimensional hypersphere with center $\mathbf{z}$ and radius $r^n(\mathbf{z})$ equal to the distance to the nth nearest measurement. Here, the volume of hypersphere is defined as

$$V\left(r^n(\mathbf{z})\right) = C_d\left(r^n(\mathbf{z})\right)^d, \tag{9}$$

where the constant C_d is defined as

$$C_d = \frac{2}{d}\frac{\pi^{d/2}}{\Gamma\left(d/2\right)}. \tag{10}$$

Here, $\Gamma(.)$ is the Gamma function. SCMDE has similar assumptions of homogeneity around the detection point like the former methods of this category. Moreover, the sparsity is assumed independent at points far from $\mathbf{z}$, thereby accounting for NHPP but with local homogeneity. It has been shown in [21] that the higher the order n of sparsity, the higher the bias in estimate in the case of a nonhomogeneous clutter process. This is due to the fact that as n increases, we tend to encounter more overlapping nonhomogenous areas, and an assumption of homogeneity in those areas could yield a biased estimate. SCMDE is similar to a k-nearest neighbor density estimation technique, and the spatial intensity estimate obtained is discontinuous in nature [19]. It has been shown in [21] and [22] that SCMDE is effective in STT because an unbiased false-alarm spatial intensity estimate is obtained at the target detections while penalizing the clutter measurements. However, it is not suitable for MTT [22] as it does not distinguish target measurements from clutter measurements. In [22] and [23], an improvement for SCMDE is proposed by calculating the clutter probability of each measurement in the hypersphere. The clutter probability $P\left(\chi_{k,j}^{0} \mid Z^{k-1}\right)$ of each jth measurement $\mathbf{z}_{k,j}$, given T_k potential tracks at time k, is computed as [22]

$$P\left(\chi_{k,j}^{0} \mid Z^{k-1}\right) = \frac{1}{1 + \sum_{\tau \epsilon T_k} \frac{P_{k,j}^{\tau}}{1 - P_{k,j}^{\tau}}}. \tag{11}$$

Here, $\chi_{k,j}^{0}$ is the event that the measurement $\mathbf{z}_{k,j}$ has not originated from any given tracks and $P_{k,j}^{\tau}$ is the prior probability that the measurement originated from the track τ described in [12]. The MTTSCMDE proposed therein computes the sparsity at any detection point as

$$\hat{\gamma}(\mathbf{z}_{k,j}) = \frac{V\left(r^{n}(\mathbf{z}_{k,j})\right)}{\sum_{l=1}^{m} C_{k,j}^{l}}. \tag{12}$$

Here, $C_{k,j}^{l}$ is the clutter probability of the lth nearest detection point computed using (11).

Even though similarities can be drawn between SCMDE and MTTSCDME based on their assumptions, there lies a significant dissimilarity between them in the method adopted to evaluate clutter spatial intensity at a point. SCMDE utilizes the number of measurements in a hypersphere in computing the sparsity estimate. These measurements can be target detections as well as clutter measurements, which may lead to a biased estimate. Whereas, MTTSCMDE uses the mean number of clutter measurements with the aid of clutter measurement probability to reduce the bias in clutter spatial intensity estimate. The mathematical distance used in SCMDE as well as MTTSCMDE is not well-defined for range (r)/bearing (θ)/range-rate ($\dot{r}$) measurement space in cases like Doppler radar. One solution could be to write the clutter spatial intensity estimate $c(r, \theta, \dot{r})$ as the product of position clutter measurement intensity $c(r, \theta)$ and clutter likelihood of range-rate measurement $p(f_d)$, as shown in [42]. However, this amounts to an assumption that the clutter distribution in range rate space is homogeneous with respect to range and bearing. This assumption may not be true for all real scenarios due to anisotropy in the field of view of range rate, range, and bearing. Another possible solution could be to use a weighted normalized distance that reflects the actual distance of measurements in nonhomogeneous measurement space, as shown in [38].

In [24], the clutter spatial intensity estimation problem is fitted into a kernel density estimation (KDE) [19] framework. A multivariate Gaussian kernel density estimator that can handle the measurement origin uncertainty as well as the continuous arrival of measurements is proposed therein to evaluate the spatial intensity of false alarms. The measurement-origin uncertainty is solved using the joint association events and their corresponding probabilities given by MTTs [8], [10]. This is possible because the measurement origin is decided in an association event. If an association event χ_i with probability $P(\chi_i)$ gives a set of clutter measurements $Z_k^C = \left\{\mathbf{z}_{k,1}, \mathbf{z}_{k,2}, \ldots, \mathbf{z}_{k,n}\right\}$, then the normalized spatial intensity estimate conditional on the association event can be evaluated as [24]

$$\lambda^{i}(\mathbf{z}) = \sum_{j=1}^{n} w_j K_H\left(\mathbf{z} - \mathbf{z}_{k,j}\right). \tag{13}$$

Here, $K_H(.)$ is the Gaussian kernel with a positive definite bandwidth matrix $\mathbf{H}$ and corresponding weight w_j. Furthermore, using the following relation between conditional expectation and unconditional expected value

$$\mathbb{E}[Z] = \mathbb{E}[\mathbb{E}[Z \mid X]], \tag{14}$$

the clutter spatial intensity at a point $\mathbf{z}$ in measurement space can be computed as

$$\hat{\lambda}(\mathbf{z}) = \sum_{i=1}^{m} P(\chi_i)\, \lambda^{i}(\mathbf{z}). \tag{15}$$

In (14), $\mathbb{E}$ denotes the expectation operator. The performance of KDE depends on the choice of bandwidth rather than the shape of the kernel used [19]. Hence, a method to compute an optimal bandwidth is given in [24], wherein the gradient of the cost function is derived using a cross-validation technique [25]. A fixed bandwidth in a KDE method affects data with a long tail distribution [19]. Hence, an adaptive kernel bandwidth matrix based on adaptive KDE [19] is adopted in [24], which ensures the kernels at lower intensity areas have a different bandwidth from that of kernels at areas with high intensity. The probability density estimate obtained by KDE inherits all the continuity and differentiability properties of the kernel used [19]. Hence, the clutter spatial intensity estimate of (15) is continuous as the Gaussian kernel has continuity and differentiability of all orders. The method in [24] does not rely on the calculation of mathematical distance unlike SCMDE and MTTSCMDE. Hence, it is applicable to nonlinear sensors like Doppler radar. It has been shown in [24] that it is effective in target tracking in a background clutter with slowly varying spatial intensity. However, a background clutter with fast varying clutter spatial intensity could mean the need to compute optimum bandwidth frequently. This could add to computational complexity in radars with higher sampling rates.

The method in [46], [49], and [50] assumes a time-invariant clutter intensity function, which is estimated using cumulative measurements collected over some window of time. Furthermore, it assumes that the number of target measurements is sparse compared to clutter measurements. Given $\mathbb{Z} = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_m\}$, the cumulative set of measurements in the period $[1, L]$, and a set C of components, the clutter intensity therein is defined as the product of the average number of clutter measurements and the spatial density function, which can be expressed as

$$\lambda(\mathbf{z}_j) = \hat{\lambda} f(\mathbf{z}_j \mid C). \tag{16}$$

Here, $\hat{\lambda}$ denotes the estimate of the average number of clutter measurements, $f(\mathbf{z}_j \mid C)$ denotes the spatial probability density function, and $\lambda(\mathbf{z}_j)$ denotes the clutter intensity at a measurement $\mathbf{z}_j$ of $\mathbb{Z}$. The spatial probability density function is represented by an N-component finite mixture model (FMM) [47] as

$$f(\mathbf{z}_j \mid C) = \sum_{i=1}^{N} w_j f(\mathbf{z}_j \mid c_i) \qquad \ni \sum_{i=1}^{N} w_j = 1, \quad (17)$$

where w_j denotes the weight of each component and $f(\mathbf{z}_j \mid c_i)$ denotes the likelihood of the measurement $\mathbf{z}_j$ with each component c_i. Gaussian mixture models were adopted in the said paper; hence, the likelihood $f(\mathbf{z}_j \mid c_i)$ is Gaussian with parameters $\theta_i = \{\mu_i, \Sigma_i\}$, wherein μ_i and Σ_i represent the mean and covariance of component c_i, respectively. Note that an implicit one-to-one mapping between measurements and components can be seen therein. Finally, $\hat{\lambda}$ and the parameters of components in (16) are estimated as follows. If $M_1, M_2, \ldots, M_L$ represent the number of measurements in each sampling time in $[1, L]$ and assuming each M_l follow a Poisson distribution, then $\hat{\lambda}$ can be given by an ML estimate expressed as

$$\hat{\lambda} = \frac{1}{L} \sum_{l=1}^{L} M_l. \quad (18)$$

Moving on, if $f(\mathbb{Z} \mid \Theta)$ given in (19) denotes the likelihood function of a parameter set, Θ, of components and by assuming the measurements in set $\mathbb{Z}$ are independent of each other, then

$$f(\mathbb{Z} \mid \Theta) = \prod_{j=1}^{m} \sum_{i=1}^{N} w_i f(\mathbf{z}_j \mid c_i) \quad (19)$$

the ML estimate for Θ can be derived by

$$\hat{\Theta} = \underset{\Theta}{argmax} \left\{ log f(\mathbb{Z} \mid \Theta) \right\}. \quad (20)$$

An EM algorithm is proposed in the same paper to solve (20) iteratively. It has been shown in [46] that the number of target measurements should be negligible when compared to clutter measurements to obtain an unbiased estimate for clutter intensity. Hence, the method has no inherent mechanism to discriminate among clutter and target measurements.

A method for updating clutter intensity online as well is proposed in [51] as an extension to [46]. It has a mechanism to distinguish clutter measurements from target measurements, which is explained below. At first, a PHD [26] filter is run to update the target states. The measurements nearest to the target states are regarded as target measurements. Hence, a set of clutter measurements for the current sampling time can be determined. The update for $\hat{\lambda}$ for the current sampling time is computed using its value in the previous sampling time and the number of clutter measurements at the current sampling time [51]. The component weights, means, and covariances are also updated based on the obtained set of clutter measurements. The set of clutter measurements is divided into subsets of clutter measurements belonging to each of the components using a χ^2 test based on the previous estimates of components [51]. Then, the new component parameters and their weights are updated using the said subset of clutter measurements. It is shown in [46] that the EM algorithm proposed therein is susceptible to giving an underestimate or overestimate of the number of components and their corresponding parameters. These issues are also addressed in [51] by adding new components, wherein the component parameters are misestimated. However, the addition of components may lead to an explosion in the number of components beyond control. Hence, some pruning and merging methods need to be adopted to reduce the nonrelevant components.

V. CLUTTER SPATIAL INTENSITY ESTIMATORS BASED ON CLUTTER GENERATORS

The methods in this section assume some unknown background targets represented using auxiliary variables called clutter generators that generate the clutter measurements. Following the random finite set (RFS) theory [26], the clutter spatial intensity estimation problem then becomes an intensity estimation problem of clutter generators. In [27] and [28], a set of clutter generators $C = \{c_1, c_2, \ldots, c_m\}$ defined in space $\mathbb{C}$, which is disjoint from the target state space $\mathbb{X}$ and measurement space $\mathbb{M}$, is assumed. Furthermore, assuming the clutter process as a Poisson mixture process, an approximate Bayesian estimator based on a PHD filter is derived for computing the intensity of clutter generators. However, the PHD filter proposed in [27] and[28] is intractable [29] as the PHD update equation requires a summation over all the partitions of measurement set at the current time, which is combinatorial in nature and no practical implementation of PHD equations is provided therein.

In [32], two assumptions about the clutter generators are made toward simplifying the above-mentioned problem. The first assumption is that each clutter generator c_i generates only one clutter measurement z_i^C and the second is that the predicted clutter generator distribution can be assumed as a Poisson point process. Then, a tractable PHD filter for the estimation of the intensity of clutter generators is proposed. Similar assumptions of one-to-one mapping between clutter generators and measurements can be seen in [29]–[31]. In [32], the clutter process is assumed to be NHPP whose intensity function is approximated by a Gaussian mixture with unknown mean and covariance. If $Z_k = \left\{\mathbf{z}_{k,1}, \mathbf{z}_{k,2}, \ldots, \mathbf{z}_{k,n}\right\}$ is the set of d-dimensional measurements and $c = \{c_1, c_2, \ldots, c_m\}$ the set of clutter generators at current time k, then the intensity function at any detection point $\mathbf{z}_{k,j} (j = 1, 2, \ldots, n)$ is given by [32]

$$c(\mathbf{z}_j) = \sum_{i=1}^{m} \frac{m_i C_1}{\sqrt{|\Sigma_i|}} \exp\left\{-\frac{1}{2}(\mathbf{z}_j - \mu_i)^T \Sigma_i^{-1}(\mathbf{z}_j - \mu_i)\right\}$$
$$= \sum_{i=1}^{m} m_i C_2 \exp\left\{-\frac{1}{2}(\mathbf{z}_j - \mu_i)^T \rho_i(\mathbf{z}_j - \mu_i)\right\}, \quad (21)$$

where $C_1 = \frac{1}{(2\pi)^{d/2}}$ and $C_2 = C_1\sqrt{|\rho_i|}$.

Here, the mean and covariance of the ith Gaussian component are $\mu_i \in R^d$ and $\Sigma_i \in R^{d\times d}$, respectively. Also, ρ_i denotes the precision matrix, i.e., the inverse of Σ_i, $\mathbf{z}_j$ denotes the detection point $\mathbf{z}_{k,j}$, and $m_i \in R^+$ represents the expected number of clutter measurements corresponding to the ith Gaussian component. Hereafter, the symbol $\mathbf{z}_j$ will be used in place of $\mathbf{z}_{k,j}$ for simplicity. Also, $c_i = \{m_i, \mu_i, \Sigma_i\}$ $(i = 1, 2, \cdot, m)$ is the ith clutter generator. All the parameters of c_i are random. Hence, the clutter generator c_i is a random point process. Again, under the assumption of a one-to-one relation between clutter generators and measurements, the clutter generator parameters reduce to $c_i = \{\mu_i, \Sigma_i\}$, as shown in [32]. Also, a Gaussian likelihood function defined below is assumed for $\mathbf{z}_j$ given c_i [32]:

$$f(\mathbf{z}_j \mid c_i) = \frac{1}{(2\pi)^{d/2}\sqrt{|\Sigma_i|}} \exp\left\{\frac{1}{2}(\mathbf{z}_j - \mu_i)^T \Sigma_i^{-1}(\mathbf{z}_j - \mu_i\right\}. \quad (22)$$

Here, μ_i and Σ_i denote the position and extension of the clutter generator c_i, respectively. Using (21) and (22), the clutter spatial intensity estimation problem becomes an estimation of Gaussian random variable with unknown mean and covariance. Furthermore, using the assumption that the state evolution of targets is statistically independent of the state evolution of clutter generator, the prediction of target PHD is decoupled from the prediction of clutter generator PHD, and the prediction equations of [26] are applied to targets and clutter generators separately [32]. The predicted PHD $D^C_{k|k-1}(c)$ for clutter generators is given as [32]

$$D^C_{k|k-1}(c) = b^C_k(c) + \int f^C_{k|k-1}(c) D^C_{k|k}(c) dc. \quad (23)$$

Here, $b^C_k(c)$ denotes the PHD of newborn clutter generators, and $f^C_{k|k-1}(c)$ denotes the state transition function for clutter generators defined in (33) and (34) of [32]. On the other hand, given the measurement set Z^k, the update equations for posterior PHD $D_{k|k}(\mathbf{x})$ of targets and posterior PHD $D^C_{k|k}(c)$ of clutter generators derived in [32] are coupled as

$$D_{k|k}(\mathbf{x}) = p_M D_{k|k-1}(\mathbf{x}) + \sum_{\mathbf{z}_j \in Z^k} \frac{P_D f(\mathbf{z}_j \mid \mathbf{x})}{L(\mathbf{z})} D_{k|k-1}(\mathbf{x})$$
$$D^C_{k|k}(c) = \sum_{\mathbf{z}_j \in Z^k} \frac{f(\mathbf{z}_j \mid c)}{L(\mathbf{z})} D^C_{k|k-1}(c). \quad (24)$$

Here, $p_M = 1 - P_D$ and $L(\mathbf{z})$ are the predicted pseudo-likelihoods for the measurement set Z^k defined as

$$L(\mathbf{z}) = P_D \int D_{k|k-1}(\mathbf{x}) f(\mathbf{z} \mid \mathbf{x}) \mathbf{dx} + \int D^C_{k|k-1}(c) f(\mathbf{z} \mid c) dc. \quad (25)$$

The pseudo-likelihood function $L(\mathbf{z})$ consists of two parts. The first part $P_D \int D_{k|k-1}(\mathbf{x}) f(\mathbf{z} \mid \mathbf{x})\mathbf{dx}$ represents the target-originated measurements and the second part $\int D^C_{k|k-1}(c) f(\mathbf{z} \mid c) dc$ represents the clutter measurements. It is evident from (25) that [32] does not need a separate filter to identify target-originated measurements. Using the idea of translated point process [43], $L(\mathbf{z})$ can be viewed as a sum of two independent translated Poisson processes: the first one denoted by $P_D \int D_{k|k-1}(\mathbf{x}) f(\mathbf{z} \mid \mathbf{x})\mathbf{dx}$ in (25) is a translated Poisson process (using basic assumption of [26]) of predicted targets with point transition density $P_D f(\mathbf{z} \mid \mathbf{x})$ and the second one represented by $\int D^C_{k|k-1}(c) f(\mathbf{z} \mid c) dc$ in (25) with point transition density $f(\mathbf{z} \mid c)$. Thus, the predicted clutter spatial intensity at a detection point $\mathbf{z}_j$ can be given by [29]

$$\hat{\lambda}(\mathbf{z}_j) = \int f(\mathbf{z} \mid c) D^C_{k|k-1}(c) dc. \quad (26)$$

Finally, using the fact that a normal-Wishart distribution is the conjugate prior of Gaussian distribution with unknown mean and covariance [34], a practical implementation for the recursion of $D^C_{k|k-1}(c)$ and $D^C_{k|k}(c)$ is shown in [32] using a normal-Wishart mixture. Similar derivations of PHD recursions for the intensity of clutter generator can be seen in [29]. It has been shown in [32] that the number of normal-Wishart mixture components grows out of bound; hence, pruning and merging techniques similar to a Gaussian mixture probability hypothesis density (GMPHD) filter [37] has to be adopted for practical implementation. In the same paper, a zero-velocity model for the dynamic evolution of clutter generators is adopted. However, in sea-based radars, the radial velocity of clutter is usually high due to the presence of moving waves on the sea surface. The radial velocity of these waves can be significant depending on the sea state that is influenced by various weather and wind conditions [45]. Hence, an accurate modeling for the clutter random process used in track-before-detect (TBD) filters [44] could be adopted to improve the accuracy of clutter spatial intensity estimate.

In [33], an extension to [32] is presented in order to use the output of standard MTTs like the joint integrated probabilistic data association (JIPDA) [10] in deriving the clutter generator PHD. The update equation (24) for clutter generator PHD $D^C_{k|k}(c)$ in [32] requires a predicted PHD $D_{k|k-1}(\mathbf{x})$ of targets. However, the standard MTTs like JIPDA readily provide the state estimates of targets as well as the association events that identify target-originated measurements. Hence, the output of JIPDA is used in [33] to derive a recursion for PHD of clutter generators. In that paper, the posterior target state outputs from standard trackers are approximated by multiBernoulli RFS (MBe RFS) [35], [36]. In addition, the possible interaction between the clutter generator and the target during prediction is neglected. Then, by using the association events and their corresponding probabilities generated by JIPDA, an approximate Bayesian clutter intensity estimator is obtained. The posterior target state pdf at time $k-1$ obtained from legacy trackers is an MBe RFS of the form [33]

$$\Pi_{k-1} = \{r_{k-1}^{i}, p_{k-1}(\mathbf{x}_{k-1})\}_{i=1}^{M_{k-1}}. \tag{27}$$

Here, r_{k-1}^{i} denotes the posterior target existence probability, $p_{k-1}(\mathbf{x}_{k-1})$ denotes the posterior pdf of the ith target state, and M_{k-1} denotes the cardinality of targets. Then, the predicted target state pdf at time k is also an MBe RFS, which can be used to generate association events. If each association event χ_i with probability $P(\chi_i)$ generates a set $Z^k(W_{\chi_i})$ of clutter measurements, then, using the similar assumptions of [32], the posterior PHD $D_{k|k}^{C}(c)$ of clutter generator can be represented by [33]

$$D_{k|k}^{C}(c) = \sum_{\chi_i} P(\chi_i) \left\{ \sum_{\mathbf{z}_j \in Z^k(W_{\chi_i})} \frac{f(\mathbf{z}_j \mid c) D_{k|k-1}^{C}(c)}{\int f(\mathbf{z} \mid c) D_{k|k-1}^{C}(c) dc} \right\}. \tag{28}$$

Here, $f(\mathbf{z}_j \mid c)$ is the likelihood function in (22). The PHD prediction for clutter generators as well as the predicted clutter spatial intensity remains the same as (23) and (26), respectively.

The methods in [32] and [33] are different from the kernel estimation-based method [24] in two aspects. The KDE-based method is a nonparametric estimation method, and as the number of measurements increases, the bandwidth of kernels decreases. In addition, no two kernels are merged at any point of time. On the other hand, the approximate Bayesian methods of [32] and [33] are parametric methods wherein the clutter area is a summation of ellipses (normal-Wishart mixture geometry). Also, the clutter generator components are merged for practical implementation. If the shape of the clutter area is close to the summation of ellipses, then [32] and [33] can give better results than [24] with fewer observations [33]. However, if the shape of the clutter area is significantly different from the summation of ellipses, then the methods in [32] and [33] can be biased.

In [39], a GMPHD-based [37] clutter spatial intensity estimator called interactive clutter measurement density estimator (ICMDE) is proposed, assuming a Poisson point process for clutter generators. Here, the assumption of one-to-one mapping of clutter generator to measurements is retained, but the likelihood function for clutter generator is assumed to be Gaussian with known covariance. Also, a nearly constant velocity model is assumed for the dynamic state evolution of clutter generators. Then, using the above assumptions, the spatial intensity function becomes a Gaussian mixture with known covariance for each component. The intensity estimation problem then reduces to the estimation of Gaussian random variable with a known covariance and unknown mean. Again, using the fact that the conjugate prior of Gaussian distribution with known covariance is also Gaussian [34], GMPHD-based recursions for the PHD of clutter generator are presented therein. The covariance required for GMPHD recursion is calculated based on the sparsity [22] of each measurement, as shown in [39]. Furthermore, ICMDE utilizes the calculated clutter probability of measurements (11) using the tracker output to distinguish target-originated measurements from clutter measurements while evaluating the sparsity of each of the measurements. Hence, ICMDE can be easily integrated to existing MTTs [10], [12]. In [40], a multiscan version of ICMDE is explained. The assumption of known covariance for the likelihood function could make it an attractive choice for practical implementation when compared to methods in [32] and [33]. However, the problem of finding the nearest neighbor detections in nonhomogenous measurement space for the computation of sparsity exists in [39]. The solutions mentioned in [42] and [38] could be adopted to circumvent it.

The nonhomogenous clutter intensity is estimated in [36] and [48] based on clutter generators but by dropping the standard Poisson assumption for clutter process. Therein, the clutter generators are modeled analogous to actual targets with separate models for births, death processes, and transition density. Furthermore, assuming that the targets and clutter generators are independent, MBe RFS recursions of [35] are applied to targets and clutter generators separately. Since clutter generators are modeled with separate models for evolution over time, both the stationary and the nonstationary clutters can be addressed by the said method.

VI. DISCUSSION ON INTERRELATION BETWEEN METHODS AND CONCLUSION

The clutter spatial intensity estimators discussed in the above sections have been categorized into three groups. The categories are decided based on the techniques used and on the assumptions made while computing the spatial intensity of clutter. The first type of methods only compute the spatial intensity of measurements in the validation gate of tracks, assuming uniform spatial intensity inside it. The second group of methods compute the spatial intensity of clutter at any detection point in measurement space. In those methods, some variation of nonparametric density estimation techniques is used to evaluate the spatial intensity of false alarms. The third group of methods assume auxiliary variables called clutter generators, disjoint from target space and measurement space. Then, using RFS theory, PHD recursion equations are used to derive the intensity for clutter generators, thereby evaluating the clutter spatial intensity

Table I
Clutter Spatial Intensity Estimators Under Different Categories

Category	Technique used	Assumptions	References
Methods based on track validation gate	-	Uniform clutter spatial intensity in validation gate	[1], [12], [14]
Methods for any detection point in measurement space	Nonparametric density estimation	1) Constant clutter spatial intensity in each cell. 2) Homogeneous Poisson process in the near proximity of measurement. 3) Nonhomogeneous clutter spatial intensity. 4) FMM approximation.	1) [11], [18], [20] 2) [18], [21]–[23], [38] 3) [24] 4) [46], [49]–[51]
Methods based on clutter generator	PHD filter/MBe RFS	1) Multiple measurements per clutter generator. 2) One measurement per clutter generator.	1) [27], [28]. 2) [29]–[33], [36], [39], [40], [48]

Table II
Pros and Cons of Clutter Spatial Intensity Estimation Methods

Method	Pros	Cons
Nonparametric clutter intensity estimation	Uniform clutter intensity for all measurements in track(s) validation gate.	1) Different clutter intensities for the same measurement in different track validation gates. 2) Unable to distinguish between target and clutter measurements.
Nonparametric clutter intensity estimation with track perceivability	1) Uniform clutter intensity for all measurements in track(s) validation gate. 2) Target measurements are distinguished from clutter measurements using track perceivability.	Different clutter intensities for the same measurement in different track validation gates.
Cluttermap method, spatial clutter map method	Nonhomogenous clutter intensity estimate obtained by dividing measurement space into cells.	1) Naive method with uniform intensity for all measurements in a cell. 2) Intensity estimate obtained has block nature and biased. 3) Unable to distinguish between target and clutter measurements.
SCMDE	1) Clutter intensity estimate for any measurement in measurement space is obtained without dividing the measurement space into cells. 2) Nonhomogenous clutter intensity estimate with local homogeneity.	1) Clutter intensity estimate obtained has block nature. 2) Unable to explicitly distinguish between target and clutter measurements. 3) Not suitable for MTT. 4) Not directly suitable for sensors with nonhomogenous measurement space.
MTTSCMDE	1) Nonhomogenous clutter intensity estimate, which in unbiased. 2) Clutter probability of measurements aids in identifying target measurements. 3) Suitable for MTT.	1) Not directly suitable for sensors with nonhomogenous measurement space. 2) Intensity estimate obtained may have block nature.
Kernel density clutter intensity estimation	1) Efficient method with unbiased and smooth clutter intensity estimate for any measurement. 2) Suitable for MTT and all type of sensors.	Efficiency comes at the cost of computational complexity of KDE.
Online FMM method	1) Recursively updates clutter intensity. 2) Discriminates between target and clutter measurements. 3) Addresses the shortcoming of estimates given by EM algorithm.	1) Nearest neighbor method to identify target measurements is not guaranteed to give correct measurements corresponding to a target. 2) Pruning and merging methods required to stop explosion of number of components. 3) Clutter intensity is assumed time invariant; hence, no transition densities are defined for components.
Integrated clutter intensity estimation	1) Recursively updating clutter intensity estimate. 2) Ease of integration with legacy trackers like JIPDA.	Use of static transition density for clutter generators.
ICMDE	1) Recursively updating clutter intensity estimate. 2) Ease of integration with legacy trackers like JIPDA and LMIPDA.	Not directly suitable for sensors with nonhomogenous measurement space.

of measurements. Table I shows the classification of the spatial intensity estimators under different categories.

The methods in [1], [14], and [12] are similar in the way they compute the clutter spatial intensity of track(s) validated measurements. However, the former has no means to discriminate between clutter and target measurements, whereas the latter two use (4) to distinguish target measurements from clutter measurements. The clutter map method and spatial clutter map estimator are based on a multivariate histogram, whereas SCMDE and MTTSCMDE are based on a k-nearest density estimation technique. The former three methods have no provision to identify target measurements, while the latter uses the clutter probability of each measurement for this purpose, hence a better choice above others in multitarget tracking. The methods similar to k-nearest neighbor density estimation use the mathematical distance to the nearest measurement in evaluating the clutter spatial intensity. They are forced to make a homogeneous assumption for clutter spatial intensity in and around the desired measurement as nonhomogeneous assumptions do not hold for the inverse of the defined mathematical distance. Moreover, they are not easily adaptable to nonlinear sensors like Doppler radar. The method based on KDE is a more general approach applicable to any type of sensors. However, the effectiveness of the approach comes at the cost of computational load of KDE.

One-to-one mapping between clutter generators and measurements can be seen throughout the methods of category three, but the clutter intensity filter recursion implementations vary among each other. The methods in [32] and [33] use a normal-Wishart mixture implementation to account for unknown mean and covariance of Gaussian mixture, whereas a Gaussian mixture implementation is used in [40] to accommodate for the known covariance. However, the former two methods cannot be directly integrated to linear multitarget (LM) tracker [12] variants, whereas the latter can be seamlessly integrated to both LM variants and JIPDA trackers. The method in [32] requires the predicted PHD of targets as shown in (24), whereas the method in [33] uses the output of JIPDA tracker to represent target PHD and then the clutter PHD is updated using the association events. Finally, the pros and cons of each of the methods are tabulated in Table II.

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