Space Based Sensor Bias Estimation in the Presence of Data Association Uncertainty

DJEDJIGA BELFADEL
RICHARD W. OSBORNE, III
YAAKOV BAR-SHALOM
KRISHNA PATTIPATI

In this paper, an approach to bias estimation in the presence of measurement association uncertainty using common targets of opportunity, is developed. Data association is carried out before the estimation of sensor angle measurement biases. Consequently, the quality of data association is critical to the overall tracking performance. Data association becomes especially challenging if the sensors are passive. Mathematically, the problem can be formulated as a multidimensional optimization problem, where the objective is to maximize the generalized likelihood that the associated measurements correspond to common targets, based on target locations and sensor bias estimates. Applying gating techniques significantly reduces the size of this problem. The association likelihoods are evaluated using an exhaustive search after which an acceptance test is applied to each solution in order to obtain the correct solution. We demonstrate the merits of this approach by applying it to a simulated tracking system, which consists of two or three satellites tracking a ballistic target. We assume the sensors are synchronized, their locations are known, and we estimate their orientation biases together with the unknown target locations.

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Authors' address: Electrical and Computer Engineering, University of Connecticut, Storrs, CT, U.S.A. (E-mail: {dbelfadel, rosborne, ybs, krishna}@engr.uconn.edu).

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I. INTRODUCTION

Data association is a crucial task in many surveillance systems, and becomes especially challenging if the sensors are passive and measure Line of Sight (LOS) angles only for the targets. Measurements from multiple sensors have to be associated to determine the biases of the sensors and the positions of the targets from which the measurements originated. In general, the goal of data association is to partition the set of measurements across sensors into a number of subsets, in which the measurements are either from the same target (i.e., having the identical origin) or false alarms. For angle-only sensors, imperfect registration leads to LOS angle measurement errors in azimuth and elevation that can be much larger than those due to measurement noise. If uncorrected, registration errors can lead to large tracking errors and potentially to the formation of multiple tracks (ghosts) on the same target [8].

Mathematically, the problem can be formulated as a multidimensional optimization problem where the objective is to maximize the generalized likelihood, based on target locations and sensor bias estimates, that the associations correspond to real targets. Any feasible solution of this problem corresponds to a potential association hypothesis. In [14], the problem was formulated as a multidimensional assignment (S-D) problem where the objective was to maximize the likelihood that the associations correspond to targets. For $S \geq 3$, the multidimensional assignment problem is NP-hard. Many suboptimal algorithms have been proposed to find an approximate solution, such as Lagrangian relaxation [11], greedy rounding adaptive search (GRASP) [15], genetic algorithms [3] and linear relaxation and rounding techniques [16]. Moreover, in many cases, it is possible to resort to gating techniques [10] which drastically reduce the number of decisions variables and make it possible to solve the problem optimally.

Even if a large part of the literature is devoted to this aspect, solving efficiently the multidimensional assignment problem is not the only challenge for data association problems. Indeed, the quality of near-optimal, or even optimal, solution may vary considerably depending on the context. In sparse configurations or with highly accurate sensors, the model behaves well and the optimal, or even an approximate solution, often has an acceptable percentage of correct associations. On the other hand, in medium or high density configurations or with sensors of low accuracy, the model behaves poorly, namely, there is ambiguity due to similarity of likelihoods. The optimal solution can have a poor association correctness while the correct solution can be suboptimal.

The optimal solution of the problem is supposed to be the most likely solution. As the complexity of the observed situations increases, the number of ambiguous elementary associations increases also. Since such associations get a high likelihood within the model, it usually happens that more than one solution can get an overall likelihood very close to the likelihood of the optimal solution. In such cases, any of these solutions, including the optimal one, could appear to be the correct association hypothesis. Therefore, it seems more reasonable to consider several candidate solutions rather than by selecting only one solution, even if it has a slightly better likelihood. The general scheme underlying our approach is based on the idea of selecting several good candidate solutions, by evaluating the likelihoods, and using a goodness of fit test to obtain the correct association hypothesis.

Space-based sensors can expand the range and effectiveness of the capabilities of a Ballistic Missile Defense System (BMDS) to counter future projected threats. Integration of space based sensors into the BMDS allows for detection and tracking of threats over a larger area than ground based sensors [1]. The Space Tracking and Surveillance System (STSS) constellation consists of two or more satellites (on known trajectories) for tracking ballistic targets. Each satellite is equipped with an IR sensor that provides the azimuth and elevation to the target. The tracking problem is made more difficult due to a constant or slowly varying bias error present in each sensor's line of sight measurements.

Maximum a posteriori (MAP) data association for concurrent bias estimation and data association based on sensor-level track state estimates was proposed in [12] and extended in [13]. Sensor calibration using in-situ celestial observations to estimate bias in space-based missile tracking was proposed in [9].

In [7] we investigated the use of the minimum possible number of moving optical sensors (three or two optical sensors to observe three or six points, respectively, on the trajectory of a single target of opportunity), under the assumption of perfect data association. In the present paper, bias estimation is investigated, in the presence of false alarms, when only targets of opportunity are available. The present problem is not amenable to the multidimensional assignment (S-D, [9]) because the number of measurements needed to obtain a solution for the sensor biases presents the sequential use of 2-D assignment and relaxation as in the S-D algorithm, i.e., in problems where S-D assignment can be used one has a first solution using the first 2 lists and then, using relaxation, the remaining lists are incorporated one at a time. In the present problem the minimum number of measurements needed for a solution is as given in equation (22) and these measurements have to be correctly associated: otherwise the residual yields "unacceptable" result. Consequently one has to find such a "correct set." After this, if one uses additional measurements from the same sensors, they have to form a set of common origin (an "extra" target point), which introduces another 3 unknowns. Thus one has to find one measurement from each of (at least) two sensors (4 scalars that add 4 equations) and a search is needed until a first such set is formed (based on the residual). Then one can proceed iteratively in this fashion by adding a measurement from another sensor or a set of 2 measurements from the same 2 sensors. For the problem considered we found that it is faster to obtain the results using directly an exhaustive search for the target points. By generating (enumerating) the set of all possible associations, which is guaranteed to contain the desired (correct association) solution, based on the association likelihoods using the target location estimates and the sensor bias estimates, an acceptance test can be applied to each solution in order to obtain the correct solution. It appears, that through the use of gating technique, the solution is obtained in a reasonable time.

We demonstrate the merits of this approach by applying it to a simulated tracking system, which consists of two or three satellites tracking a ballistic target. We assume the sensors are synchronized, their locations are known, and we estimate their orientation biases. We investigate the use of the minimum possible number of space-based sensors (which can not be less than two). Two cases are considered. In the first case, we use three optical sensors to estimate three points on the (unknown) trajectory of a single target of opportunity simultaneously with the biases of the three optical sensors [5]. In the second case, we estimate the position of six points on the trajectory of a single target of opportunity simultaneously with the biases of two space-based optical sensors [4].

Section II presents the problem formulation and solution in detail. Section III describes the simulations performed and gives the results. Finally, Section IV gives the conclusions.

II. PROBLEM FORMULATION

Assume there are N_S synchronized moving passive sensors, with known positions in the Earth Centred Inertial (ECI) Coordinate System at times t_i,

$$\boldsymbol{\xi}_s(t_i) = [\xi_s(t_i), \eta_s(t_i), \zeta_s(t_i)]', \quad s = 1, 2, \ldots, N_S \tag{1}$$

and N_t target locations (target trajectory at N_t time instants of a single target) at

$$\mathbf{x}(t_i) = [x(t_i), y(t_i), z(t_i)]' \quad i = 1, 2, \ldots, N_t \tag{2}$$

also in ECI coordinates. We assume that each sensor sees all the target locations (same physical target at different times).[1]

The rotation between the ECI and a sensor frame is described by $\phi_s + \phi_s^{\mathrm{n}}$, $\rho_s + \rho_s^{\mathrm{n}}$, $\psi_s + \psi_s^{\mathrm{n}}$ of sensor s as roll, pitch, and yaw respectively, where ϕ_s^{n} is the nominal roll angle, ϕ_s is the roll bias, etc. Each angle defines a rotation about a prescribed axis, in order to align the sensor frame axes with the ECI axes. The xyz rotation sequence is chosen, which is accomplished by first rotating about the x axis by ϕ_s^{n}, then rotating about the y axis by ρ_s^{n}, and finally rotating about the z axis by ψ_s^{n}. The operations needed to transform the position of a given target location at t_i expressed in ECI

[1]This can also be different targets at a common time or at different times, as long as the sensors are synchronized.

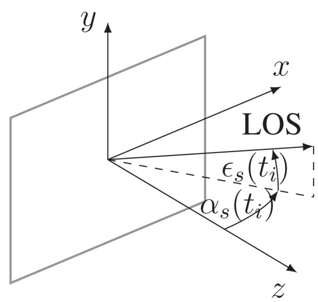

Fig. 1. Optical sensor coordinate system with the origin in the center of the focal plane.

coordinates into the sensor s coordinate system (based on its nominal orientation) is

$$\mathbf{x}_s^{\mathrm{n}}(t_i) = T(\boldsymbol{\omega}_s(t_i))(\mathbf{x}(t_i) - \boldsymbol{\xi}_s(t_i))$$
$$i = 1,2,\ldots,N_t, \quad s = 1,2,\ldots,N_S \tag{3}$$

where $\boldsymbol{\omega}_s(t_i) = [\phi_s^{\mathrm{n}}(t_i), \rho_s^{\mathrm{n}}(t_i), \psi_s^{\mathrm{n}}(t_i)]'$ is the nominal orientation of sensor s at times t_i, $T(\boldsymbol{\omega}_s(t_i))$ is the appropriate rotation matrix, and the translation $(\mathbf{x}(t_i) - \boldsymbol{\xi}_s(t_i))$ is the difference between the vector position of the target at time t_i and the vector position of the sensor s at time t_i, both expressed in ECI coordinates. The superscript "n" in (3) indicates that the rotation matrix is based on the nominal sensor orientation.

As shown in Figure 1, the azimuth angle $\alpha_s(t_i)$ is the angle in the sensor's xz plane between the sensor's z axis and the projection of the line of sight to the target onto the xz plane, while the elevation angle $\epsilon_s(t_i)$ is the angle between the line of sight to the target and its projection onto the xz plane, i.e.,

$$\begin{bmatrix} \alpha_s(t_i) \\ \epsilon_s(t_i) \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\dfrac{x_s(t_i)}{z_s(t_i)}\right) \\ \tan^{-1}\left(\dfrac{y_s(t_i)}{\sqrt{x_s^2(t_i) + z_s^2(t_i)}}\right) \end{bmatrix} \tag{4}$$

The model for the biased noise-free LOS measurements is then

$$\begin{bmatrix} \boldsymbol{\alpha}_s^b(t_i) \\ \boldsymbol{\epsilon}_s^b(t_i) \end{bmatrix} = \begin{bmatrix} g_1(\mathbf{x}(t_i), \boldsymbol{\xi}_s(t_i), \boldsymbol{\omega}_s(t_i), \mathbf{b}_s) \\ g_2(\mathbf{x}(t_i), \boldsymbol{\xi}_s(t_i), \boldsymbol{\omega}_s(t_i), \mathbf{b}_s) \end{bmatrix}$$
$$\stackrel{\Delta}{=} \mathbf{g}[\mathbf{x}(t_i), \boldsymbol{\xi}_s(t_i), \boldsymbol{\omega}_s(t_i), \mathbf{b}_s] \tag{5}$$

where g_1 and g_2 denote the sensor Cartesian coordinates-to-azimuth/elevation angle mapping that can be found by inserting (3) and (4) into (5), and the bias vector of sensor s is

$$\mathbf{b}_s = [\phi_s, \rho_s, \psi_s]' \tag{6}$$

For a given target, each sensor provides the noisy LOS measurements

$$\mathbf{z}_s(t_i) = \mathbf{g}[\mathbf{x}(t_i), \boldsymbol{\xi}_s(t_i), \boldsymbol{\omega}_s(t_i), \mathbf{b}_s] + \mathbf{w}_s(t_i) \tag{7}$$

where

$$\mathbf{w}_s(t_i) = [w_s^{\alpha}(t_i), w_s^{\epsilon}(t_i)]' \tag{8}$$

The measurement noises $\mathbf{w}_s(t_i)$ are zero-mean, white Gaussian with

$$R_s = \begin{bmatrix} (\sigma_s^{\alpha})^2 & 0 \\ 0 & (\sigma_s^{\epsilon})^2 \end{bmatrix} \tag{9}$$

and are assumed mutually independent. We shall assume, for simlicity, $\sigma_s^{\alpha} = \sigma_s^{\epsilon} = \sigma$.

The problem is to estimate the bias vectors for all sensors and the locations of the targets of opportunity. We shall obtain the maximum likelihood (ML) estimate of the augmented parameter vector

$$\boldsymbol{\theta} = [\mathbf{x}(t_1)', \ldots, \mathbf{x}(t_{N_t})', \mathbf{b}_1', \ldots, \mathbf{b}_{N_S}']' \tag{10}$$

consisting of the (unknown) target locations and sensor biases, by maximizing the likelihood function (LF) of $\boldsymbol{\theta}$.

III. SOLUTION

It will be assumed that there is a single target at different (unknown) locations (2), observed at times t_i, $i = 1,\ldots,N_t$. The set of measurements from sensor s at time t_i is

$$Z_s(t_i) = \{\mathbf{z}_s(l,t_i)\}_{l=1}^{n_{s,i}} \quad i = 1,2,\ldots,N_t, \quad s = 1,2,\ldots,N_S \tag{11}$$

and it contains the noisy measurement from the target and clutter points or false alarms (assumed to be spatially and temporally white); the total number of measurements at sensor s at time t_i is denoted as $n_{s,i}$. The problem consists of selecting the measurement $l_{s,i}$ deemed from the target, i.e., one from each of the $N_s N_t$ lists. Due to the high accuracy of the IR spaced based sensors, we assume that each target is detected by the sensors at any given time t_i, i.e., the probability of detection $P_D = 1$. The likelihood function (LF) of θ for a particular set of selected measurements (one from each sensor s and time t_i) assumed target-originated

$$\mathcal{L} = \{l_{s,i}\} \tag{12}$$

based on the entire set of measurements

$$\mathbf{Z} = \{Z_s(t_i) \quad i = 1,2,\ldots,N_t, \quad s = 1,2,\ldots,N_S\} \tag{13}$$

is

$$\Lambda(\boldsymbol{\theta}; \mathcal{L}, Z_{\mathcal{L}}) = \prod_{i=1}^{N_t} \prod_{s=1}^{N_S} p(\mathbf{z}_s(l_{s,i}, t_i) \mid \boldsymbol{\theta}) \tag{14}$$

where $Z_{\mathcal{L}}$ is the set of selected measurements, and

$$p[\mathbf{z}_s(l_{s,i}, t_i) \mid \boldsymbol{\theta}] = \prod_{i=1}^{N_t} \prod_{s=1}^{N_S} \mathcal{N}(\mathbf{z}_s(l_{s,i}, t_i); \mathbf{h}_{si}(\boldsymbol{\theta}), R_s) \tag{15}$$

and we use the compact notation

$$\mathbf{h}_{si}(\boldsymbol{\theta}) \stackrel{\Delta}{=} \mathbf{g}(\mathbf{x}(t_i), \boldsymbol{\xi}_s(t_i), \boldsymbol{\omega}_s(t_i), \mathbf{b}_s) \tag{16}$$

Note that each $\mathcal{L}$ consists of an $N_S N_t$-tuple. The ML estimate of $\boldsymbol{\theta}$ for a certain $\mathcal{L}$ is

$$\hat{\boldsymbol{\theta}}^{\mathrm{ML}}(\mathcal{L}) = \arg\max_{\boldsymbol{\theta}} \Lambda(\boldsymbol{\theta}; \mathcal{L}, Z_{\mathcal{L}}) \tag{17}$$

and

$$\hat{\boldsymbol{\theta}}^{\mathrm{ML}} = \hat{\boldsymbol{\theta}}^{\mathrm{ML}}(\mathcal{L}^{\mathrm{ML}}) \tag{18}$$

where

$$\mathcal{L}^{\mathrm{ML}} = \arg\max_{\mathcal{L}} \Lambda(\hat{\theta}^{\mathrm{ML}}(\mathcal{L}); \mathcal{L}, Z_{\mathcal{L}}) \tag{19}$$

i.e., the final estimate (18) of (10) is based on the most likely assignment (19). The final (generalized) likelihood to be used for acceptance testing is

$$\begin{aligned}\hat{\Lambda}(\mathcal{L}) &= \Lambda(\hat{\boldsymbol{\theta}}^{\mathrm{ML}}(\mathcal{L}^{\mathrm{ML}}); \mathcal{L}^{\mathrm{ML}}, Z_{\mathcal{L}^{\mathrm{ML}}}) \\ &= \prod_{i=1}^{N_t} \prod_{s=1}^{N_S} \mathcal{N}(\mathbf{z}_s(l_{s,i}, t_i); \mathbf{h}_s[\hat{\theta}^{\mathrm{ML}}(\mathcal{L}^{\mathrm{ML}}), t_i], R_s)\end{aligned} \tag{20}$$

Solving (17) amounts to a nonlinear LS (NLS) problem. While there are many methods to obtain $\hat{\boldsymbol{\theta}}$, the iterated least squares (ILS) technique is preferred since it is easy to implement (no Hessian involved) and provides an (approximate) covariance matrix for its estimate at the same time. In order to find the MLE, one has to solve a nonlinear least squares problem for the exponent in (15). This will be done using a numerical search via the ILS technique [2].

A. Gating Region (Validation Region)

Validation gates are set up for selecting the candidate measurements originated from the target with high probability for each t_i. Measurements outside the validation regions can be ignored reasonably because the probabilities of them being from the corresponding target are quite low according to the true measurement statistical characterization. After enumerating the set of all possible associations, i.e., generating all full tuples (of length N_S) with one measurement from each of the N_s lists, *the maximum cross range error* is used in gating to prune unlikely associations. If a candidate association fails in the gating test, there is no need to use it in the likelihood cost. The calculation of the gate is recursive. Beginning with the measurement $\mathbf{z}_1(l_{1,i}, t_i)$ from the first sensor (list), we take one measurement from each list at time t_i. If the measurement from the second list $\mathbf{z}_2(l_{2,i} t_i)$ falls inside the gate bounded by the cone with angle ($4\sigma +$ max bias), around the $\mathbf{z}_1(l_{1,i} t_i)$, this measurement is incorporated in the tuple for time t_i, which advances to the next list. Only full tuples (consisting of N_S LOS measurements), are to be considered. If no measurement of a particular sensor appears in any validated tuple at t_i, then none of these tuples carry information about the biases of this sensor. Consequently, none of these tuples (from t_i) will be used in the estimation of the N_S sensor biases. This is repeated for each t_i and then (16) can be carried out. Consequently, the CPU time spent in the cost computation can be reduced via the gating process.

B. Number of Hypotheses

The total number of hypotheses (combinations) for a scenario of N_t target locations and N_S sensors (assuming no missed detections) is

$$N_H = \prod_{i=1}^{N_t} \prod_{s=1}^{N_S} n_{s,i} \tag{21}$$

For example, in the case of the 2 sensors and 6 target locations, with medium clutter density, in a particular run, assume $n_{s,i}$ (number of clutter points plus the measurement from the target) as: $2,1,2,1,3,3$ for $s = 1$ and $1,5,2,2,1,2$ for $s = 2$; then the total number of hypotheses is 1440. The size of the search problem can be reduced considerably by applying gating in order to prevent implausible associations. In the previous example, only, 14% (201) passed the gating: then, this problem can be solved exactly by using an exhaustive search of modest size.

C. Requirements for bias estimability

First requirement for bias estimability. For a given target location we have a two-dimensional measurement from each sensor (the two LOS angles to the target). We assume that each sensor sees all the target locations at common times. Stacking together each measurement of N_t target locations seen by N_S sensors results in an overall measurement vector of dimension $2N_tN_S$. Given that the position and bias vectors of each target are three-dimensional, and knowing that the number of equations (size of the stacked measurement vector) has to be at least equal to the number of parameters to be estimated (target locations and biases), we must have

$$2N_tN_S \geq 3(N_t + N_S) \tag{22}$$

This is a necessary condition but not sufficient because (18) has to have a unique solution, i.e., the parameter vector has to be estimable. This is guaranteed by the second requirement.

Second requirement of bias estimability. This is the invertibility of the Fisher Information Matrix (FIM). In order to have parameter observability, the FIM must be invertible. If the FIM is not invertible (i.e., it is singular), then the CRLB (the inverse of the FIM) will not exist—the FIM will have one or more infinite eigenvalues, which means total uncertainty in a subspace of the parameter space, i.e., ambiguity [2].

For the examples of bias estimability discussed in the sequel, to estimate the biases of 3 sensors (9 bias components) we need 3 target locations (9 position components), i.e., the search is in an 18-dimensional space, while for 2 sensors (6 bias components) we need at least 6 target locations (18 position components) in order to meet the necessary requirement (22). As stated previously, the FIM must be invertible, so the rank of the FIM has to be equal to the number of parameters to be estimated ($9 + 9 = 18$, or $6 + 18 = 24$, in the previous examples). The full rank of the FIM is a necessary and sufficient condition for estimability.

D. Iterated Least Squares for maximization of the LF of $\boldsymbol{\theta}$

Given the estimate $\hat{\boldsymbol{\theta}}^j$ after j iterations, the ILS estimate after the $(j+1)$th iteration will be

$$\hat{\boldsymbol{\theta}}^{j+1} = \hat{\boldsymbol{\theta}}^j + [(H^j)' R^{-1} H^j]^{-1} (H^j)' R^{-1} [\mathbf{z} - \mathbf{h}(\hat{\boldsymbol{\theta}}^j)] \tag{23}$$

where

$$\mathbf{z} = [z_1(t_1)', \ldots, z_s(t_1)', \ldots, z_s(t_i)', \ldots, z_{N_S}(t_{N_t})']' \quad (24)$$

$$\mathbf{h}(\hat{\boldsymbol{\theta}}^j) = [h_{11}(\hat{\boldsymbol{\theta}}^j)', \ldots, h_{is}(\hat{\boldsymbol{\theta}}^j)', \ldots, h_{N_t N_S}(\hat{\boldsymbol{\theta}}^j)'] \quad (25)$$

$$R = \begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & R_{N_S} \end{bmatrix} \quad (26)$$

where R_s is the measurement noise covariance matrix of sensor s, and

$$H^j = \left. \frac{\partial \mathbf{h}(\boldsymbol{\theta}^j)}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^j} \quad (27)$$

is the Jacobian matrix of the vector consisting of the stacked measurement functions (25) w.r.t. (10) evaluated at the ILS estimate from the previous iteration j. In this case, the Jacobian matrix is, with the iteration index omitted for conciseness,

$$H = [H_{11} \quad H_{21} \cdots H_{N_t 1} \quad H_{12} \cdots H_{N_t N_S}]' \quad (28)$$

where

$$H_{is} = \begin{bmatrix} \frac{\partial g_{1_s}(t_i)}{\partial x(t_1)} & \frac{\partial g_{2_s}(t_i)}{\partial x(t_1)} \\ \frac{\partial g_{1_s}(t_i)}{\partial y(t_1)} & \frac{\partial g_{2_s}(t_i)}{\partial y(t_1)} \\ \frac{\partial g_{1_s}(t_i)}{\partial z(t_1)} & \frac{\partial g_{2_s}(t_i)}{\partial z(t_1)} \\ \vdots & \vdots \\ \frac{\partial g_{1_s}(t_i)}{\partial x(t_{N_t})} & \frac{\partial g_{2_s}(t_i)}{\partial x(t_{N_t})} \\ \frac{\partial g_{1_s}(t_i)}{\partial y(t_{N_t})} & \frac{\partial g_{2_s}(t_i)}{\partial y(t_{N_t})} \\ \frac{\partial g_{1_s}(t_i)}{\partial z(t_{N_t})} & \frac{\partial g_{2_s}(t_i)}{\partial z(t_{N_t})} \\ \frac{\partial g_{1_s}(t_i)}{\partial \psi_1} & \frac{\partial g_{2_s}(t_i)}{\partial \psi_1} \\ \frac{\partial g_{1_s}(t_i)}{\partial \rho_1} & \frac{\partial g_{2_s}(t_i)}{\partial \rho_1} \\ \frac{\partial g_{1_s}(t_i)}{\partial \phi_1} & \frac{\partial g_{2_s}(t_i)}{\partial \phi_1} \\ \vdots & \vdots \\ \frac{\partial g_{1_s}(t_i)}{\partial \psi_{N_S}} & \frac{\partial g_{2_s}(t_i)}{\partial \psi_{N_S}} \\ \frac{\partial g_{1_s}(t_i)}{\partial \rho_{N_S}} & \frac{\partial g_{2_s}(t_i)}{\partial \rho_{N_S}} \\ \frac{\partial g_{1_s}(t_i)}{\partial \phi_{N_S}} & \frac{\partial g_{2_s}(t_i)}{\partial \phi_{N_S}} \end{bmatrix} \quad (29)$$

The appropriate partial derivatives are given in the appendix.

E. Initialialization

In order to perform the numerical search via ILS, an initial estimate $\hat{\boldsymbol{\theta}}^0$ is required. Assuming that the biases are null, the LOS measurements from the first and the second sensor $\alpha_1(t_i)$, $\alpha_2(t_i)$ and $\epsilon_1(t_i)$ can be used to solve for each initial Cartesian target position, in ECI coordinates, using (30)–(32).

$$x(t_i)^0 = \frac{\xi_2(t_i) - \xi_1(t_i) + \zeta_1(t_i)\tan\alpha_1(t_i) - \zeta_2(t_i)\tan\alpha_2(t_i)}{\tan\alpha_1(t_i) - \tan\alpha_2(t_i)} \quad (30)$$

$$y(t_i)^0 = \frac{\tan\alpha_1(t_i)(\xi_2(t_i) + \tan\alpha_2(t_i)(\zeta_1(t_i) - \zeta_2(t_i))) - \xi_1(t_i)\tan\alpha_2(t_i)}{\tan\alpha_1(t_i) - \tan\alpha_2(t_i)} \quad (31)$$

$$z(t_i)^0 = \eta_1(t_i) + \tan\epsilon_1(t_i) \left| \frac{(\xi_1(t_i) - \xi_2(t_i))\cos\alpha_2(t_i) + (\zeta_2(t_i) - \zeta_1(t_i))\sin\alpha_2(t_i)}{\sin(\alpha_1(t_i) - \alpha_2(t_i))} \right| \quad (32)$$

F. Cramér-Rao Lower Bound

In order to evaluate the efficiency of the estimator, the CRLB must be calculated. The CRLB provides a lower bound on the covariance matrix of an unbiased estimator as [1]

$$E\{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})'\} \geq J(\boldsymbol{\theta})^{-1} \quad (33)$$

where J is the Fisher Information Matrix (FIM), $\boldsymbol{\theta}$ is the true parameter vector to be estimated, and $\hat{\boldsymbol{\theta}}$ is the estimate. The FIM is

$$J(\boldsymbol{\theta}) = E\{[\nabla_{\boldsymbol{\theta}} \ln\Lambda(\boldsymbol{\theta})][\nabla_{\boldsymbol{\theta}} \ln\Lambda(\boldsymbol{\theta})]'\}|_{\boldsymbol{\theta}=\boldsymbol{\theta}_{\text{true}}} \quad (34)$$

where the gradient of the log-likelihood function is

$$\lambda(\boldsymbol{\theta}) \stackrel{\Delta}{=} \ln\Lambda(\boldsymbol{\theta}) \quad (35)$$

$$\nabla_{\boldsymbol{\theta}}\lambda(\boldsymbol{\theta}) = \sum_{i=1}^{N_t}\sum_{s=1}^{N_S} H_{is}' R_s^{-1}(\mathbf{z}_s(t_i) - \mathbf{h}_{si}(\boldsymbol{\theta})) \quad (36)$$

which, when plugged into (34), gives

$$\begin{aligned} J(\boldsymbol{\theta}) &= \sum_{i=1}^{N_t}\sum_{s=1}^{N_S} H_{is}'(R_s^{-1})H_{is}|_{\boldsymbol{\theta}=\boldsymbol{\theta}_{\text{true}}} \\ &= H'(R^{-1})H|_{\boldsymbol{\theta}=\boldsymbol{\theta}_{\text{true}}} \end{aligned} \quad (37)$$

IV. SIMULATIONS

We simulate a space based system tracking a ballistic missile. The missile and satellite trajectories are generated using System Tool Kit (STK).[2] The target modeled represents a ballistic missile with a flight time of about 20 minutes. STK provides the target and sensor positions in three dimensional Cartesian coordinates at 1 s intervals. The target launch time is chosen so that the

[2]STK Systems Tool Kit are registered trademarks of Analytical Graphics, Inc.

satellite based sensors were able to follow the missile trajectory throughout its flight path.

Any association $N_S N_t$-tuple that passes the gating test, falls into one of the following three categories:

- Completely correct (CC) association: The measurements in an association tuple have identical origin and there is no clutter measurement associated.
- Partially correct (PC) association: There are at least 2 measurements with common origin, and the rest may be from different origins or clutter measurements.
- Completely incorrect (CI) association: In an association tuple, there does not exist a pair of measurements that come from the same origin.

A. Statistical Acceptance test (Goodness of Fit)

In order to obtain the correct association, the Sum of the Normalized Square Residuals (SNSR) is used as a measure of the goodness of fit, which is defined as the minimized value of the log likelihood function (20), multiplied by 2 for convenience

$$\lambda^\star(\hat{\theta}^{\mathrm{ML}}(\mathcal{L}^{\mathrm{ML}})) = \sum_{i=1}^{N_t}\sum_{s=1}^{N_S}\Big([\mathbf{z}_s(l_{s,i},t_i) - \mathbf{h}_{si}(\hat{\theta}^{\mathrm{ML}}(\mathcal{L}^{\mathrm{ML}}))]' \times R_s^{-1}[\mathbf{z}_s(l_{s,i},t_i) - \mathbf{h}_{si}(\hat{\theta}^{\mathrm{ML}}(\mathcal{L}^{\mathrm{ML}}))]\Big) \tag{38}$$

This is similar to the linear least squares case (LS), under the Gaussian noise assumptions, where the fitting error was shown to be Chi-square distributed in [2].

In the present nonlinear LS problem, a Monte Carlo simulation is used to confirm the validity of this result, by summing up the fitting errors from N runs with independent random variables, with n_z being the number of measurements and n_x is the number of parameters, the total error obtained is Chi-square distributed with $N(n_z - n_x)$ degrees of freedom.

For the three sensor case ($n_x = 18$), the sample average SNSR over 100 Monte Carlo runs was evaluated using $n_z = 24$ LOS measurements yielding 5.71. The 99% upper limit of the probability region is, based on the $100(n_z - n_x) = 600$ degrees of freedom Chi-square distribution (divided by 100), approximately 6.83. Similar results were obtained for the two sensor case ($n_x = 24$): the sample average SNSR over 100 Monte Carlo runs was evaluated using $n_z = 28$ LOS measurements yielding 4.13. The 99% upper limit of the probability region is, based on the $100(n_z - n_x) = 400$ degrees of freedom Chi-square distribution (divided by 100), approximately 4.68.

The statistical acceptance test of an association, in a particular run, is based on data from single run, which can be used with real data, and does not require knowledge of the true parameter. Then

$$\lambda^\star(\hat{\theta}^{\mathrm{ML}}(\mathcal{L}^{\mathrm{ML}})) \sim \chi^2_{n_z - n_x} \tag{39}$$

Namely, $\lambda^\star$ should be, with 99% probability, below the threshold $\chi^2_{n_z-n_x}(0.01)$ denoted as τ. Given an association tuple, if its SNSR (38) is less than the threshold τ, then this association is accepted, otherwise it is rejected.

For the three sensor case ($n_x = 18$), three scenarios are considered, in the first scenario, the SNSR is evaluated using $n_z = 30$ LOS measurements. The 99% upper limit of the probability region is 26.6, based on the $n_z - n_x = 12$ degrees of freedom Chi-square distribution ($\tau = 26.6$). In the second scenario, the SNSR is evaluated using $n_z = 24$ LOS measurements. The 99% upper limit of the probability region is 16.8, based on the $n_z - n_x = 6$ degrees of freedom Chi-square distribution ($\tau = 16.8$). In the third scenario, we evaluate the SNSR using an 18 LOS measurements, in this case ($\tau = 0$). Practically, in this case one has 18 unknowns and 18 nonlinear equations.

For the two sensor case ($n_x = 24$), three scenarios are considered, in the first scenario, the SNSR is evaluated using $n_z = 32$ LOS measurements. The 99% upper limit of the probability region is 20.1, based on the $n_z - n_x = 8$ degrees of freedom Chi-square distribution ($\tau = 20.1$). In the second scenario, the SNSR is evaluated using $n_z = 28$ LOS measurements. The 99% upper limit of the probability region is 13.3, based on the $n_z - n_x = 4$ degrees of freedom Chi-square distribution ($\tau = 13.3$). In the third scenario, we evaluate the SNSR using 24 LOS measurements ($\tau = 0$).

B. Three-Sensor Case

We simulated three space based optical sensors at various known orbits observing a target at three points in time at unknown locations. In this case, an 18-dimensional parameter vector is to be estimated. Figure 2 shows each target position observed by the sensors (Figure 3 gives an image of this). All the sensors are assumed to have the same accuracy, detection probability $P_D = 1$ and the expected number of false measurements at each sensor at each time is assumed to be 3. As discussed in the previous section, the three sensor biases are roll, pitch and yaw angle offsets. The biases for each sensor were set to $0.5° = 8.72$ mrad. We ran 100 Monte Carlo runs. The horizontal and vertical fields-of-view of each sensor are assumed to be 60°. The measurement noise standard deviation σ_s (identical across sensors for both azimuth and elevation measurements, $\sigma_s^\alpha = \sigma_s^\epsilon = \sigma$) was assumed to be 30 μrad.

1) Description of the Scenarios. The sensors are assumed to provide LOS angle measurements. We denote by $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \boldsymbol{\xi}_3$ the 3D Cartesian sensor locations, and $\mathbf{x}(t_1), \mathbf{x}(t_2), \mathbf{x}(t_3)$ the 3D Cartesian target locations (all in ECI). The three target locations were chosen from a

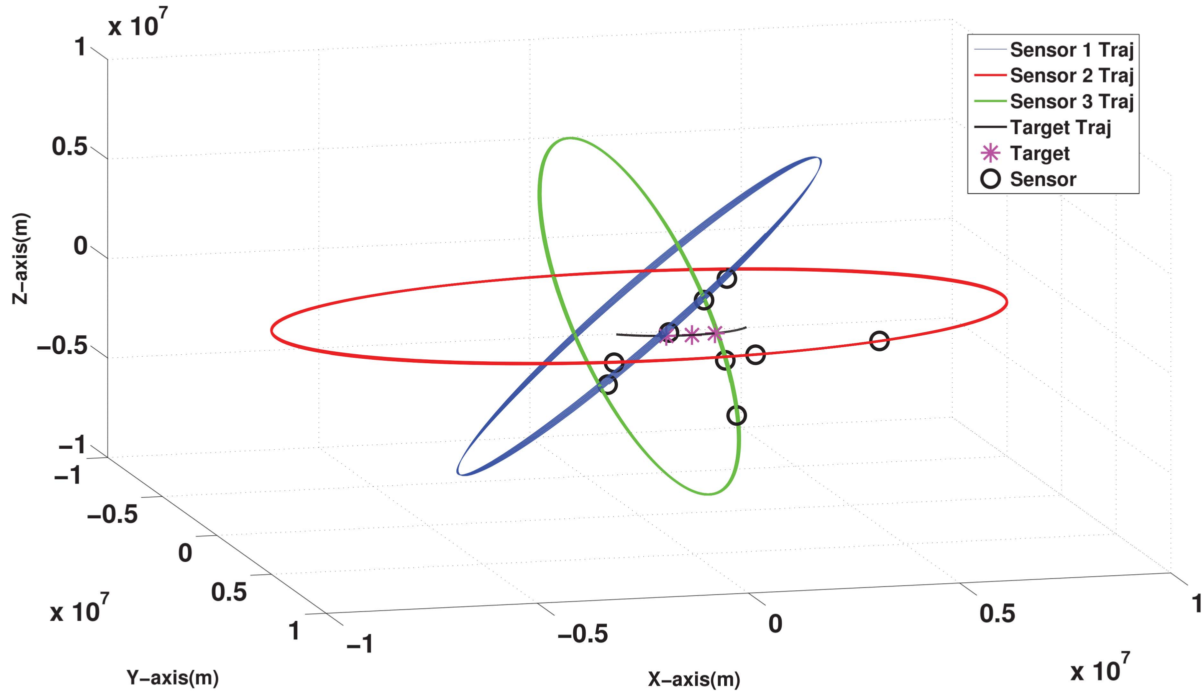

Fig. 2. Target and satellite trajectories for the three-sensor case

TABLE I
Sensor positions (km).

	ξ_1	η_1	ζ_1	ξ_2	η_2	ζ_2	ξ_3	η_3	ζ_3
Time 1	1,235	158	6,927	5,549	1,116	6,285	6,499	−279	−5,407
Time 2	1,062	−174	6,955	3,061	2,993	7,295	7,897	−719	−2,944
Time 3	887	−507	6,963	112	4,418	7,212	8,389	−1,074	−143

trajectory of a ballistic target as follows (in km)

$$\mathbf{x}(t_1) = [7{,}518 \quad -1{,}311 \quad -1{,}673]' \tag{40}$$

$$\mathbf{x}(t_2) = [7{,}942 \quad -509 \quad -1{,}375]' \tag{41}$$

$$\mathbf{x}(t_3) = [7{,}988 \quad 317 \quad -1{,}012]' \tag{42}$$

Table I summarizes the sensor positions (in km).

The statistical acceptance of an association hypothesis is carried out as discussed in Sec. IV-A. The SNSR is evaluated for each validated association hypothesis. Three scenarios are considered, in the first scenario, the SNSR is evaluated using $n_z = 30$ LOS measurements. The 99% upper limit of the probability region is 26.6, based on the $n_z - n_x = 12$ degrees of freedom Chi-square distribution ($\tau = 26.6$). In the second scenario, the SNSR is evaluated using $n_z = 24$ LOS measurements. The 99% upper limit of the probability region is 16.8, based on the $n_z - n_x = 6$ degrees of freedom Chi-square distribution ($\tau = 16.8$). In the third scenario, we evaluate the SNSR using an 18 LOS measurements, in this case ($\tau = 0$). Practically, in this case one has 18 unknowns and 18 nonlinear equations and the problem is not solvable unless $P_D = 1$, in this case, we set $\tau = 0.01$ to account for numerical imprecisions. For the first scenario, the SNRS of the completely correct (CC) association is 5.66. The SNSR of the partially correct (PC) associations and the completely incorrect (CI) associations are of the order of 10^9. For the second scenario, the SNSR of the completely correct (CC) association is 6.12. The SNSR of the partially correct (PC) associations and the completely incorrect (CI) associations are of the order of 10^9. For the last scenario, the SNSR of the completely correct (CC) association is $0.23 \cdot 10^{-24}$. The SNSR of the partially correct (PC) associations and the completely incorrect (CI) associations are of the order of 10^9.

The RMS bias errors for the correct association, are summarized in Table II, for the three scenarios in the three sensors case. The value of the σ_{CRLB} was calculated using (37) and they were provided by the ILS [6].

C. Two-Sensor Case

We simulated two space-based optical sensors at various known orbits observing a target at six (unknown) locations (which is equivalent to viewing six different targets at unknown locations). In this case, a 24-dimensional parameter vector is to be estimated. As shown in Figure 4, each target position can be observed by all sensors. All the sensors are assumed to have the

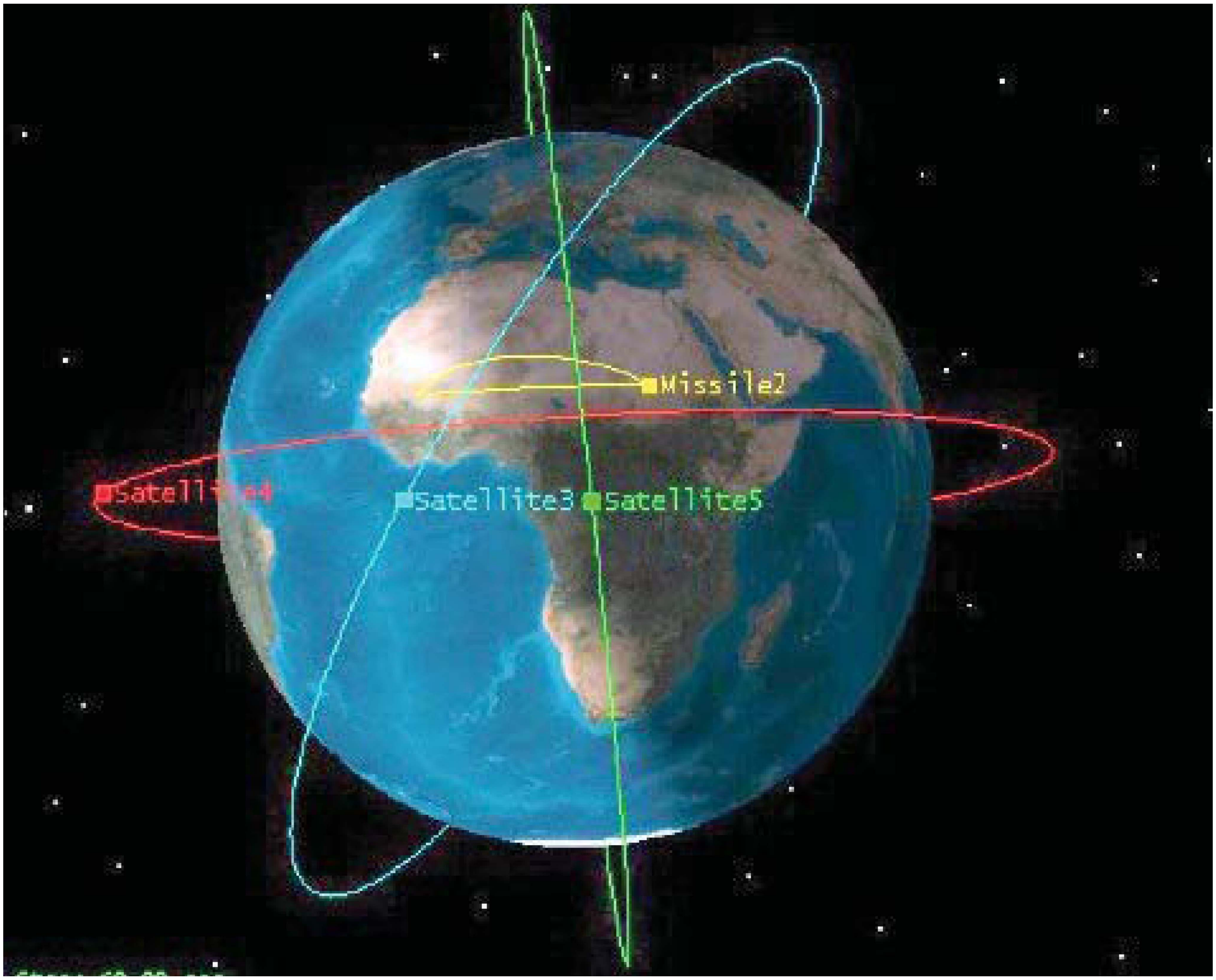

Fig. 3. Target and satellite trajectories for the three-sensor case

same accuracy, detection probability $P_D = 1$ and the expected number of false measurements at each sensor at each time is assumed to be 3. As discussed in the previous section, the three sensor biases were roll, pitch and yaw angle offsets. All the biases for each sensor were set to $0.5^\circ = 8.72$ mrad. The measurement noise standard deviation σ_s (identical across sensors for both azimuth and elevation measurements) was assumed to be 30 μrad.

1) Description of the Scenarios. The sensors are assumed to provide LOS angle measurements. We denote by $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2$ the 3D Cartesian sensor positions at six different times, and $\mathbf{x}(t_1)$, $\mathbf{x}(t_2)$, $\mathbf{x}(t_3)$, $\mathbf{x}(t_4)$, $\mathbf{x}(t_5)$, $\mathbf{x}(t_6)$ the six 3D Cartesian target locations (all in ECI). The six target locations were chosen from a trajectory of a ballistic target as follows (in km)

$$\mathbf{x}(t_1) = [-1{,}167 \quad -5{,}782 \quad 3{,}028]' \tag{43}$$

$$\mathbf{x}(t_2) = [-1{,}054 \quad -6{,}027 \quad 3{,}436]' \tag{44}$$

$$\mathbf{x}(t_3) = [-922 \quad -6{,}148 \quad 3{,}772]' \tag{45}$$

$$\mathbf{x}(t_4) = [-774 \quad -6{,}155 \quad 4{,}036]' \tag{46}$$

$$\mathbf{x}(t_5) = [-611 \quad -6{,}056 \quad 4{,}228]' \tag{47}$$

$$\mathbf{x}(t_6) = [-435 \quad -5{,}852 \quad 4{,}344]' \tag{48}$$

Table III summarizes the sensor positions.

The statistical acceptance is done as follows. The SNSR is evaluated for each validated association hypothesis. Three scenarios were considered. In the first scenario, the SNSR is evaluated using $n_z = 32$ LOS measurements. The 99% upper limit of the probability region is 20.8, based on the 8 degrees of freedom Chi-square distribution ($\tau = 20.8$). In the second scenario, the SNSR is evaluated using $n_z = 28$ LOS measurements. The 99% upper limit of the probability region is 13.3, based on the 4 degrees of freedom Chi-square distribution ($\tau = 13.3$). In the third scenario, we evaluate the SNSR using $n_z = 24$ LOS measurements, Practically, in this case one has 24 unknowns and 24 nonlinear equations and the problem is not solvable unless $P_D = 1$, in this case, we set $\tau = 0.01$ to account for numerical imprecisions. For

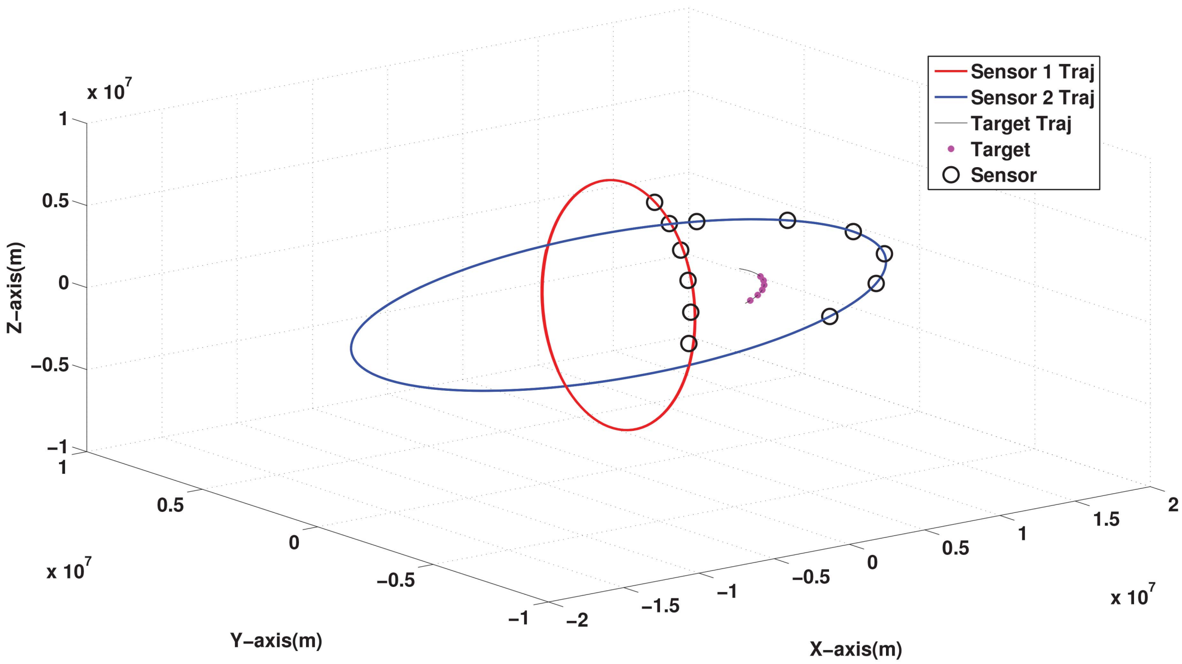

Fig. 4. Target and satellite trajectories for the two-sensor case

TABLE II
Sample average bias RMSE over 100 Monte Carlo runs and the corresponding bias standard deviation from the CRLB (σ_{CRLB})(μrad) (Three-sensor case).

Scenario		First Sensor			Second Sensor			Third Sensor		
		ψ	ρ	ϕ	ψ	ρ	ϕ	ψ	ρ	ϕ
1	RMSE	79.493	35.943	71.858	50.758	26.681	159.936	65.475	38.605	122.921
	σ_{CRLB}	78.365	39.332	85.466	50.407	25.728	152.354	69.317	38.452	133.942
2	RMSE	67.209	37.311	79.951	49.890	22.072	145.564	55.912	31.129	125.762
	σ_{CRLB}	68.909	36.620	82.351	48.584	24.235	143.217	62.641	34.364	126.637
3	RMSE	86.245	39.679	97.153	53.311	25.623	164.339	77.544	38.196	148.291
	σ_{CRLB}	78.349	39.337	85.473	50.401	25.729	152.355	69.320	38.459	133.963

the first scenario, the SNSR of the completely correct (CC) association is 6.47. The SNSR of the partially correct (PC) associations and the completely incorrect (CI) associations are of the order of 10^{10}. For the second scenario, the SNSR of the completely correct (CC) association is 7.12. The SNSR of the partially correct (PC) associations and the completely incorrect (CI) associations are of the order of 10^{10}. For the last scenario, the SNRS of the completely correct (CC) association is $0.42 \cdot 10^{-24}$. The SNSR of the partially correct (PC) associations and the completely incorrect (CI) associations are of the order of 10^{10}.

The RMS bias errors for the correct association, are summarized in Table IV, for the three scenarios in the two sensors case.

TABLE III
Sensor positions (km).

	ξ_1	η_1	ζ_1	ξ_2	η_2	ζ_2
t_1	187	−1,439	6,886	−3,966	−5,969	8,519
t_2	−902	−2,786	6,400	123	−7,238	8,458
t_3	−1,934	−3,951	5,494	4,195	−7,436	7,145
t_4	−2,840	−4,858	4,229	7,646	−6,533	4,774
t_5	−3,559	−5,447	2,687	9,965	−4,664	1,698
t_6	−4,046	−5,680	968	10,810	−2,105	−1,630

V. CONCLUSIONS

In this paper we presented an approach to bias estimation in the presence of measurement association uncertainty using common targets of opportunity. The association likelihoods are evaluated, following gating, using an exhaustive search after which a statistical acceptance test is applied to each solution in order to discriminate the correct solution from the incorrect associations. Using simulated space based tracking systems consisting of two or three satellites tracking a ballistic target, we showed that this approach performs well.

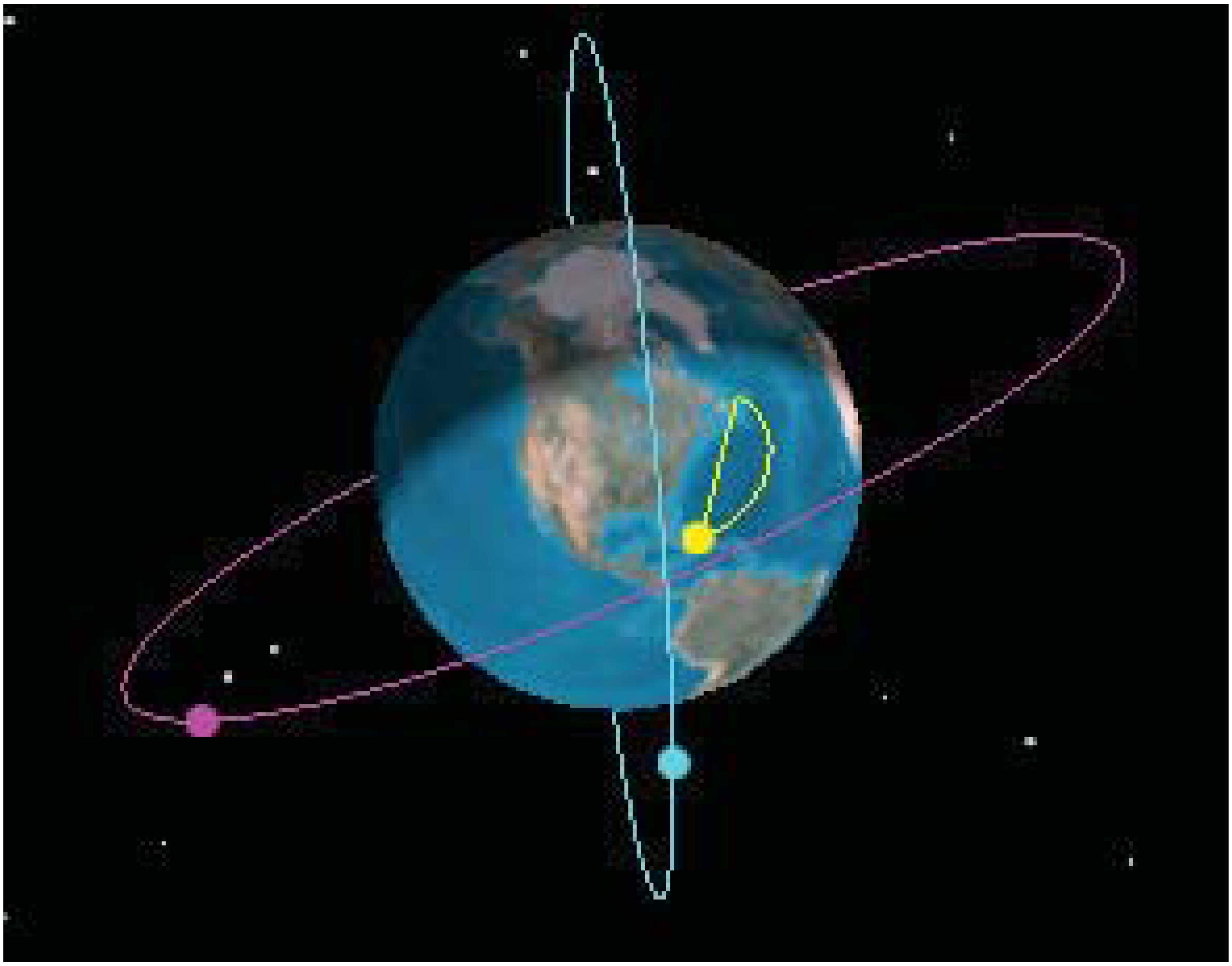

Fig. 5. Target and satellite trajectories for the two-sensor case

TABLE IV
Sample average bias RMSE over 100 Monte Carlo runs and the corresponding bias standard deviation from the CRLB (σ_{CRLB})(μrad) (Two-sensor case).

		First Sensor			Second Sensor		
Scenario		ψ	ρ	ϕ	ψ	ρ	ϕ
1	RMSE	128.469	139.761	164.244	74.097	43.693	166.525
	σ_{CRLB}	133.688	150.919	165.933	73.772	46.724	164.050
2	RMSE	143.732	148.461	173.969	80.755	49.571	173.860
	σ_{CRLB}	133.609	151.170	165.929	73.865	46.622	164.23
3	RMSE	149.383	168.707	180.788	82.082	52.476	181.479
	σ_{CRLB}	133.784	151.194	177.097	74.251	46.727	170.014

Another significance of this work is the formulation of a measure of the goodness of fit (Sum of the Normalized Square Residuals—(SNSR)) for the nonlinear least squares case, under Gaussian noise assumptions. Similarly, to the linear least squares case, where the fitting error was shown to be Chi-square distributed [2], we showed that this can be used in the nonlinear LS, thus providing a statistical test that selects the correct associations.

APPENDIX A PARTIAL DERIVATIVES

The appropriate partial derivatives of (29) are

$$\frac{\partial g_{1_s}(t_i)}{\partial x(t_k)} = \frac{\partial g_{1_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial x(t_k)} + \frac{\partial g_{1_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial x(t_k)} + \frac{\partial g_{1_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial x(t_k)} \tag{49}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial y(t_k)} = \frac{\partial g_{1_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial y(t_k)} + \frac{\partial g_{1_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial y(t_k)} + \frac{\partial g_{1_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial y(t_k)} \tag{50}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial z(t_k)} = \frac{\partial g_{1_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial z(t_k)} + \frac{\partial g_{1_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial z(t_k)} + \frac{\partial g_{1_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial z(t_k)} \tag{51}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial \psi_k} = \frac{\partial g_{1_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial \psi_k} + \frac{\partial g_{1_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial \psi_k} + \frac{\partial g_{1_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial \psi_k} \tag{52}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial \rho_k} = \frac{\partial g_{1_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial \rho_k} + \frac{\partial g_{1_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial \rho_k} + \frac{\partial g_{1_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial \rho_k} \tag{53}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial \phi_k} = \frac{\partial g_{1_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial \phi_k} + \frac{\partial g_{1_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial \phi_k} + \frac{\partial g_{1_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial \phi_k} \tag{54}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial x(t_k)} = \frac{\partial g_{2_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial x(t_k)} + \frac{\partial g_{2_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial x(t_k)} + \frac{\partial g_{2_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial x(t_k)} \tag{55}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial y(t_k)} = \frac{\partial g_{2_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial y(t_k)} + \frac{\partial g_{2_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial y(t_k)} + \frac{\partial g_{2_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial y(t_k)} \tag{56}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial z(t_k)} = \frac{\partial g_{2_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial z(t_k)} + \frac{\partial g_{2_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial z(t_k)} + \frac{\partial g_{2_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial z(t_k)} \tag{57}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial \psi_k} = \frac{\partial g_{2_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial \psi_k} + \frac{\partial g_{2_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial \psi_k} + \frac{\partial g_{2_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial \psi_k} \tag{58}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial \rho_k} = \frac{\partial g_{2_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial \rho_k} + \frac{\partial g_{2_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial \rho_k} + \frac{\partial g_{2_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial \rho_k} \tag{59}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial \phi_k} = \frac{\partial g_{2_s}(t_i)}{\partial x_s(t_i)}\frac{\partial x_s(t_i)}{\partial \phi_k} + \frac{\partial g_{2_s}(t_i)}{\partial y_s(t_i)}\frac{\partial y_s(t_i)}{\partial \phi_k} + \frac{\partial g_{2_s}(t_i)}{\partial z_s(t_i)}\frac{\partial z_s(t_i)}{\partial \phi_k} \tag{60}$$

Given that (3) can be written as

$$\mathbf{x}_s(t_i) = \begin{bmatrix} x_s(t_i) \\ y_s(t_i) \\ z_s(t_i) \end{bmatrix} = T_s(\mathbf{x}(t_i) - \boldsymbol{\xi}_s) = \begin{bmatrix} T_{s_{11}} & T_{s_{12}} & T_{s_{13}} \\ T_{s_{21}} & T_{s_{22}} & T_{s_{23}} \\ T_{s_{31}} & T_{s_{32}} & T_{s_{33}} \end{bmatrix} \begin{bmatrix} x(t_i) - \xi_s \\ y(t_i) - \eta_s \\ z(t_i) - \zeta_s \end{bmatrix} \tag{61}$$

therefore

$$x_s(t_i) = T_{s_{11}}(x(t_i) - \xi_s) + T_{s_{12}}(y(t_i) - \eta_s) + T_{s_{13}}(z(t_i) - \zeta_s) \tag{62}$$

$$y_s(t_i) = T_{s_{21}}(x(t_i) - \xi_s) + T_{s_{22}}(y(t_i) - \eta_s) + T_{s_{23}}(z(t_i) - \zeta_s) \tag{63}$$

$$z_s(t_i) = T_{s_{31}}(x(t_i) - \xi_s) + T_{s_{32}}(y(t_i) - \eta_s) + T_{s_{33}}(z(t_i) - \zeta_s) \tag{64}$$

and

$$\begin{aligned} \frac{\partial x_s(t_i)}{\partial x(t_k)} &= T_{s_{11}}, \quad \frac{\partial x_s(t_i)}{\partial y(t_k)} = T_{s_{12}}, \quad \frac{\partial x_s(t_i)}{\partial y(t_k)} = T_{s_{13}} \\ \frac{\partial y_s(t_i)}{\partial x(t_k)} &= T_{s_{21}}, \quad \frac{\partial y_s(t_i)}{\partial y(t_k)} = T_{s_{22}}, \quad \frac{\partial y_s(t_i)}{\partial y(t_k)} = T_{s_{23}} \\ \frac{\partial z_s(t_i)}{\partial x(t_k)} &= T_{s_{31}}, \quad \frac{\partial z_s(t_i)}{\partial y(t_k)} = T_{s_{32}}, \quad \frac{\partial z_s(t_i)}{\partial y(t_k)} = T_{s_{33}} \end{aligned} \tag{65}$$

$$\frac{\partial x_s(t_i)}{\partial \psi_k} = \frac{\partial T_{s_{11}}}{\partial \psi_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{12}}}{\partial \psi_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{13}}}{\partial \psi_k}(z(t_i) - \zeta_s) \tag{66}$$

$$\frac{\partial x_s(t_i)}{\partial \rho_k} = \frac{\partial T_{s_{11}}}{\partial \rho_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{12}}}{\partial \rho_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{13}}}{\partial \rho_k}(z(t_i) - \zeta_s) \tag{67}$$

$$\frac{\partial x_s(t_i)}{\partial \phi_k} = \frac{\partial T_{s_{11}}}{\partial \phi_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{12}}}{\partial \phi_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{13}}}{\partial \phi_k}(z(t_i) - \zeta_s) \tag{68}$$

$$\frac{\partial y_s(t_i)}{\partial \psi_k} = \frac{\partial T_{s_{21}}}{\partial \psi_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{22}}}{\partial \psi_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{23}}}{\partial \psi_k}(z(t_i) - \zeta_s) \tag{69}$$

$$\frac{\partial y_s(t_i)}{\partial \rho_k} = \frac{\partial T_{s_{21}}}{\partial \rho_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{22}}}{\partial \rho_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{23}}}{\partial \rho_k}(z(t_i) - \zeta_s) \tag{70}$$

$$\frac{\partial y_s(t_i)}{\partial \phi_k} = \frac{\partial T_{s_{11}}}{\partial \phi_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{22}}}{\partial \phi_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{23}}}{\partial \phi_k}(z(t_i) - \zeta_s) \tag{71}$$

$$\frac{\partial z_s(t_i)}{\partial \psi_k} = \frac{\partial T_{s_{31}}}{\partial \psi_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{32}}}{\partial \psi_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{33}}}{\partial \psi_k}(z(t_i) - \zeta_s) \tag{72}$$

$$\frac{\partial z_s(t_i)}{\partial \rho_k} = \frac{\partial T_{s_{31}}}{\partial \rho_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{32}}}{\partial \rho_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{33}}}{\partial \rho_k}(z(t_i) - \zeta_s) \tag{73}$$

$$\frac{\partial z_s(t_i)}{\partial \phi_k} = \frac{\partial T_{s_{31}}}{\partial \phi_k}(x(t_i) - \xi_s) + \frac{\partial T_{s_{32}}}{\partial \phi_k}(y(t_i) - \eta_s) + \frac{\partial T_{s_{33}}}{\partial \phi_k}(z(t_i) - \zeta_s) \tag{74}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial x_s(t_i)} = \frac{z_s(t_i)}{z_s(t_i)^2 + x_s(t_i)^2} \tag{75}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial y_s(t_i)} = 0 \tag{76}$$

$$\frac{\partial g_{1_s}(t_i)}{\partial z_s(t_i)} = -\frac{x_s(t_i)}{x_s(t_i)^2 + z_s(t_i)^2} \tag{77}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial x_s(t_i)} = -\frac{x_s(t_i)y_s(t_i)}{\sqrt{(x_s(t_i)^2 + z_s(t_i)^2)}(x_s(t_i)^2 + y_s(t_i)^2 + z_s(t_i)^2)} \tag{78}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial y_s(t_i)} = \frac{\sqrt{x_s(t_i)^2 + z_s(t_i)^2}}{x_s(t_i)^2 + y_s(t_i)^2 + z_s(t_i)^2} \tag{79}$$

$$\frac{\partial g_{2_s}(t_i)}{\partial z_s(t_i)} = -\frac{z_s(t_i) y_s(t_i)}{(x_s(t_i)^2 + y_s(t_i)^2 + z_s(t_i)^2)\left(\sqrt{x_s(t_i)^2 + z_s(t_i)^2}\right)} \tag{80}$$

$$\frac{\partial T_{s_{11}}}{\partial \psi_k} = -\sin\psi_k \cos\rho_k \tag{81}$$

$$\frac{\partial T_{s_{12}}}{\partial \psi_k} = -\sin\psi_k \sin\rho_k \sin\phi_k - \cos\psi_k \cos\phi_k \tag{82}$$

$$\frac{\partial T_{s_{13}}}{\partial \psi_k} = -\sin\psi_k \sin\rho_k \cos\phi_k + \cos\psi_k \sin\phi_k \tag{83}$$

$$\frac{\partial T_{s_{21}}}{\partial \psi_k} = \cos\psi_k \cos\rho_k \tag{84}$$

$$\frac{\partial \mathrm{T}_{s_{22}}}{\partial \psi_{\mathrm{k}}} = \cos\psi_k \sin\rho_k \sin\phi_k - \sin\psi_k \cos\phi_k \tag{85}$$

$$\frac{\partial T_{s_{23}}}{\partial \psi_k} = \cos\psi_k \sin\rho_k \cos\phi_k + \sin\psi_k \sin\phi_k \tag{86}$$

$$\frac{\partial T_{s_{31}}}{\partial \psi_k} = 0 \tag{87}$$

$$\frac{\partial T_{s_{32}}}{\partial \psi_k} = 0 \tag{88}$$

$$\frac{\partial T_{s_{33}}}{\partial \psi_k} = 0$$

$$\frac{\partial T_{s_{11}}}{\partial \rho_k} = -\cos\psi_k \sin\rho_k \tag{89}$$

$$\frac{\partial T_{s_{12}}}{\partial \rho_k} = \cos\psi_k \cos\rho_k \sin\phi_k \tag{90}$$

$$\frac{\partial T_{s_{13}}}{\partial \rho_k} = \cos\psi_k \cos\rho_k \cos\phi_k \tag{91}$$

$$\frac{\partial T_{s_{21}}}{\partial \rho_k} = -\sin\psi_k \sin\phi_k \tag{92}$$

$$\frac{\partial T_{s_{22}}}{\partial \rho_k} = \sin\psi_k \cos\rho_k \sin\phi_k \tag{93}$$

$$\frac{\partial T_{s_{23}}}{\partial \rho_k} = \sin\psi_k \cos\rho_k \cos\phi_k \tag{94}$$

$$\frac{\partial T_{s_{31}}}{\partial \rho_k} = -\cos\phi_k \tag{95}$$

$$\frac{\partial T_{s_{32}}}{\partial \rho_k} = -\sin\rho_k \sin\phi_k \tag{96}$$

$$\frac{\partial T_{s_{33}}}{\partial \rho_k} = -\sin\rho_k \cos\phi_k \tag{97}$$

$$\frac{\partial T_{s_{11}}}{\partial \phi_k} = 0 \tag{98}$$

$$\frac{\partial T_{s_{12}}}{\partial \phi_k} = \cos\psi_k \sin\rho_k \cos\phi_k + \sin\psi_k \sin\phi_k \tag{99}$$

$$\frac{\partial T_{s_{13}}}{\partial \phi_k} = -\cos\psi_k \sin\rho_k \sin\phi_k + \sin\psi_k \cos\phi_k \tag{100}$$

$$\frac{\partial T_{s_{21}}}{\partial \phi_k} = 0 \tag{101}$$

$$\frac{\partial T_{s_{22}}}{\partial \phi_k} = \sin\psi_k \sin\rho_k \cos\phi_k - \cos\psi_k \sin\phi_k \tag{102}$$

$$\frac{\partial T_{s_{23}}}{\partial \phi_k} = -\sin\psi_k \sin\rho_k \sin\phi_k - \cos\psi_k \cos\phi_k \tag{103}$$

$$\frac{\partial T_{s_{31}}}{\partial \phi_k} = 0 \tag{104}$$

$$\frac{\partial T_{s_{32}}}{\partial \phi_k} = \cos\psi_k \cos\phi_k \tag{105}$$

$$\frac{\partial T_{s_{33}}}{\partial \phi_k} = -\cos\rho_k \sin\phi_k \tag{106}$$

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Djedjiga Belfadel is an Assistant Professor in the Electrical and Computer Engineering department at Fairfield University, Fairfield, CT. She obtained her B.S., degrees from the University of Mouloud Mammeri in 2003, her M.S., degrees from the University of New Haven in 2008, and her Ph.D. degree from University of Connecticut in 2015, all in electrical engineering. From 2009 to 2011, she worked, as an Electrical Engineer, at Evax Systems Inc. in Branford, Connecticut. Her research interests include target tracking, data association, sensor fusion, machine vision, and other aspects of estimation.

Richard W. Osborne, III obtained his B.S., M.S., and Ph.D. degrees in electrical engineering from the University of Connecticut in 2004, 2007, and 2012, respectively. From 2012–2014 he was an Assistant Research Professor in the Electrical Engineering department at the University of Connecticut, Storrs, CT. From 2014–2015 he was a Senior Research Engineer at BAE Systems, Inc. in Burlington, MA, and since 2015, he has been a Senior Research Engineer at United Technologies Research Center in East Hartford, CT. His academic interests include adaptive target tracking, information/sensor fusion, perception/computer vision, autonomous systems, and other aspects of estimation.

Yaakov Bar-Shalom received the B.S. and M.S. degrees from the Technion in 1963 and 1967 and the Ph.D. degree from Princeton University in 1970, all in EE. From 1970 to 1976 he was with Systems Control, Inc., Palo Alto, California. Currently he is Board of Trustees Distinguished Professor in the Dept. of Electrical and Computer Engineering and Marianne E. Klewin Professor in Engineering at the University of Connecticut. His current research interests are in estimation theory, target tracking and data fusion. He has published over 550 papers and book chapters. He coauthored/edited 8 books, including Tracking and Data Fusion (YBS Publishing, 2011), He has been elected Fellow of IEEE for "contributions to the theory of stochastic systems and of multitarget tracking." He served as Associate Editor of the IEEE Transactions on Automatic Control and Automatica. He was General Chairman of the 1985 ACC. He served as Chairman of the Conference Activities Board of the IEEE CSS and member of its Board of Governors. He served as General Chairman of FUSION 2000, President of ISIF in 2000 and 2002 and Vice President for Publications during 2004–13. In 1987 he received the IEEE CSS Distinguished Member Award. Since 1995 he is a Distinguished Lecturer of the IEEE AESS. He is corecipient of the M. Barry Carlton Award for the best paper in the IEEE TAESystems in 1995 and 2000. In 2002 he received the J. Mignona Data Fusion Award from the DoD JDL Data Fusion Group. He is a member of the Connecticut Academy of Science and Engineering. In 2008 he was awarded the IEEE Dennis J. Picard Medal for Radar Technologies and Applications, and in 2012 the Connecticut Medal of Technology. He has been listed by academic.research.microsoft (top authors in engineering) as #1 among the researchers in Aerospace Engineering based on the citations of his work. He is the recipient of the 2015 ISIF Award for a Lifetime of Excellence in Information Fusion. This award has been renamed in 2016 as the Yaakov Bar-Shalom Award for a Lifetime of Excellence in Information Fusion.

Krishna R. Pattipati received the B. Tech. degree in electrical engineering with highest honors from the Indian Institute of Technology, Kharagpur, in 1975, and the M.S. and Ph.D. degrees in systems engineering from UConn, Storrs, in 1977 and 1980, respectively. He was with ALPHATECH, Inc., Burlington, MA from 1980 to 1986. He has been with the department of Electrical and Computer Engineering at UConn, where he is currently the Board of Trustees Distinguished Professor and the UTC Chair Professor in Systems Engineering. Dr. Pattipati's research activities are in the areas of proactive decision support, uncertainty quantification, smart manufacturing, autonomy, knowledge representation, and optimization-based learning and inference. A common theme among these applications is that they are characterized by a great deal of uncertainty, complexity, and computational intractability. He is a cofounder of Qualtech Systems, Inc., a firm specializing in advanced integrated diagnostics software tools (TEAMS, TEAMS-RT, TEAMS-RDS, TEAMATE), and serves on the board of Aptima, Inc. Dr. Pattipati was selected by the IEEE Systems, Man, and Cybernetics (SMC) Society as the Outstanding Young Engineer of 1984, and received the Centennial Key to the Future award. He has served as the Editor-in-Chief of the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART B from 1998 to 2001, Vice-President for Technical Activities of the IEEE SMC Society from 1998 to 1999, and as Vice-President for Conferences and Meetings of the IEEE SMC Society from 2000 to 2001. He was co-recipient of the Andrew P. Sage Award for the Best SMC Transactions Paper for 1999, the Barry Carlton Award for the Best AES Transactions Paper for 2000, the 2002 and 2008 NASA Space Act Awards for "A Comprehensive Toolset for Model-based Health Monitoring and Diagnosis," and "Real-time Update of Fault-Test Dependencies of Dynamic Systems: A Comprehensive Toolset for Model-Based Health Monitoring and Diagnostics," and the 2003 AAUP Research Excellence Award at UCONN. He also won the best technical paper awards at the 1985, 1990, 1994, 2002, 2004, 2005 and 2011 IEEE AUTOTEST Conferences, and at the 1997, 2004 Command and Control Conference. He is an elected Fellow of IEEE and of the Connecticut Academy of Science and Engineering.