Augmented State Integrated Probabilistic Data Association Smoothing for Automatic Track Initiation in Clutter

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We introduce a fixed lag smoother algorithm based on the integrated probabilistic data association (IPDA) algorithm. IPDA jointly estimates both the target state and its existence. In this paper the joint density of target state and existence is extended for fixed lag smoothing. The proposed smoothing algorithm is also tested against various multiple target tracking parameters like state RMS estimation, number of true target detected, number of false target confirmed and target termination time and simulation results are also presented in the paper.

Manuscript received November 7, 2004; revised July 27, 2005 and January 30, 2006.

Refereeing of this contribution was handled by Dr. Jean Dezert.

The authors would also like to acknowledge the contribution of Center for Autonomous Systems (CAS), UTS for its contribution towards the reasearch work.

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1. INTRODUCTION

Smoothing within the state estimation context is technically defined as a process where the current measurements are used to improve the estimates of the past states of the object of interest. In the target tracking problem, this corresponds to estimating the past target states and associated tracker performance parameters using current measurements. Formally, one can define the track estimation problem as follows. Let $x(t_k)$ denote the target state at time k and $y^k = \{y(1), y(2), \ldots, y(k)\}$ denote the measurement sequence up to time k where $y(i)$ denotes the measurement at time i. The target estimation problem can then be defined as the problem of computing the conditional mean estimate of the target state

$$\hat{x}(t_{k-L|k}) = E[x(t_{k-L}) \mid y^k] \tag{1}$$

and its associated error covariance

$$P_{k-L|k} = E[(x(t_{k-L}) - \hat{x}(t_{k-L|k}))(x(t_k) - \hat{x}(t_{k-L|k}))^T \mid y^k] \tag{2}$$

where $L = 0$, $L < 0$ and $L > 0$ are for three types of estimation namely filtering, prediction and smoothing respectively.

Smoothing algorithms were shown to provide significant performance improvements in terms of RMS errors in several important tracking problems like maneuvering target tracking using IMM smoothers by Helmick et al. [13]–[14], tracking in clutter using PDA smoothers by Mahalanabis et el. [17] and using IMMPDA smoothing for maneuvering target tracking in clutter [12]. More recently, the augmented state smoothing framework was used for dealing with out of sequence measurements by Challa et al. [11], [25].

One of the very important assumptions made in all these efforts is the fact that the target exists. However, in reality, target existence must first be established before using one of the above methods. Several techniques to achieve this are available in literature, like the heuristic M out of N detections method [6] and the Bayesian approaches like IMMPDA and IPDA.

IMMPDA, when used in the context of automatic track formation [2], uses two models—one that assumes that the target is "observable" by the sensor with a detection probability of P_D, $0 < P_D \leq 1$, and the other assumes that the target is not "observable" with probability of detection $P_D = 0$. The algorithm uses a probability measure (the model probability) for each of the models and estimates the "true target probability." If that probability crosses a suitable threshold, a decision on target's presence (existence) is taken.

Another effective algorithm to solve automatic track formation in clutter, referred to as Integrated Probabilistic Data Association (IPDA), is proposed by Musicki and Evans [22]–[23]. Many of its variants for use in difficult environments are proposed in [18], [21]. IPDA models target existence as a random event satisfying Markovian properties between existence and nonexistence states and provides a mechanism to associate a probability measure (the target existence probability) to it. Similarly to the method of IMMPDA, the target existence probability is estimated along with the target states and if the existence probability crosses a threshold, a decision on target's presence (existence) is taken.

IMMPDA and IPDA are Bayesian approaches and are amenable for treatment within the smoothing framework. In this paper, we focus on the IPDA based approach for target existence and develop a new algorithm for automatic track initiation in clutter within the augmented state smoothing framework. We investigate the effect of smoothed track existence probability on tracker performance measures, e.g. true/false track discrimination, by comparing its performance with the standard IPDA algorithm. The flow chart of the algorithm is also presented in this paper. Simulation results are also provided, where the improvements in true/false track statistics are found to be significant with a potential to improve all higher layer functions of tracking systems like situation and threat assessment.

The paper is organized as follows. Following the introduction, Section 2 formulates the automatic track formulation problem as conceptualized by IPDA. The theory of the augmented state IPDA smoother is described in Section 3. The complete mathematical derivation of the algorithm is carried out in Section 4. The flow chart of the algorithm is presented in Section 5. The simulation scenario and results are presented in Section 6. Conclusions are drawn in Section 7.

2. PROBLEM FORMULATION

The target tracking algorithm starts with a priori knowledge of the target dynamic model. Each target within the surveillance region is assumed to follow the dynamic equation

$$x_k = Fx_{k-1} + Gw_k \tag{3}$$

where

- the target state x_k consists of kinematic states e.g. position, velocity etc.,
- F is the state transition matrix,
- w_k is the noise (called "process noise"). w_k is assumed normally distributed with mean zero and variance Q. It is also assumed that $E\{w_k w_j\} = 0$ if $k \neq j$.

A measurement model is defined as

$$y_k = Hx_k + v_k \tag{4}$$

where

- H is the state to measurement transition matrix,
- v_k is noise (called "measurement noise"). v_k is assumed to be normally distributed with mean zero and variance R. It is also assumed that $E\{v_k v_j\} = 0$ if $k \neq j$. Moreover $E\{w_i v_j\} = 0$ for any i,j.

IPDA takes "track existence" as a random event and finds the probability of the event to solve the problem of "automatic track maintenance." IPDA models the existence of a track as a two state random variable, E_k, where

- $E_k = 1$ refers to the event that the track exists at time $t = k$,
- $E_k = 0$ refers to the event that the track does not exist at time $t = k$.

A target or track can also switch between these two states according to a predefined switching probability matrix which is

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{10} \\ \Gamma_{01} & \Gamma_{00} \end{bmatrix} \tag{5}$$

where

$$\Gamma_{ij} = p(E_k = j \mid E_{k-1} = i), \qquad i,j \in 0,1. \tag{6}$$

In the rest of the text, $E_k = 1$ will be denoted as E_k and $E_k = 0$ will be denoted as $\bar{E}_k$ with the definition of Γ_{ij} considered to be understood as defined by (6).

IPDA solves the uncertainty in "target existence" automatically by estimating

$$p(E_{k-L} \mid y^k) \tag{7}$$

where again $L = 0$, $L < 0$ and $L > 0$ are for filtering, prediction and smoothing types of estimation respectively. The state is estimated with the condition that the target exists and thus the state estimation is redefined by the introduction of a conditional parameter as

$$p(x_{k-L} \mid E_{k-L}, y^k). \tag{8}$$

3. FIXED LAG AUGMENTED STATE IPDA (AS-IPDA) SMOOTHING

In an augmented approach for a lag of N, the target dynamic model and measurement equation of (3) and (4) respectively will be replaced by the augmented vectors

$$\mathcal{X}_k = \mathcal{F}\mathcal{X}_{k-1} + \mathcal{G}w_k \tag{9}$$

$$\mathcal{Y}_k = \mathcal{H}\mathcal{X}_k + v_k \tag{10}$$

where

$$\mathcal{X}_k = [x_k \;\; x_{k-1} \cdots x_{k-N}]^T \tag{11}$$

$$\mathcal{F} = \begin{bmatrix} F & 0 & 0 & \cdots & 0 \\ I & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \tag{12}$$

$$\mathcal{Y}_k = [y_k] \quad (13)$$

$$\mathcal{H} = [H \ 0 \cdots 0]. \quad (14)$$

The noise variance matrix Q_k will also be adjusted to

$$\mathcal{Q} = \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}. \quad (15)$$

But the IPDA concept suggests that along with the target's state, its existence event also needs to be augmented. Based on that conceptual framework, two possible combinations of target state and its existence are possible at any given instant of time. These are:

- $C_k^1 = \{x_k, E_k\}$, the target exists and so does its state,
- $C_k^2 = \{\phi, \bar{E}_k\}$, the target does not exist and so neither does its state.

Thus for an entire fixed lag of N, the augmentation can be carried out in the following manner

$$[C_k^a \ C_{k-1}^b \cdots C_{k-N}^d]^T \quad (16)$$

where $a, b, \ldots, d \in 1, 2$. This suggests that there can be more than one augmented hypothesis possible.

From the implementation point of view of IPDA, when a target goes out of existence, it remains that way for all future time. In that context, the transition matrix of (5) can be made more specific

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{10} \\ 0 & 1 \end{bmatrix}. \quad (17)$$

All published results of IPDA follow this transition matrix.

Therefore not all the combinations of C_ks are valid in (16). Thus at any time, eliminating the impossible hypotheses, there remain $N+2$ permissible augmented hypotheses. These are

- Hypothesis 1:

$$\mathbf{H}_k = [\mathcal{X}_k, \mathcal{E}_k] = \begin{bmatrix} x_k, E_k \\ x_{k-1}, E_{k-1} \\ \vdots \\ x_{k-N}, E_{k-N} \end{bmatrix} \quad (18)$$

- Hypothesis m:

$$\mathbf{H}_k^m = [\mathcal{X}_k^m, \mathcal{E}_k^m] = \begin{bmatrix} \phi, \bar{E}_k \\ \vdots \\ \phi, \bar{E}_{k-m} \\ x_{k-m-1}, E_{k-m-1} \\ \vdots \\ x_{k-N}, E_{k-N} \end{bmatrix} \quad (19)$$

where $m = 0, 1, 2, \ldots, N-1$.

- Hypothesis n:

$$\mathbf{H}_k^n = [\mathcal{X}_k^n, \mathcal{E}_k^n] = \begin{bmatrix} \phi, \bar{E}_k \\ \vdots \\ \phi, \bar{E}_{k-N} \end{bmatrix}. \quad (20)$$

Except for hypothesis one, the other hypotheses assume that the target does not exist at the current time. It is also shown in the Appendix that hypotheses $\mathbf{H}_k^m$ and $\mathbf{H}_k^n$ also do not contribute in the state update. This is also supported by the fact that as these hypotheses refer to the scenario that at the current time the target does not exist, measurements at the current time will therefore contain no information about the target. Thus the smoothing of a track is concerned only with the first hypothesis.

Thus the underlying Bayesian approach for developing an IPDA smoothing algorithm reduces to the calculation of the probability density,

$$p(\mathcal{X}_k, \mathcal{E}_k \mid y^k) = p(\mathcal{X}_k \mid \mathcal{E}_k, y^k) p(\mathcal{E}_k \mid y^k). \quad (21)$$

Furthermore the existence probabilities at each time instant are readily given by,

$$p(E_k) = p(\mathcal{E}_k \mid y^k) \quad (22)$$

$$p(E_{k-d}) = p(\mathcal{E}_k \mid y^k) + \sum_{j=0}^{d-1} p(\mathcal{E}_k^j \mid y^k) \quad (23)$$

where $d = 1, 2, \ldots, N$.

The conditional state estimate of (21) and the existence probabilities of (22) and (23) together solve the IPDA smoothing problem. In the next section, the calculation of these estimates will be carried out.

4. DERIVATION OF AS-IPDA SMOOTHING

In this section, one iteration of the state estimate and existence probability estimates are derived separately for clarity.

4.1 Conditional State Estimate

The conditional state estimate $p(\mathcal{X}_k \mid \mathcal{E}_k, y^k)$ can be expanded through Bayes' Theorem:

$$\begin{aligned} p(\mathcal{X}_k \mid \mathcal{E}_k, y^k) &= p(\mathcal{X}_k \mid \mathcal{E}_k, y_k, y^{k-1}) \\ &= \frac{p(y_k \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}).p(\mathcal{X}_k \mid \mathcal{E}_k, y^{k-1})}{p(y_k \mid \mathcal{E}_k, y^{k-1})} \\ &= \frac{\text{Likelihood} \times \text{Prediction}}{\text{Normalization}}. \end{aligned} \quad (24)$$

The three terms likelihood, prediction and normalization will be derived step by step. The a priori target state is assumed known (either from a previous iteration or from initialization of the tracks) with a Gaussian distribution having mean $\hat{\mathcal{X}}_{k-1|k-1}$ and covariance $\mathcal{P}_{k-1|k-1}$.

Likelihood

As there is uncertainty involved about the origin of measurements, the measurements that fall within a validating gate are used for the track state update. The volume of the elliptical validation region is V_k.

To calculate the likelihood, the assumptions made in the literature are

- the probability mass function of the number of false measurements conditioned on the past measurement history at time k is a Poisson distribution,

$$P(m_k \mid y^{k-1}) \equiv P_0(m_k) = \frac{\lambda^{m_k} e^{-\lambda}}{m_k!} \tag{25}$$

where λ is the expected number of validated measurements and is given by

$$\lambda = \begin{cases} 0 & \text{if} \quad m_k = 0 \\ m_k - P_D P_G p(E_k \mid y^{k-1}) & \text{if} \quad m_k > 0 \end{cases} \tag{26}$$

where m_k is the number of validated measurements at the time $t = k$. P_D and P_G denote the detection probability and gate probability respectively while $p(E_k \mid y^{k-1})$ can be obtained from $p(E_{k-1} \mid y^{k-1})$ by using the Markov Transition Probability as

$$p(E_k \mid y^{k-1}) = \Gamma_{11} p(E_{k-1} \mid y^{k-1}) \tag{27}$$

- the hypotheses that
—α_0: all validated measurements are false measurement or clutter
—α_i: the ith validated measurement is target originated and all others are false measurements
These are complementary sets and hence the following conditional probabilities can be defined
  1) No validated measurement is target originated given the target exists

$$P(\alpha_0 \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) = 1 - P_D P_G. \tag{28}$$

  2) The ith validated measurement is target originated given the target exists

$$P(\alpha_i \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) = \frac{P_D P_G}{m_k}. \tag{29}$$

Based on the above defined parameters, the likelihood in (24) is calculated as

$$\begin{aligned}
&p(y_k \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \\
&\quad = \sum_{i=0}^{m_k} p(y_k \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}, y^{k-1}, \alpha_i, m_k) \\
&\qquad .P(m_k \mid y^{k-1}).P(\alpha_i \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \\
&\quad = \left(\frac{1}{V_k}\right)^{m_k} P_0(m_k) P(\alpha_0 \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \\
&\qquad + \sum_{i=1}^{m_k} \left(\frac{1}{V_k}\right)^{m_k - 1} P_0(m_k - 1) \times p(y_k(i) \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}, \alpha_i) \\
&\qquad \times P(\alpha_i \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \\
&\quad = \left(\frac{1}{V_k}\right)^{m_k} P_0(m_k) P(\alpha_0 \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \\
&\qquad + \left(\frac{1}{V_k}\right)^{m_k - 1} \frac{m_k}{\lambda} P_0(m_k) \sum_{i=1}^{m_k} p(y_k(i) \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}, \alpha_i) \\
&\qquad .P(\alpha_i \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \\
&\quad = \left(\frac{1}{V_k}\right)^{m_k} P_0(m_k) \\
&\qquad \times \left\{ 1 - P_D P_G + P_D P_G \frac{V_k}{\lambda} \sum_{i=1}^{m_k} p(y_k(i) \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}, \alpha_i) \right\}
\end{aligned} \tag{30}$$

where V_k is the volume of the measurement validation gate at time $t = k$.

Under the assumption of Gaussian measurement noise, the likelihood of the ith validated measurement is also Gaussian and hence the likelihood term within the summation sign is a Gaussian PDF,

$$p(y_k(i) \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}, \alpha_i) \approx \mathcal{N}(y_k(i); \mathcal{H}\mathcal{X}_k, \mathcal{R}). \tag{31}$$

Therefore the expression for likelihood from (30) is

$$\begin{aligned}
&p(y_k \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \\
&= \left(\frac{1}{V_k}\right)^{m_k} P_0(m_k) \left\{ 1 - P_D P_G + P_D P_G \frac{V_k}{\lambda} \sum_{i=1}^{m_k} \mathcal{N}(y_k(i); \mathcal{H}\mathcal{X}_k, \mathcal{R}) \right\}.
\end{aligned} \tag{32}$$

Prediction

Given the linear process and measurement equations of (25) and (26), the prediction can be directly derived from Kalman filter theory and is given as

$$p(\mathcal{X}_k \mid \mathcal{E}_k, y^{k-1}) = \mathcal{N}(\mathcal{X}_k; \hat{\mathcal{X}}_{k|k-1}, \mathcal{P}_{k|k-1}) \tag{33}$$

where

$$\hat{\mathcal{X}}_{k|k-1} = \mathcal{F}\hat{\mathcal{X}}_{k-1|k-1} \tag{34}$$

$$\mathcal{P}_{k|k-1} = \mathcal{F}\mathcal{P}_{k-1|k-1}\mathcal{F}^T + \mathcal{Q}. \tag{35}$$

Normalization

From (24), the normalization is

$$\begin{aligned}
&p(y_k \mid \mathcal{E}_k, y^{k-1}) = \delta \\
&\quad = \int_{\mathcal{X}_k} p(y_k \mid \mathcal{X}_k, \mathcal{E}_k, y^{k-1}) \times p(\mathcal{X}_k \mid \mathcal{E}_k, y^{k-1}) d\mathcal{X}_k \\
&\quad = \int_{\mathcal{X}_k} \left(\frac{1}{V_k}\right)^{m_k} P_0(m_k) \\
&\qquad \times \left\{ 1 - P_D P_G + P_D P_G \frac{V_k}{\lambda} \sum_{i=1}^{m_k} \mathcal{N}(y_k(i); \mathcal{H}\mathcal{X}_k, \mathcal{R}) \right\} \\
&\qquad \times \mathcal{N}(\mathcal{X}_k; \hat{\mathcal{X}}_{k|k-1}, \mathcal{P}_{k|k-1}) d\mathcal{X}_k.
\end{aligned} \tag{36}$$

In (36)

$$
\begin{aligned}
&\mathcal{N}(y_k(i);\mathcal{H}\mathcal{X}_k,\mathcal{R})\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k-1},\mathcal{P}_{k|k-1})\\
&\quad=\frac{\mathcal{N}(y_k(i);\mathcal{H}\mathcal{X}_k,\mathcal{R})\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k-1},\mathcal{P}_{k|k-1})}{\mathcal{N}(y_k(i);H\hat{\mathcal{X}}_{k|k-1},\mathcal{S})}\\
&\qquad\times\mathcal{N}(y_k(i);H\hat{\mathcal{X}}_{k|k-1},\mathcal{S})\\
&\quad=\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k}(i),\mathcal{P}_{k|k}(i))\times\mathcal{N}(y_k(i);H\hat{\mathcal{X}}_{k|k-1},\mathcal{S})
\end{aligned}
\tag{37}
$$

where

$$
\begin{aligned}
\mathcal{S}&=\mathcal{H}\mathcal{P}_{k|k-1}\mathcal{H}^T+\mathcal{R}\\
\mathcal{K}&=\mathcal{P}_{k|k-1}\mathcal{H}^T(\mathcal{S})^{-1}\\
\hat{\mathcal{X}}_{k|k}(i)&=\hat{\mathcal{X}}_{k|k-1}+\mathcal{K}(y_k(i)-\mathcal{H}\hat{\mathcal{X}}_{k|k-1})\\
\mathcal{P}_{k|k}(i)&=(\mathcal{I}-\mathcal{K}\mathcal{H})\mathcal{P}_{k|k-1}.
\end{aligned}
$$

Therefore (36) becomes

$$
\begin{aligned}
&p(y_k\mid\mathcal{E}_k,y^{k-1})=\delta\\
&\quad=\int_{\mathcal{X}_k}\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)\\
&\qquad\times\left\{1-P_DP_G+P_DP_G\frac{V_k}{\lambda}\sum_{i=1}^{m_k}\mathcal{N}(y_k(i);H\hat{\mathcal{X}}_{k|k-1},\mathcal{S})\right\}\\
&\qquad\times\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k}(i),\mathcal{P}_{k|k}(i))d\mathcal{X}_k\\
&\quad=\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)\\
&\qquad\times\left\{1-P_DP_G+P_DP_G\frac{V_k}{\lambda}\sum_{i=1}^{m_k}\mathcal{N}(y_k(i);H\hat{\mathcal{X}}_{k|k-1},\mathcal{S})\right\}.
\end{aligned}
\tag{38}
$$

Now putting all the respective expressions in (24), the conditional state estimate becomes

$$
\begin{aligned}
&p(\mathcal{X}_k\mid\mathcal{E}_k,y_k)\\
&\quad=\frac{1}{\delta}\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k-1},\mathcal{P}_{k|k-1})\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)\\
&\qquad\times\left\{1-P_DP_G+P_DP_G\frac{V_k}{\lambda}\sum_{i=1}^{m_k}\mathcal{N}(y_k(i);\mathcal{H}\mathcal{X}_k,\mathcal{R})\right\}\\
&\quad=\frac{1}{\delta}\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)(1-P_DP_G)\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k-1},\mathcal{P}_{k|k-1})\\
&\qquad+\sum_{i=1}^{m_k}\frac{1}{\delta}\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)P_DP_G\frac{V_k}{\lambda}\\
&\qquad\times\mathcal{N}(y_k(i);\mathcal{H}\mathcal{X}_k,\mathcal{R})\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k-1},\mathcal{P}_{k|k-1}).
\end{aligned}
\tag{39}
$$

Using (37), (39) can be reduced to

$$
\begin{aligned}
&p(\mathcal{X}_k\mid\mathcal{E}_k,y_k)\\
&\quad=\frac{1}{\delta}\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)(1-P_DP_G)\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k-1},\mathcal{P}_{k|k-1})\\
&\qquad+\sum_{i=1}^{m_k}\frac{1}{\delta}\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)P_DP_G\frac{V_k}{\lambda}\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k}(i),\mathcal{P}_{k|k}(i))\\
&\qquad\times\mathcal{N}(y_k(i);\mathcal{H}\hat{\mathcal{X}}_{k|k-1},\mathcal{S})\\
&\quad=\beta_k(0)\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k-1},\mathcal{P}_{k|k-1})\\
&\qquad+\sum_{i=1}^{m_k}\beta_k(i)\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k}(i),\mathcal{P}_{k|k}(i))\\
&\quad=\sum_{i=0}^{m_k}\beta_k(i)\times\mathcal{N}(\mathcal{X}_k;\hat{\mathcal{X}}_{k|k}(i),\mathcal{P}_{k|k}(i))
\end{aligned}
\tag{40}
$$

where

$$
\beta_k(0)=\frac{1}{\delta}\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)(1-P_DP_G)\tag{41}
$$

$$
\beta_k(i)=\frac{1}{\delta}\left(\frac{1}{V_k}\right)^{m_k}P_0(m_k)P_DP_G\frac{V_k}{\lambda}\mathcal{N}(y_k(i);H\hat{\mathcal{X}}_{k|k-1},\mathcal{S})\tag{42}
$$

and taking

$$
\hat{\mathcal{X}}_{k|k}(0)=\hat{\mathcal{X}}_{k|k-1}\tag{43}
$$

$$
\mathcal{P}_{k|k}(0)=\mathcal{P}_{k|k-1}.\tag{44}
$$

From (40), the estimates of state and covariance are derived as

$$
\hat{\mathcal{X}}_{k|k}=\sum_{i=0}^{m_k}\beta_k(i)\hat{\mathcal{X}}_{k|k}(i)\tag{45}
$$

$$
\mathcal{P}_{k|k}=\sum_{i=0}^{m_k}\beta_k(i)\mathcal{P}_{k|k}(i)+\sum_{i=0}^{m_k}\beta_k(i)\hat{\mathcal{X}}_{k|k}(i)\hat{\mathcal{X}}_{k|k}(i)^T-\hat{\mathcal{X}}_{k|k}\hat{\mathcal{X}}_{k|k}^T.\tag{46}
$$

The expressions in (45) and (46) give the state estimate and its covariance matrix conditioned on the target existence.

4.2 Existence Probability Estimate

Smoothing of the existence probability requires two steps,

- first, calculation of the probabilities of the augmented existence hypotheses,
- second, from there calculation of existence probabilities at each time instant using (22) and (23).

Here the derivation of the probabilities is shown in details.

Hypothesis 1:

$$
\begin{aligned}
p(\mathcal{E}_k \mid y^k) &= p(\mathcal{E}_k \mid y_k, y^{k-1}) \\
&= \frac{p(y_k \mid \mathcal{E}_k, y^{k-1}).p(\mathcal{E}_k \mid y^{k-1})}{\Delta} \\
&= \frac{1}{\Delta}\delta \times [p(E_k,\ldots,E_{k-N},E_{K-N-1} \mid y^{k-1}) \\
&\quad + p(E_k,\ldots,E_{k-N},\bar{E}_{K-N-1} \mid y^{k-1})] \\
&= \frac{1}{\Delta}\delta[p(E_k \mid E_{k-1}).\ldots.p(E_{k-N} \mid E_{k-N-1}).p(E_{k-N-1} \mid y_{k-1})] \\
&\quad + p(E_k \mid E_{k-1}).\ldots.p(E_{k-N} \mid \bar{E}_{k-N-1}).p(\bar{E}_{k-N-1} \mid y_{k-1})] \\
&= \frac{1}{\Delta}\delta \times (\Gamma_{11})^{N+1} \times p(E_{k-N-1} \mid y_{k-1}) \qquad (47)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta = {} & p(y_k \mid \mathcal{E}_k, y^{k-1}).p(\mathcal{E}_k \mid y^{k-1}) \\
& + \sum_{m=0}^{N-1} p(y_k \mid \mathcal{E}_k^m, y^{k-1}).p(\mathcal{E}_k^m \mid y^{k-1}) \\
& + p(y_k \mid \mathcal{E}_k^n, y^{k-1}).p(\mathcal{E}_k^n \mid y^{k-1}). \qquad (48)
\end{aligned}
$$

Hypothesis m:

$$
\begin{aligned}
&p(\mathcal{E}_k^m \mid y^k) \\
&\quad = p(\mathcal{E}_k^m \mid y_k, y^{k-1}) \\
&\quad = \frac{p(y_k \mid \mathcal{E}_k^m, y^{k-1}).p(\mathcal{E}_k^m \mid y^{k-1})}{\Delta} \\
&\quad = \frac{1}{\Delta} p(y_k \mid \bar{E}_k, y^{k-1}) \\
&\quad\quad .p(\bar{E}_k,\ldots,\bar{E}_{k-m},E_{k-m-1},\ldots,E_{k-N} \mid y^{k-1}) \\
&\quad = \frac{1}{\Delta}\left(\frac{1}{V_k}\right)^{m_k} P_0(m_k) \\
&\quad\quad [p(\bar{E}_k,\ldots,\bar{E}_{k-m},E_{k-m-1},\ldots,E_{k-N},E_{k-N-1} \mid y^{k-1}) \\
&\quad\quad + p(\bar{E}_k,\ldots,\bar{E}_{k-m},E_{k-m-1},\ldots,E_{k-N},\bar{E}_{k-N-1} \mid y^{k-1})] \\
&\quad = \frac{1}{\Delta}\left(\frac{1}{V_k}\right)^{m_k} P_0(m_k)(\Gamma_{00})^m.\Gamma_{10}.(\Gamma_{11})^{N-m} p(E_{k-N-1} \mid y^{k-1}). \qquad (49)
\end{aligned}
$$

Hypothesis n:

$$
\begin{aligned}
p(\mathcal{E}_k^n \mid y^k) &= p(\mathcal{E}_k^n \mid y_k, y^{k-1}) \\
&= \frac{p(y_k \mid \mathcal{E}_k^n, y^{k-1}).p(\mathcal{E}_k^n \mid y^{k-1})}{\Delta} \\
&= \frac{1}{\Delta} p(y_k \mid \bar{E}_k, y^{k-1}).p(\bar{E}_k,\ldots,\bar{E}_{k-N} \mid y^{k-1}) \\
&= \frac{1}{\Delta}\left(\frac{1}{V_k}\right)^{m_k} P_0(m_k)[p(\bar{E}_k,\ldots,\bar{E}_{k-N},E_{k-N-1} \mid y^{k-1}) \\
&\quad + p(\bar{E}_k,\ldots,\bar{E}_{k-N},\bar{E}_{k-N-1} \mid y^{k-1})] \\
&= \frac{1}{\Delta}\left(\frac{1}{V_k}\right)^{m_k} P_0(m_k)[\Gamma_{00}^N.\Gamma_{10} p(E_{k-N-1} \mid y^{k-1}) \\
&\quad + \Gamma_{00}^{N+1}(1 - p(E_{k-N-1} \mid y^{k-1}))]. \qquad (50)
\end{aligned}
$$

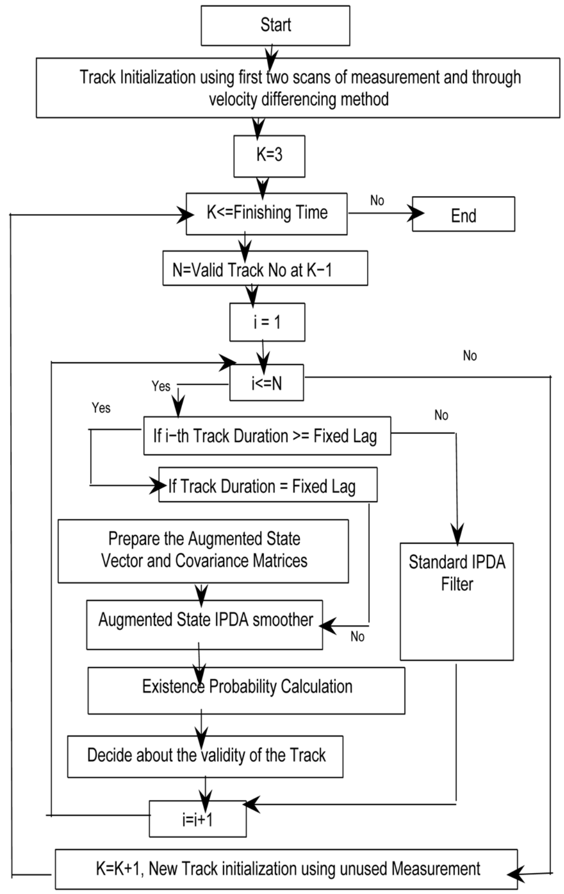

Fig. 1. Flow chart of AS-IPDA smoothing.

Both in Hypothesis m and Hypothesis n, the target does not exist at the current time $t = k$ and so by definition

$$
P(\alpha_0 \mid \bar{E}_k, y^{k-1}) = 1
$$

$$
P(\alpha_i \mid \bar{E}_k, y^{k-1}) = 0 \qquad \text{for} \quad i = 1,2,\ldots,m_k.
$$

Therefore the likelihood,

$$
p(y_k \mid \bar{E}_k, y^{k-1}) = \left(\frac{1}{V_k}\right)^{m_k} \qquad (51)
$$

is used in the derivation of (49) and (50).

Thus (47)–(50) give the probabilities of the augmented existence hypotheses. From these expressions, the track existence probability at each time step (of the entire lag of N) can be obtained by using (22) and (23).

5. ALGORITHM FLOW CHART

In this section the proposed smoothing algorithm is converted into a flow chart for direct implementation. The flow chart is given in Fig. 1.

6. SIMULATION RESULT

Simulations were carried out to investigate the performance of the proposed AS-IPDA smoother with standard IPDA filter. The simulation scenario consists of nonmaneuvering targets moving in a 500 m long and

200 m wide two-dimensional surveillance region. The target dynamic state is assumed to consist of position and velocity in each axis. The state transition matrix is defined as

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where is the sampling period $T = 1$ s.

The sensor receives the position of each target. Hence the state to measurement conversion matrix is defined as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{52}$$

The process noise is zero mean with covariance $E[w(k)w(j)'] = Q$ where

$$Q = q \begin{bmatrix} T^4/4 & T^3/2 & 0 & 0 \\ T^3/2 & T^2 & 0 & 0 \\ 0 & 0 & T^4/4 & T^3/2 \\ 0 & 0 & T^3/2 & T^2 \end{bmatrix}$$

where $q = 0.25$. The sensor introduces an error with variance 3 m in either coordinate axis.

The number of clutter points is generated according to a Poisson distribution with a density of $1.0 \times 10^{-4}/\text{scan/m}^2$. The clutter points are also uniformly distributed within the whole surveillance region. The tracks are initiated by two-point differencing assuming a maximum velocity of 50 m/s. The detection probability is 0.90.

The target existence transition matrix used is

$$\Gamma = \begin{bmatrix} 0.98 & 0.02 \\ 0 & 1 \end{bmatrix}$$

The measurement validation gate threshold is 9, which ensures a gating probability of $P_G = 0.99$. If the existence probability of a track equals or goes above 0.9, i.e

$$p(E_k \mid y^k) \geqslant 0.9$$

the track is moved from tentative to confirmed while if

$$p(E_k \mid y^k) < 0.05$$

the track is terminated.

The track-to-track association threshold is taken to be 0.005. The parameters are tested using 1000 Monte-Carlo run (for true/false track discrimination 400 runs were used) where each time the target reappears with state $[100\text{ m}\ \ 25\text{ m/s}\ \ 100\text{ m}\ \ 5\text{ m/s}]^T$. Also a default fix lag of three was used for the smoother.

6.1 Termination Time Detection

In this simulation scenario the simulation is carried out for 40 scans while the single target is dropped at the 30th scan. The remaining 10 scans consist of only clutter. Two different switching matrices are used for comparison purposes—one with $\Gamma_{11} = 0.98$ and the other with $\Gamma_{11} = 0.90$ where, by definition, $\Gamma_{10} = 1 - \Gamma_{11}$. The results for these two cases are shown in Table I and Table II.

TABLE I
$\Gamma_{11} = 0.98$, Actual Termination Time = 30

Filter Detection	Smoother Detection			
	Lag 1	Lag 2	Lag 3	Lag 4
34	33	32	31	30

TABLE II
$\Gamma_{11} = 0.9$, Actual Termination Time = 30

Filter Detection	Smoother Detection			
	Lag 1	Lag 2	Lag 3	Lag 4
33	32	32	32	31

6.2 Target State Estimation

The filter and smoother are compared in terms of RMS error in position and velocity for both axes. A detection of probability of 0.90 was used. The results are shown in Figs. 2–5.

6.3 True/False Discrimination

For the simulation of the smoothing performance in terms of true/false track detection, the scenario is set as a 400 run of the simulation. A single target reappears at the beginning of each run. The smoother uses a fixed lag of three. At each time instant, the tracks that are updated with true target originated measurements are considered as true tracks. The detected number of confirmed true tracks against time is shown in the Fig. 6. Also the number of false tracks that are confirmed by both algorithms are shown in Fig. 7.

7. CONCLUSION

In this paper, a fixed lag smoother is derived to solve the target existence uncertainty problem in clutter. Besides providing better state estimation, the smoother performs better in distinguishing between true and false tracks as well as determining true targets' termination time compared to a standard filter algorithm. The proposed algorithm extends automatic track initiation into the smoothing type of estimation and establishes the immediate gain in estimation. Applications that take higher level decisions or make strategies shall obtain a great advantage due to such an improvement by using smoothers such as the proposed one.

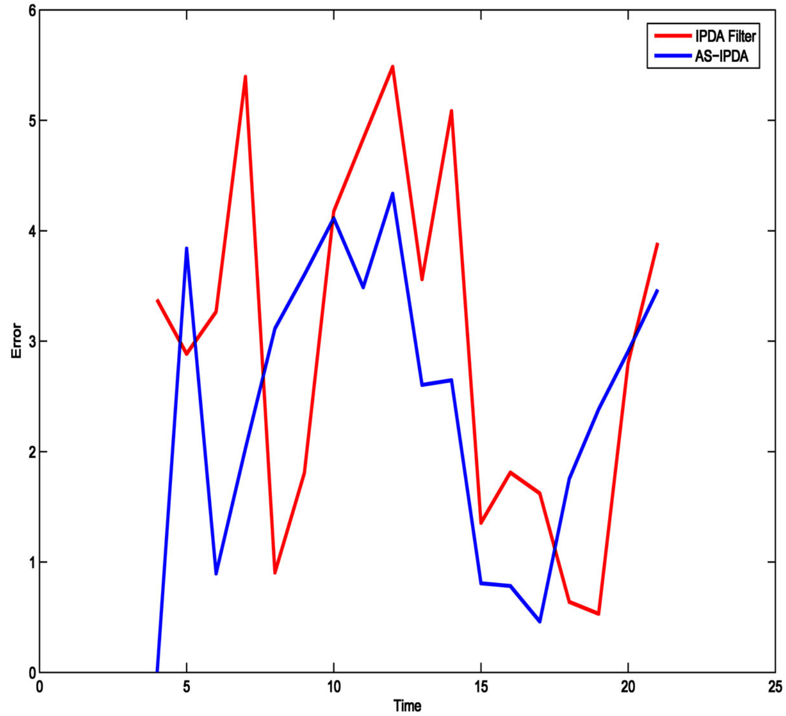

Fig. 2. RMS error comparison of x-direction position using 1000 Monte Carlo runs with detection probability 0.90.

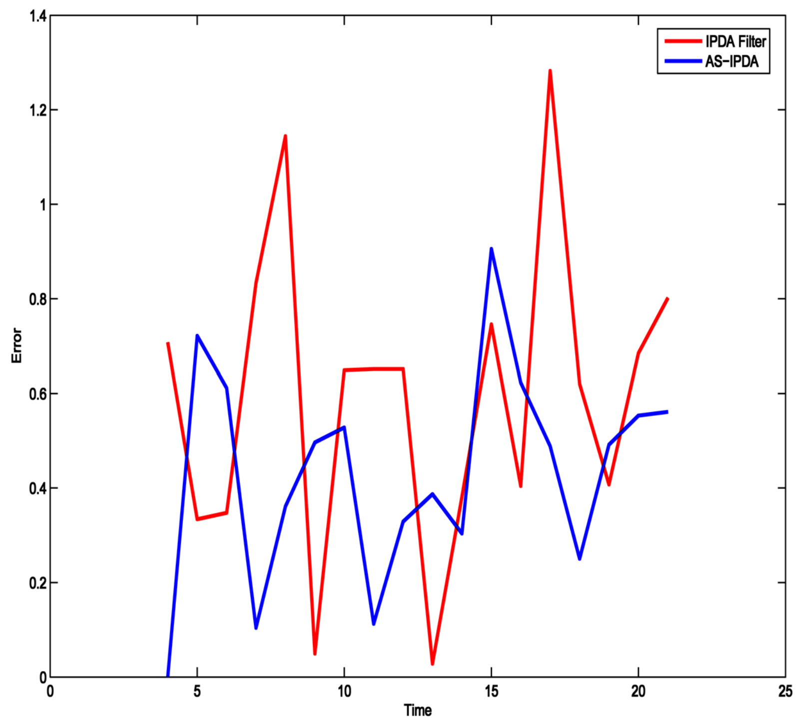

Fig. 3. RMS error comparison of x-direction velocity using 1000 Monte Carlo runs with detection probability 0.90.

APPENDIX. THE HYPOTHESES $\mathbf{H}_k^m$ and $\mathbf{H}_k^n$ DO NOT CONTRIBUTE TO THE STATE UPDATE

$$\begin{aligned} p(\mathcal{X}_k^m \mid \mathcal{E}_k^m, y^k) &= p(\mathcal{X}_k^m \mid \mathcal{E}_k^m, y_k, y^{k-1}) \\ &= \frac{p(y_k \mid \mathcal{X}_k^m, \mathcal{E}_k^m, y^{k-1}).p(\mathcal{X}_k^m \mid \mathcal{E}_k^m, y^{k-1})}{p(y_k \mid \mathcal{E}_k^m, \mathcal{Y}^{k-1})} \\ &= \frac{\text{Likelihood} \times \text{Prediction}}{\text{Normalization}}. \end{aligned} \tag{53}$$

Under the assumption that the target does not exist at time $t = k$, the likelihood term in (53) reduces to,

$$p(y_k \mid \mathcal{X}_k^m, \mathcal{E}_k^m, y^{k-1}) = p(y_k \mid \bar{E}_k, y^{k-1}) = \left(\frac{1}{V_k}\right)^{m_k} P_0(m_k). \tag{54}$$

The prediction term is

$$p(\mathcal{X}_k^m \mid \mathcal{E}_k^m, y^{k-1}) = p(x_{k-m-1}, \ldots, x_{k-N} \mid E_{k-m-1}, \ldots, E_{k-N}, y^{k-1}). \tag{55}$$

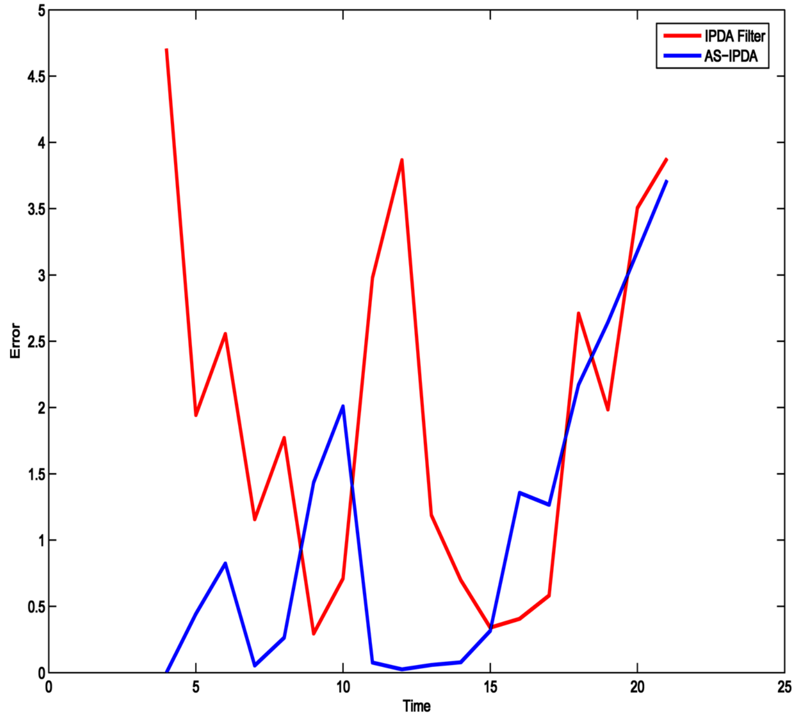

Fig. 4. RMS error comparison of y-direction position using 1000 Monte Carlo runs with detection probability 0.90.

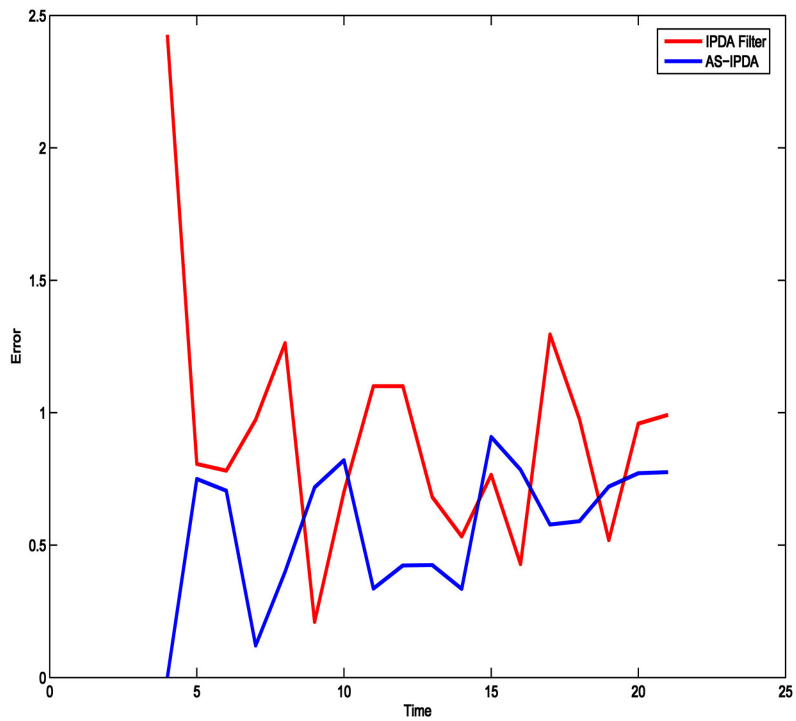

Fig. 5. RMS error comparison of y-direction velocity using 1000 Monte Carlo runs with detection probability 0.90.

Lastly the normalization is

$$p(y_k \mid \mathcal{E}_k^m, y^{k-1}) = \int_{\mathcal{X}_k^m} p(y_k \mid \mathcal{X}_k^m, \mathcal{E}_k^m, y^{k-1}) \times p(\mathcal{X}_k^m, \mathcal{E}_k^m, y^{k-1}))d\mathcal{X}_k^m = \left(\frac{1}{V_k}\right)^{m_k} P_0(m_k). \quad (56)$$

As the likelihood and normalization terms are the same, (53) can be simplified as

$$p(\mathcal{X}_k^m \mid \mathcal{E}_k^m, y^k) = p(x_{k-m-1},\ldots,x_{k-N} \mid E_{k-m-1},\ldots,E_{k-N}, y^{k-1}) \times p(\mathcal{X}_k^m \mid \mathcal{E}_k^m, y^{k-1}). \quad (57)$$

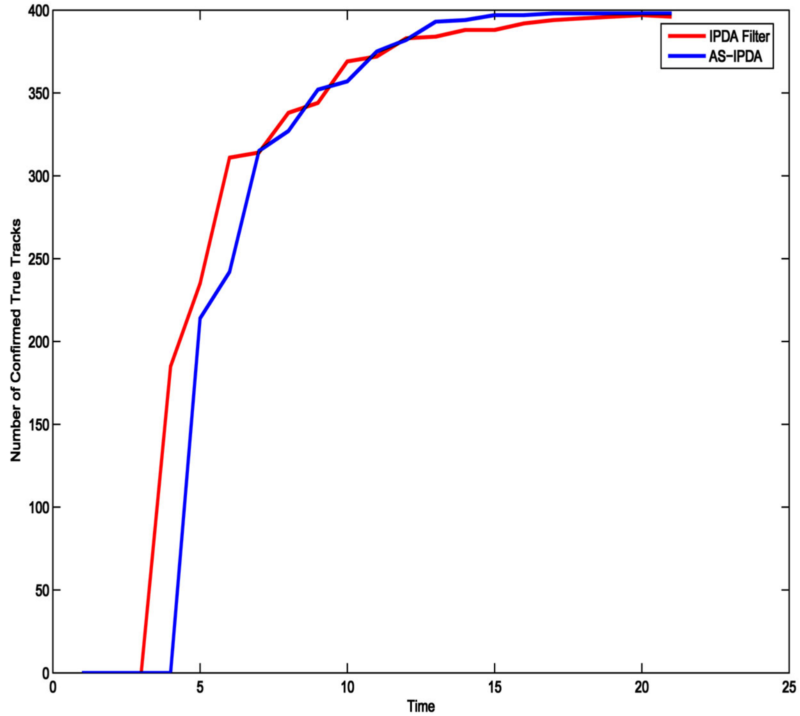

Fig. 6. Number of confirmed true tracks.

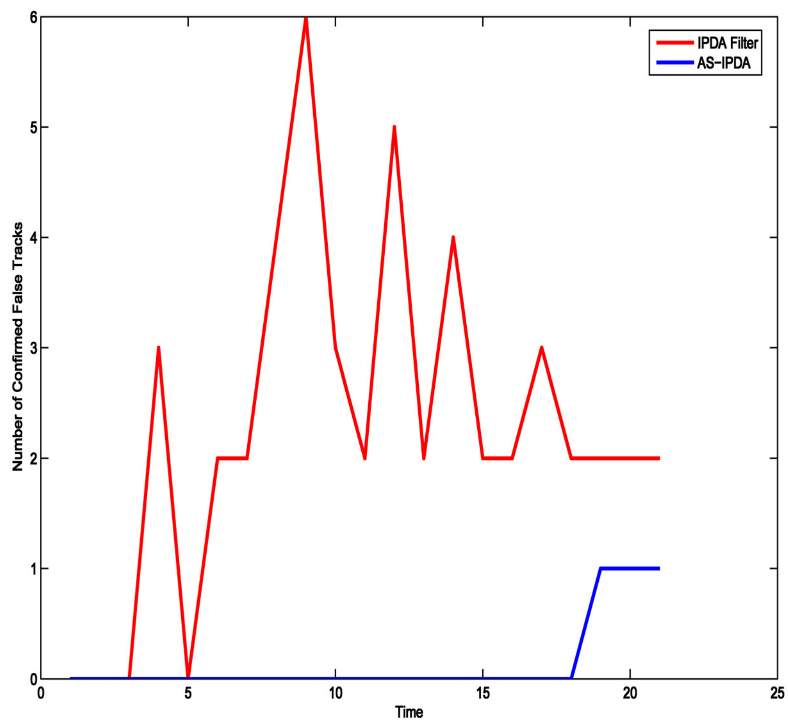

Fig. 7. Number of confirmed false tracks.

Thus (57) shows that the hypotheses $\mathbf{H}_k^m$ and $\mathbf{H}_k^n$, where the target does not exist at current time $t = k$, do not contribute to the update of the state and covariance. So if the target does not exist at current time, the previous smoothed or filtered values are retained as is.

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