Emitter Localization under Multipath Propagation
Using SMC-Intensity Filters

Christoph Degen, Felix Govaers, Wolfgang Koch
SDF Dept.
Fraunhofer FKIE
Wachtberg, Germany
Email: {christoph.degen,felix.govaers,wolfgang.koch}@fkie.fraunhofer.de

Abstract—The passive non-cooperative localization and tracking of mobile terminals in urban scenarios, called blind mobile localization (BML), is a highly demanding task which occurs for instance in safety, emergency and security applications with non-subscribed phone user locations. In this paper it is demonstrated how Sequential Monte Carlo (SMC) implementations of intensity filters (e.g. PHD, iFilter) can be used to solve the tracking and localization task. Novel schemes for extracting state estimates, based on a new generalized grouping of particles-approach, are presented and compared to a mean computation in a single-target scenario. Finally, a ray-tracing simulation is used to demonstrate numerically that an SMC-iFilter using a generalized grouping of particles approach can successfully solve the problem of BML.

I. INTRODUCTION

BML is of highest interest for tasks of civilian security and safety in urban scenarios. Possible fields of application are terror-defense, safety- and emergency scenarios where the object of interest is the location of a non-subscribed phone user. While cooperative scenarios are investigated extensively (see [1], [2]) and sophisticated localization methods exist, BML is hindered by two aspects which complicate the problem. Firstly, “blind” implies that the mobile terminal neither cooperates with the observer station nor with a base station of the cellular network. Therefore, the observer station has to determine the location of the mobile terminal by only inspecting the transmitted electromagnetic waves. Secondly, “urban” implies a densely built-up area. Thus, due to physical propagation effects like reflection, diffraction and scattering the observer station receives multiple signals that have traveled along different multipaths. Existing BML-data fusion algorithms [3], [4] need to be enhanced considering these factors.

Since conventional direction finding approaches use a single direction of arrival (DoA) for the localization, multipaths deteriorate the localization result or make it even impossible to estimate the position of the mobile station. To overcome these limitations ray-tracing simulations can be applied to the problem. Ray-tracers are well established in the field of network planning and the system design of mobile communication systems. Based on a city map, the ray-tracer predicts the set of multipaths and produces a set of DoA and relative time of arrival (RToA) measurements received by the observer station.

In [3] the problem of BML is approached by incorporating the prediction of the ray-tracer into the fusion algorithm. For a given mobile station-observer station geometry the fusion algorithm possesses the measured multipaths as well as the predicted multipaths of the ray tracing simulation. After computing the cost (or distance) matrix of normalized distances between the path-parameters for the two sets of predicted and measured multipaths, a standard assignment algorithm like the Munkres algorithm [5] determines the best match between the two sets. Based on this assignment, likelihood functions for three localization algorithms [3], [4] need to be enhanced considering these factors.
In this paper SMC-implementations of filters that propagate the intensity or Probability Hypothesis Density (PHD) like the iFilter [7] or the PHD-filter [8] are applied to the BML task. Such filters are referred to as SMC-intensity filters in this work. Different definitions of the measurement or observation space of an SMC-intensity filter within the BML framework are presented. Afterwards, a state estimation scheme called grouping of particles [8] is generalized for processing multipath measurements in an SMC-intensity filter. Finally, the generalized grouping of particles approach is compared in a numerical evaluation (w.r.t. accuracy) to a basic state extraction scheme, using a single target scenario and a ray-tracing simulation.

This paper is organized as follows. Section II describes the problem formulation. A definition of the probabilistic likelihood function from [3] is given in II-A. In II-B different observation space definitions are presented. Issues concerning the different definitions, the problem of target state estimation and the connection between observation and target space are discussed in II-C. In section III the generalized grouping of particles is developed. Section IV considers the compared methods for state estimation. It divides into a simple mean computation for a single-target scenario (IV-A) and two newly developed generalized grouping of particles approaches (IV-B,IV-C). A numerical evaluation using the SMC-iFilter can be found in section V, before the conclusions are drawn and future work is presented in section VI.

II. FORMULATION OF THE PROBLEM

The problem of BML is comprised of numerous challenging tasks. This paper focuses on the sub-task of achieving an SMC-implementation of an intensity filter for BML.

A. Likelihood Function

To update the particle weights within an SMC-intensity filter an appropriate likelihood function must be used. A suitable likelihood function is defined in this section, following the considerations in [4] and [3, chapter 4.4]. Let a multipath be defined by its DoA \( \varphi \in [-\pi, \pi] \) and its RToA \( \tau \in [0, \tau_{\text{max}}] \)

\[
\zeta := \begin{bmatrix} \varphi \\ \tau \end{bmatrix}^T,
\]

where \( \tau_{\text{max}} \) denotes the maximal RToA. Let a measured multipath be denoted by \( z^k \), \( k \in \{1,...,K\} \), where \( K \) is the total number of measured multipaths. Then the measurement set is defined by the set of measured multipaths

\[
Z := \{z^k\}_{k=1}^K = \{\eta^k + \nu^k\}_{k=1}^K,
\]

where \( \eta^k \) is the true \( k\)th measured multipath and \( \nu^k := [w^k_\varphi, w^k_\tau]^T \) is the \( k\)th measurement noise, \( w^k_\varphi \sim \mathcal{N}(0, \sigma^2_\varphi) \), \( w^k_\tau \sim \mathcal{N}(0, \sigma^2_\tau) \) and \( \sigma^2_\varphi \) and \( \sigma^2_\tau \) be the noise variances and \( C^k := \text{diag}(\sigma^2_\varphi, \sigma^2_\tau) \) defines the covariance matrix. Let \( \xi := [\xi_x, \xi_y]^T \) be the hypothetical emitter position and let \( h^k, k \in \{1,...,M\} \) be a predicted multipath of the ray-tracing simulation, where \( M \) is the total number of predicted multipaths. Then the set of predicted multipaths with respect to a hypothetical emitter position \( \xi \) and a fixed observer station location is defined by

\[
h_\xi := \{h^k\}_{k=1}^M.
\]

Possessing a set of measured and predicted multipaths, the possible data interpretations are denoted by \( E^K_{i_1,...,i_M} \), where

\[
i_j := \begin{cases} 
0, & \text{no association, measured multipath is not detected} \\
 k \in \{1,...,K\}, & j\text{-th predicted multipath is associated with measured multipath } k 
\end{cases}
\]

is an index for each predicted multipath, which defines its assignment. Let the probability of detection of a specific multipath be denoted by \( P_D \in [0,1] \) and the clutter density by \( \lambda_\Phi \). Let \( n \in \{0,...,\min(M,K)\} \) be the number of assigned multipaths of a specific data interpretation \( E^K_{i_1,...,i_M} \). Then the probabilistic likelihood function is defined by

\[
p(Z|\xi) := \sum_{E^K_{i_1,...,i_M}} P_D^n \cdot (1 - P_D)^{M-n} \cdot \lambda_\Phi^{K-n} \cdot \prod_{j \in I} \mathcal{N}(h^k_j, z^j, C^{i_j}),
\]

where \( I := \{j \in \{1,...,M\} | i_j \neq 0\} \), i.e., the index set of the assigned predicted multipaths. Depending on the number of measured and predicted multipaths the computational effort to sum over all data interpretations can be enormous and grows exponentially. Therefore, the likelihood function has to be approximated according to [3, chapter 4.5.2]: First, an optimal assignment for \( \min(M,K) \) multipath-assignments is computed. Then the assigned pairs are sorted with respect to their normalized distances. The corresponding values of the Gaussian-distribution are stored in descending order in \( \nu \in \mathbb{R}^{\min(M,K) \times 1} \). Then the approximated likelihood function is given by

\[
p(Z|\xi) \approx \sum_{n=1}^{\min(M,K)} P_D^n \cdot (1 - P_D)^{M-n} \cdot \lambda_\Phi^{K-n} \cdot \prod_{j=1}^n \nu_j.
\]
is required to assign the multipaths. Assume such a pre-processing algorithm is available. Then the derived measurement set contains a one-to-one correspondence between the observation and the target space (see figure 2) as it is assumed in the SMC-implementations from [8] and [7]. Therefore, a target state estimation using the conventional grouping of particles approach is convenient and delivers a target state estimate for each measurement, where each measurement is a class of multipaths (if the threshold-test from [8] is passed). Furthermore, the approximation of the probabilistic likelihood function (6) can be used for the update. Hence, the only adaption to be made for applying SMC-intensity filters with observation space 1 to BML is to deliver an appropriate pre-processing algorithm for the target-class assignment of multipaths. Unfortunately, it is not clear to the authors how to do such a pre-processing, since typically the multipaths are not spatially related in the parameter space, which is given by $[-180^\circ, 180^\circ] \times \mathbb{R}^+$ and occur randomly. However, in a single-target scenario with perfect detection and no false alarms, observation space 1 delivers a possibility to easily apply an SMC-intensity filter to BML and can thus be considered as a benchmark for the proposed estimation approaches from IV. Note, that an extended target approach [9] is not appropriate for the problem, since the measurements which belong to a specific target do not possess a spatial relation in the measurement space. Thus, the spatial measurement distribution defined in [9] cannot be defined for the considered problem of BML.

On the contrary observation space 2 does not presume an additional assignment and defines each measured multipath as a single measurement. This definition can be seen as an iterative approach of information processing. Since a mobile station location is represented by a set of multipaths, a single multipath contributes only a part of the complete measurement information to the filter by adjusting the weights via an appropriate likelihood function (see figure 3). Due to do the updating for each multipath separately the following definition of the likelihood function is used for a single multipath.

$$p(z^k|\xi) := \begin{cases} \mathcal{N}(h^k_\xi; z^{\xi}_j, C^{(j)}_\xi), & \text{if } \exists j \in \{1, ..., M\} \text{ such that } i_j = k \\ 0, & \text{otherwise} \end{cases}$$

for each $z^k \in Z, \ k \in \{1, ..., K\}$. The assignment is done via Munkres Algorithm between the set of measured multipaths $Z$ and the set of predicted multipaths $h_\xi$ of $\xi$. Therefore, the index $i_j$ denotes the assigned measured multipath of the $j$th predicted multipath from $h_\xi$. Thus, iteratively updating using (9) with all elements from $Z$ delivers the unlabeled particle set. Due to processing the unsorted multipaths there is no information available for the intensity filter about the relation between the measurements and therefore the ordinary grouping of particles approach delivers a target state estimate for each measured multipath. Hence, for a single target several target states might be estimated. Moreover, these estimates are suboptimal, since a mobile station location is represented by
be seen as a probability of existence. After a threshold-test is passed an estimate for measurement \( k \) is computed by

\[
\hat{y}_k := \frac{1}{\sum_{i=1}^{N} w_{k,i}} \sum_{i=1}^{N} w_{k,i} x_i.
\]

Within an SMC-version of an arbitrary intensity filter that uses observation space 2, this estimation procedure is done for each multipath and therefore might yield several sub-optimal target state estimates for one target (depending on the number of passed threshold-tests). Therefore a generalization of this scheme to sets of measurements for an arbitrary SMC-intensity filter is considered next.

Since a target is represented by a set of measurements the aim is not only to compute the probability that particle \( x_i \) occurred due to measurement \( z^k \) as it is done in the original grouping of particles but to compute the probability that particle \( x_i \) is created due to a specific subset of measurements \( \{z^1, \ldots, z^L\} \subset Z \), \( 0 \leq L \leq K \). This idea is similar to the approach proposed in [3, chapter 7.2.2], where within an MHT-algorithm subsets of measurements are scored and used to estimate the target location. This is visualized in figure 4, where a subset of multipaths delivers one estimate for a target. After knowing how to compute this probability it is easy to assess the different subsets and develop procedures for the BML task that offer one state estimate for one target. This fused state estimate is then more accurate than the single multipath estimates. First consider \( p(x_i|z^1, \ldots, z^L) \). If Bayes Theorem is applied twice and it is assumed that the elements of \( \{z^k\}_{k=1}^{L} \) are statistically independent the following holds

\[
p(x_i|z^1, \ldots, z^L) = \frac{p(z^1|x_i) \cdot p(x_i|z^2, \ldots, z^L)}{p(z^1|z^2, \ldots, z^L)}
\]

and hence

\[
p(x_i|z^1, \ldots, z^L) = \prod_{j=1}^{2} \left( \frac{p(z^j|x_i)}{p(z^j)} \right) \frac{p(z^3|x_i)p(x_i)}{p(z^3)}.
\]

It can then be easily shown by induction (apply Bayes Theorem once in the induction step) that

\[
p(x_i|z^1, \ldots, z^L) = \prod_{j=1}^{L-1} \left( \frac{p(z^j|x_i)}{p(z^j)} \right) \frac{p(z^L|x_i)p(x_i)}{p(z^L)}
\]

holds and thus

\[
p(x_i|z^1, \ldots, z^L) = \prod_{j=1}^{L} \left( \frac{p(z^j|x_i)}{p(z^j)} \right) p(x_i).
\]

The generalized grouping of particles weights can be computed using (18) for each subset of the measurement set. Now, the generalized grouping of particles weight for a particle \( x_i \) and a set of measurements are defined exemplary in terms of the iFilter. First, let \( Z \) be the current measurement set and
consider analogously to [3, chapter 7.2.2] the set of subsets of the set of measured multipaths, i.e.,

\[ \mathcal{J} := \{ J \subset Z \}. \tag{19} \]

Due to the binomial series the number of subsets is given by

\[ \sum_{i=0}^{|Z|} \binom{|Z|}{i} = 2^{|Z|}, \tag{20} \]

where \(| \cdot | : \mathcal{J} \to \mathbb{N}\) determines the cardinality of an element from \(\mathcal{J}\). Hence the number of subsets of \(Z\) grows exponentially with the number of elements of \(Z\).

For an arbitrary subset \(J \in \mathcal{J}\) and particle \(x_i, i \in N\), where \(N\) is the number of persistent particles (present in the previous iteration, i.e., not newborn in the current iteration, see [7]), the generalized grouping of particles weight is defined as

\[ w_{J,i} := \prod_{j=1}^{|J|} \left( \frac{p(z^j|x_i)}{\lambda(z^j)} \right) \cdot p^D(x_i) \cdot w_i, \tag{21} \]

where \(p(z^j|x_i)\) is the single-multipath likelihood function defined in (9), \(\lambda(z^j)\) is the predicted measurement intensity of measurement \(z^j\), \(p^D(x_i)\) is the detection probability and \(w_i\) the weight of particle \(x_i\). To use the proposed scheme within an SMC-PHD implementation \(\lambda(z)\) is simply replaced by \(\mathcal{L}(z)\) (see (25) in [8]) for all \(j \in \{1, \ldots, |J|\}\).

IV. METHODS FOR STATE EXTRACTION

In the following, approaches for state estimation in a single target scenario are presented. Additionally to a basic mean computation two enhanced algorithms, that utilize the generalized grouping of particles equations from the previous section are presented. These state extraction algorithms work in a similar way as the algorithms from [7] and [8] which use the ordinary grouping of particle approach.

A. Mean Computation

Consider a single-target scenario. The easiest method to do a state extraction is to omit the grouping of particles step from [7] and to perform a simple mean value computation of the particle set instead. To compute the desired estimate, the normalized weights after the update of the intensity function are used to determine

\[ \hat{y} = \frac{1}{W} \sum_{i=1}^N w_i \cdot x_i, \tag{22} \]

where \(N\) is the number of particles from the previous iteration (without the newborn particles from the current iteration), \(W := \sum_{i=1}^N w_i\) and \(w_i\) are the weights of the particles \(x_i, i \in 1, \ldots N\).

B. Generalized Mean Computation

When considering a single-target scenario it is implicitly assumed that the obtained set of measured multipaths does not bear any confusion in target assignment, which means that only one (subset of multipaths represents the present target. An approach, which has a low computational complexity is to consider only one subset of the measurement set, namely the set itself. Therefore \(w_{Z,i}\) is investigated for each particle \(x_i\). After computing each grouping of particles weight,

\[ W_Z := \sum_{i=1}^N w_{Z,i} \tag{23} \]

is determined, which can be interpreted as a probability of existence for the target based on the set of received multipaths (see also [7]). Note, that due to its definition \(0 \leq W_Z \leq 1\). The respective proof can be found in the Appendix. If \(W_Z > \tau\) holds, where \(\tau \in [0, 1]\) is a threshold for target existence, the target state estimate is given by

\[ \hat{y} = \frac{1}{\sum_{i=1}^N w_{Z,i}} \sum_{i=1}^N w_{Z,i} \cdot x_i. \tag{24} \]

C. Generalized Ranking Estimation

The second generalized grouping of particles approach presented restricts itself not only to the whole measurement-set but evaluates the grouping of particles weights from (21) for several subsets. Note, that due to the exponential growth of the number of subsets with the number of measurements, a consideration of all subsets is computational feasible only for a restricted number of multipaths. To filter out measurements that occur due to clutter the original grouping of particles scheme is evaluated first. For each measured multipath a probability of existence is determined according to (10) and (11). Then,

\[ \hat{Z} := \{ z^k \in Z | W_k > \tau \}, \tag{25} \]

is defined, where \(\tau \in [0, 1]\). Note, that a determination of a target state estimate could now be done for each measured multipath without any extra computational burden. Afterwards, the measurements are sorted w.r.t. to their probability of existence and saved in descending order in \(Z_{sorted}\). The \(r\)-best are taken out and all possible subsets except for the empty set and the subsets which only contain one element (the corresponding estimates would be the estimates generated by the ordinary grouping of particles) are created , i.e. for all \(J \in \mathcal{J}\)

\[ \mathcal{J}_{fused} := \{ J \subset \{ z^1, \ldots, z^r \} | |J| \neq 0, 1 \} \tag{26} \]

\(w_{J,i}\) is computed. Then,

\[ \mathcal{J}_{fused} := \{ J \subset \{ z^1, \ldots, z^r \} | W_j = \sum_{i=1}^N w_{J,i} > \tau_{fused} \} \tag{27} \]

is defined, where \(\tau_{fused} \in [0, 1]\). Finally, the element of \(\mathcal{J}_{fused}\) with the highest probability of existence represents the target state estimate for the single-target scenario.
Note, that the proposed approach is an approximation of the best representing subset, since only the \( r \)-best (in terms of the probability of existence for the single multipaths) measurements are used for the estimation. An alternative approximation would be to do restrict the cardinality of the considered subsets (see [3, chapter 7.2.3]). For both approximations better results are expected in a single-target scenario without clutter and measurement failure when applying the generalized mean computation from IV-B. This is due to the correct measurement being not known a priori (no clutter, no mis-detection). However, the generalized ranking approach bears several advantages compared to the generalized mean methodology. First, it is obvious that in a real-world scenario clutter and measurement-failure is inevitable. The generalized mean approach therefore incorporates false measurements into its estimation since the measurement set contains false measurements. The generalized ranking on the contrary does not assume to “know” the correct measurement subset, but it determines the best measurement representation for the target by computing the probabilities of existence for a specific number of subsets (the considered number depends on the computational capacity and determines how accurate the approximation of the correct subset is). Hence, it is expected that the generalized ranking approach is more stable and more accurate under the influence of clutter than the generalized mean. Moreover, the generalized ranking approach offers the possibility to extend the presented iFilter-adaption to a multi-target scenario (see VI).

V. Numerical Evaluation

To verify the applicability of the proposed methods a linear single-target scenario is considered in an urban environment. An SMC-iFilter implementation from [7] is used as a representative of the class of SMC-intensity filters. First, a database for a fixed observer station position and a grid of mobile station locations is generated, using a ray-tracing simulation for a specific city-map (see figure 1). Afterwards, a linear ground-truth for a target with constant velocity is created. For each time-step the lower-left grid point of the box, in which the target is located, is determined and the corresponding multipaths from the database are stored as the measurements of the respective iteration. For this evaluation neither clutter nor measurement noise is added to the measurement set and perfect detection of the single multipaths is assumed. Note, that the influence of model-mismatches, i.e., the difference between multipath creation in reality and the synthetic database can only be investigated for real-world measurements which is not in the scope of this paper.

In the following the target is moving linearly with constant velocity \( v = 2.4 \frac{m}{s} \) (see the ground-truth in figure 5). Moreover, the detection probability of the probabilistic likelihood function is set to \( p^D(x) = 0.75 \) and the clutter intensity of the likelihood function is defined as \( \lambda_\phi = 0.001 \). The measurement noise in DoA is set to \( \sigma_\varphi = 10^\circ \) and in RToA to \( \sigma_\tau = \frac{10m}{c_{\text{light}}} \), where \( c_{\text{light}} \) denotes electromagnetic propagation. For each iteration and each multipath the same covariance matrix \( C := \text{diag} \left[ \sigma_\varphi^2 \sigma_\tau^2 \right] \) is used. For the iFilter-implementation the following detection probabilities are defined: The detection probability in \( S \) is set to \( p^D(x) = 0.75 \) for each particle \( x_i \) and the detection probability in \( S_\phi \) is defined as \( p^D(\phi) := 0.3 \). The transition probability from \( S_\phi \) to \( S \) is set to \( \Psi(x|\phi) = 0.2 \), the transition probability in \( S_\phi \) is defined as \( \Psi(\phi|x) = 0.2 \) and the transition probability from \( S \) to \( S_\phi \) is given by \( \Psi(\phi|x) = 0.1 \). The thresholds for target existence of the ordinary and the generalized grouping of particles are set to 0. This is done to make the generalized grouping of particles approaches comparable to the simple mean computation which is implemented without any threshold criterion for target existence. Furthermore, the maximal number of particles is restricted to 1500 and new particles are created uniformly over the considered city-map, where the number of newborn particles is determined according to [7].

To assess the different approaches w.r.t. accuracy 100 Monte-Carlo runs of the presented scenario are performed. The results are shown in figure 6 and 7 in terms of the position and velocity RMSE. For the considered scenario, the ordinary mean computation can be seen as a benchmark for the proposed generalized grouping of particles methodologies, since the measurement conditions are perfect. That is no measurement noise, no clutter and no mis-detection imply that the received set of measurements perfectly represents the target. Therefore, the functionality of the proposed algorithms for state extraction can be verified by comparing the RMSE-values. For most of the times the best result in terms of accuracy is delivered by the generalized mean computation approach which computes the generalized grouping of particles.
weights for the whole measurement set. Analogously to the ordinary mean computation the generalized mean computation profits from the a priori knowledge that the received set of multipaths is correct. Any approximation of this set by leaving out specific multipaths deteriorates the result. Due to the approach of only taking the \( r \)-best measurements into account (in this simulation \( r = 5 \), the maximal number of received multipaths was 20) the set of measurements for the state estimation is permanently weakened in terms of missing true multipaths. Therefore, it is remarkable that even though the set of measurements is reduced, the approach of generalized ranking is capable to deliver almost the same (and for some iterations an even better) accuracy as the ordinary mean computation, that does not suffer from crossing out true measurements. This capability is mainly due to the individual weighting of measurements and sets of multipaths by the predicted measurement intensity in the procedure. This is also why the generalized mean computation delivers the best result of the compared methodologies. However, it is expectable that the influence of clutter and mis-detection delivers a completely different result. Due to the scoring of measurements in the upstreamed ordinary grouping of particles step, clutter measurements should be provided with lower probabilities of existence than the true measurements. Therefore, the generalized ranking approach favors measurements which belong to true targets towards clutter measurements. This should surely improve the performance of the generalized ranking methodology compared to the generalized mean computation and the ordinary mean computation in a cluttered single-target scenario with mis-detection and measurement noise.

VI. CONCLUSION AND FUTURE WORK

In this paper the application of SMC-intensity filters to the problem of BML is presented. First, two methods of defining the observation space are discussed. Due to the absence of an appropriate pre-processing procedure to assign each multipath to a target-class, observation space 2 is used, which defines a measurement to be a multipath. The definition of a measurement as a single multipath creates the problem that an application of the ordinary grouping of particles scheme presented in [7] and [8] does not deliver a single estimate for the target, but one estimate for each multipath. Thus, a generalization of the grouping of particles scheme for sets of measurements is presented, resulting in the generalized grouping of particles weights. Possessing the definition of these weights two approaches for a single-target scenario are formulated and compared in a clutter- and noise-free scenario with perfect detection to a basic mean computation approach. The results of the numerical evaluation show that the proposed approaches deliver comparable (for the generalized ranking) and most of the time even more accurate results (for the generalized mean computation) which motivates the application of intensity filters to more complex BML-scenarios.

Future work will investigate the influence of clutter, measurement noise and mis-detection as well as the applicability of the presented approach to a multi-target scenario, where another level of assignment comes into play. The correct subsets for each target has to be determined additionally to the assignment problem between measured and predicted multipaths, which has to be solved to evaluate the probabilistic likelihood function. The further development of methods that utilize the generalized grouping of particles weights should deliver appropriate algorithms to solve this problem using SMC-versions of intensity filters.

ACKNOWLEDGMENT

The authors would like to thank AWE Communications from Böblingen/Germany for providing a latest version of their ray-tracing simulation “WinProp” to evaluate the developed algorithms.

This work was supported by the Federal Ministry of Education and Research of Germany (BMBF), within the Project EiLT:

**APPENDIX**

The fact that the definition of $W_j$ as a probability of existence makes sense for an arbitrary subset of the measurement set $Z$ is proved exemplary in terms of the SMC-iFilter in the following. Therefore, let $x_i$ be an arbitrary particle, $J \subseteq Z$ and $N$ be the number of particles. Then it obviously holds

$$w_{J,i} := \prod_{j=1}^{[J]} \left( \frac{p(z_j | x_i)}{\lambda(z_j)} \right) \cdot p^D(x_i) \cdot w_i \geq 0,$$

(28)

since each factor of $w_{J,i}$ is $\geq 0$ and thus

$$\sum_{i=1}^{N} w_{J,i} \geq 0.$$  

(29)

Hence it remains to show that $\sum_{i=1}^{N} w_{J,i} \leq 1$. Since

$$\lambda(z_j) := \sum_{i=1}^{N + N_{new}} p(z_j | x_i) p^D(x_i) w_i + p(z_j | \phi) p^D(\phi) f(\phi),$$

(30)

where $p(z_j | \phi) \geq 0$ is the likelihood function for elements of $S_{\phi}$, $p^D(\phi) \geq 0$ the detection probability for elements from $S_{\phi}$ and $f(\phi) \geq 0$ the intensity on $S_{\phi}$ (for details we refer to \cite{7} and \cite{10}). $N_{new}$ is the number of newly created particles. Hence it holds

$$\lambda(z_j) \geq \sum_{i=1}^{N} p(z_j | x_i) p^D(x_i) w_i$$

(31)

and thus

$$\hat{w}_{J,i} := \prod_{j=1}^{[J]} \left( \frac{p(z_j | x_i)}{\sum_{i=1}^{N} p(z_j | x_i) p^D(x_i) w_i} \right) \cdot p^D(x_i) \cdot w_i \geq w_{J,i}.$$  

(32)

(33)

Therefore, it suffices to show that $\sum_{i=1}^{N} \hat{w}_{J,i} \leq 1$, since then $\sum_{i=1}^{N} w_{J,i} \leq \sum_{i=1}^{N} \hat{w}_{J,i} \leq 1$ obviously follows. Consider

$$\sum_{i=1}^{N} \hat{w}_{J,i} = \sum_{i=1}^{N} \prod_{j=1}^{[J]} \left( \frac{p(z_j | x_i) p^D(x_i) w_i}{\sum_{i=1}^{N} p(z_j | x_i) p^D(x_i) w_i} \right)$$

(34)

$$= \sum_{i=1}^{N} \left( \prod_{j=1}^{[J]} \frac{p(z_j | x_i) p^D(x_i) w_i}{\sum_{i=1}^{N} p(z_j | x_i) p^D(x_i) w_i} \right)$$

(35)

$$= \sum_{i=1}^{N} \prod_{j=1}^{[J]} \frac{p(z_j | x_i) p^D(x_i) w_i}{\sum_{j=1}^{N} p(z_j | x_i) p^D(x_i) w_i},$$

(36)

(36) holds since the denominator is independent of index $i$. Hence it remains to show that for arbitrary $J$, $I \in \mathbb{N}$ and $\{x_{i,j}\}_{i \in \{1,..,I\}, j \in \{1,..,J\}}$ with $x_{i,j} \geq 0$

$$\sum_{i=1}^{I} \prod_{j=1}^{J} x_{i,j} \leq \prod_{j=1}^{J} \sum_{i=1}^{I} x_{i,j}$$

holds for all $i \in \{1,..,I\}$, $j \in \{1,..,J\}$. (37) is now proved by induction over $I \in \mathbb{N}$ for an arbitrary $J > 0$. For $I = 1$ (37) is trivial. Assume now that (37) is true for an arbitrary $I > 1$. Then it has to be shown that it also valid for $I + 1$.

$$\sum_{i=1}^{I+1} \prod_{j=1}^{J} x_{i,j} \leq \sum_{j=1}^{J} \sum_{i=1}^{I} x_{i,j} + \sum_{j=1}^{I} x_{I+1,j}$$

(38)

$$\sum_{i=1}^{I} \prod_{j=1}^{J} x_{i,j} \leq \sum_{j=1}^{J} \sum_{i=1}^{I} x_{i,j},$$

(39)

$$\prod_{j=1}^{J} x_{I+1,j},$$

(40)

where (40) is true because $\prod_{j=1}^{J} x_{I+1,j}$ can be written as the product w.r.t. $j$ of the summands with $i = I + 1$ from $\sum_{i=1}^{N} x_{i,j}$. Therefore, it is proven that

$$0 \leq \sum_{i=1}^{N} w_{J,i} \leq 1,$$

(41)

and hence $W_j$ makes sense as a probability of existence.

\[\square\]

**References**


