Track-to-Track Fusion With Missing Information: Empirical Study

Tracking a Variety of Target Types

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Abstract—Copious research has been done comparing various track-to-track methods via their calculated (theoretical) covariance. What has not been studied is how these various track-to-track methods actually perform (i.e., their empirical variance) against a variety of target types. This paper will compare best of breed, centralized measurement fusion, track to track (TTT) without memory (with and without cross correlation), information matrix fusion, and TTT with memory to see which method empirically has the lowest covariance against a variety of targets. These targets are a discrete white noise acceleration, constant velocity, and constant acceleration. This study will further investigate how a track-to-track fusion center handles missing information, particularly estimating the process noise values ($\sigma_a$) for the sensor trackers and what happens to the relative quality of the estimates as the variance assumed at the fusion center is mismatched with those from the sensor trackers.

track-to-track fusion; empirical comparison analysis; covariance consistency; tracking a variety of target types

I. INTRODUCTION

There are a variety of track-to-track (TTT) fusion methods in the literature. Many of these papers focus on the theoretical performance when the assumed target model matches the true target behavior. However, when tracking a variety of targets, how should the track filters be “tuned” to handle different target motion behaviors, and how do these various track-to-track methods stack up against differing targets? It is well-known that under ideal conditions, centralized measurement fusion (CMF) is the best way to combine the data from the various sensors. Namely, all the sensor measurements from the multiple sensors are sent to a central fusion center (CFC) to produce the overall tracking solution. However, due to system requirements, restrictions, or communication constraints, this centralized approach is often quite difficult, or even impossible, to realize. If the assumed process noise and cross-correlation among the sensors trackers [1] is known and available at the CFC, the CMF method can be re-constructed [8] [5]. But this information is not always available at the CFC. For example, often the sensor trackers are already operational, so it is difficult or costly to change them. The data they provide becomes the only data the CFC can work with. Unfortunately, some of these sensor tracking systems do not provide any track information other than the state estimate and covariance. Clearly, if the sensor trackers provided the sensor measurements, they could be used to create the CMF tracking solution at the CFC. The CFC could (optimally) combine the sensor data by ignoring the state estimates from the sensor trackers and just use the measurements from all the sensors to achieve centralized tracking. This approach assumes that the communication bandwidth and persistence are there to ensure the CFC receives all the measurements from all the sensors.

So the question arises of how to perform the track-to-track fusion when the sensor trackers only provide their state estimate and covariance. There are basically two categories of approaches to address this problem at the CFC: 1) estimate the missing information, or 2) treat the missing information as negligible and ignore it. If the estimation of the missing information at the CFC can be done well, it would seem that this would be the preferred approach. However, there is another issue to contend with. The sensor trackers are attempting to track all kinds of targets executing different motion behaviors. Thus, the CFC must deal with the assumptions made by these upstream sensor trackers and fuse the data to account for the different target motions.

This study will look at a few track-to-track fusion methods comparing whether it is better to estimate the missing information or ignore it. The study will determine the actual mean error and variance via Monte Carlo analysis and compare these results to the calculated values. Thus, this study is analyzing the covariance consistency between the theoretical and empirical results. The comparisons are performed against three target motion types: discrete white noise acceleration (DWN), constant velocity, and constant acceleration. For this study, it will be assumed that the communication bandwidth can support receiving all sensor tracker updates. It will also be assumed that there are two identical sensors and that their track updates are time-synchronized.

The track-to-track methods considered for this study fall into three categories: selection (a.k.a., best-of-breed (BoB)), TTT with memory, and TTT without memory. These three methods will be explained in the next section.

It should be noted, however, that for any track-to-track fusion method, the first step is to (correctly) associate the tracks from each sensor tracking system so all the tracks tracking the same target are grouped together. This step can be quite difficult in some scenarios due to the various measurement and association errors that each sensor tracker
brings to its tracking solution. Although this is a crucially important step, it will be ignored in this study. This paper will consider a single target with no false alarms; so perfect association is assumed.

II. THE FILTER METHODS

As mentioned, three categories of TTT filter methods were used for this comparison.

A. Track Selection

The first category is the selection method. With this method, the CFC simply selects the sensor track with the smallest covariance at the update time. This method is also known as best-of-breed (BoB) due to only selecting and using the best track. The main advantage of this method is that since no merging of data occurs, there is no need to account for cross-correlation or any other additional sensor track information at the CFC. But the main disadvantage of BoB is that none of the other sensor tracks are used so this additional information is simply ignored. In this study, it is assumed that both sensors are identical and time-synchronized, so the BoB method was handled by selecting the state estimate from sensor 1 (since they will always have the same covariance).

B. TTT Without Memory

In a TTT without memory (TT~M) method, the tracks from the sensor trackers are combined at the CFC using the state estimate and covariance from each sensor track. The combined estimate only persists for the current update. On the next sensor track update the previous combined estimate is discarded and a new estimate is formed. This method was studied in [2]. This method can be implemented with (TT~M+X) or without (TT~M~X) accounting for the cross-correlation. Calculation of the cross-correlation is given in [1]. The exact algorithm that uses the cross-correlation (TT~M+X) or without (TT~M~X) accounting for the cross-correlation. Calculation of the cross-correlation is given in [8]. In their paper, they assumed the cross-correlation was provided to the CFC and they compared the TT~M+X to a CMF based on the calculated variances. Their theoretical results showed that TT~M+X performs slightly worse than CMF. In this study, the cross-correlation is not provided to the CFC, so it must be estimated to use this method. The other option is to ignore the cross-correlation and use the TT~M~X at the CFC. Thus, this study will compare the effect of estimating the cross-correlation versus ignoring it (i.e., treating it as zero). Furthermore, instead of comparing the calculated variances, this study will compare the empirical variances derived via simulated measurements.

C. TTT With Memory

The last category of TTT methods is TTT with memory (TT+M). This method maintains the combined state estimate and covariance from update to update at the CFC. Like the TT~M method, it can be implemented with (TT+M+X) or without (TT+M~X) accounting for the cross-correlation. Not accounting for the cross-correlation is effectively treating it as zero (or at least, negligible). Both using and ignoring the cross-correlation methods were used in this study. For the case when the cross-correlation is estimated, the information matrix fusion (IMF) method, as provided in [5], was used for this study. Thus, the TT+M+X is really IMF. As pointed out in [8], this method is identical to CMF under full-rate conditions. Since it is assumed that all sensor track updates are reported to the CFC, the full-rate condition is upheld. However, this equivalence to the CMF is valid only if the process noise used by the sensor trackers is available at the CFC (and the true target motion agrees with the motion model assumed). In this study, it is assumed that that process noise information is not available, so it must be estimated. Regardless of which of these two TT+M methods is used, the process noise used by the sensor trackers must be available to the CFC. So again, this study compares which TT+M approach is better when needing to estimate the process noise at the CFC.

III. THE COMPARISON METHOD

For this comparison, it was assumed that there were two sensors trackers. The sensors were assumed to be identical and the track updates time-synchronized. The measurement error for the sensors was \( \sigma_x = \sigma_y = 30 \) m. The measurement sampling rate was \( T = 5 \) s. Each sensor tracker used a 2D (single-model) Kalman filter. The filters were tuned for a discrete white noise acceleration (DWNA) target with an assumed process noise \( q = 4 \) m/s\(^2\). Thus, \( \sigma_a = 2 \) m/s\(^2\). These sensors trackers remained fixed over all analysis.

Three different target motion behaviors were considered: DWNA, constant velocity (CV), and constant acceleration (CA). For all motion models, the target was initially at the origin with a speed of \( s = 15 \) m/s and course of \( \theta = 45 \) deg.

The DWNA target motion model used was as described in [6] and [3]. The (true) process noise of the target was set to agree with the process noise assumed by the sensor trackers, i.e., \( q = 4 \) m/s\(^2\).

For the constant velocity motion, the target continued to move along at its initial course and speed for the entire scenario. The constant acceleration motion had the target continually accelerating at \( a_x = a_y = 2 \) m/s\(^2\) for the entire scenario duration. Since the acceleration was the same in both \( x \) and \( y \), the target maintained the same initial course over the scenario. Note that for the CV and CA target motion, the (true) process noise of the target was zero, but the sensor trackers did not change.

Keep in mind that for this study, there are two process noise parameters; one used by both sensor trackers, and the other assumed by the CFC. Let \( \sigma_a \) be the process noise used by the sensor trackers and \( \tilde{\sigma}_a \) be the process noise used at the CFC. The process noise for the sensor trackers was \( \sigma_a = 2 \) m/s\(^2\). For each target motion type, the empirical results were plotted as a function of the process noise assumed at the CFC, \( \tilde{\sigma}_a \). This process noise was varied from 0.25 - 4 m/s\(^2\), in steps of 0.25 m/s\(^2\).

Fig. 1 compares the calculated variances for all the TTT methods considered in this study. As can be seen in the figure, CMF is not the minimum variance. The TTT methods that do not include the cross-correlation produce the minimum variance, albeit an overly optimistic one. Furthermore, the position and velocity variances for CMF are increasing as \( \tilde{\sigma}_a \) increases, however, the position variance is bounded by the measurement variance while the velocity variance has no bound. As mentioned in [8], when \( \tilde{\sigma}_a = \sigma_a = 2 \), the CMF and
IMF are equivalent. This is shown in the figure with the CMF and IMF (i.e., TT+M+X) intersecting at this sigma value. At the same sigma value, the TT~M+X is slightly less accurate, again in agreement with [8]. Notice that TT+M+X becomes erratic calculating negative variance when $\sigma_a < 1.5$. Since the TT+M+X method is really the IMF method, these negative variances are likely due to the information matrix becoming (nearly) non-invertible when $\sigma_a$ is small (relative to $\sigma_0$). The calculated position variance for the BoB method is not shown in Fig. 1 because it is much higher than the other methods with a constant value of 746.3m$^2$. It is constant because BoB, like TT~M~X, is not dependent on $\sigma_a$. However, the calculated velocity variance for the BoB method is shown. It is less than velocity variance for CMF when $\sigma_a > 2.5$. These results raised suspicion in the consistency of the calculated variance. It is this suspicion that helped motivate this study.

To address the results shown, Monte Carlo analysis was used to compute the empirical (i.e., actual) position and velocity variances for all the TTT methods and then compared to their theoretical (i.e., calculated) variances. The empirical results were based on 10,000 Monte Carlo samples.

To help clarify the results, the analysis was repeated for each of the three target motions. Therefore, there will be a section for each target motion. Keep in mind that the (true) target motion is the only difference among the next three sections. The analysis for all target motions was performed immediately after the 20th track update. The selection of this update was somewhat arbitrary but served as reasonable point where the trackers would be (nearly) at steady state yet remained realistic to an actual TTT scenario, such as maritime surveillance. However, covariance analysis performed at other update times yielded similar results so the results presented in this paper are representative.

### A. Discrete White Noise Acceleration (DWNA) Target

The first target motion type considered is one that can be modeled as discrete white noise acceleration. Fig. 2 plots the position and velocity variances for BoB and CMF, as a function of the process noise assumed at the CFC. Fig. 3 plots the variances for TT~M. The variances for TT+M are plotted in Fig. 4. The solid lines are the empirical (i.e., actual) variances and the dotted lines are the theoretical (i.e., calculated) variances. BoB is independent of the selection of $\sigma_a$. The minor fluctuations seen in Fig. 2 for BoB are due to variations in the random samples. BoB does much worse than any other method as long as $\sigma_a < 1.5$. When $\sigma_a > 1.5$, CMF and TT+M methods break down yielding large variances.

As expected, Fig. 2 shows the CMF empirical and theoretical variances intersect at $\sigma_a = \sigma_0 = 2$. When $\sigma_a < 2$, the calculated variances are much lower than the actual variances for both position and velocity. As $\sigma_a$ increases beyond 2, the empirical and theoretical position variances remain equal and flat while the velocity variances do not. Both the theoretical and empirical velocity variances continue to increase but the actual increase is much slower than what CMF calculates.

![Figure 2 – Empirical and theoretical error variance for CMF and BoB methods against a DWNA target.](image)
Notice no differentiation is made between the empirical variances for TT~M~X and TT~M+X in Fig. 3. This is because they are identical for this problem. They are identical due to the assumption in this study that both sensors are identical with the same process noise parameter and time-synchronized. If these assumptions were altered, the two TT~M empirical variances would be different. Like the CMF method, the empirical and theoretical variances for TT~M+X agree when $\sigma_a = \sigma_a = 2$. The calculated variances for TT~M~X are much smaller than the actual variance. Therefore, for both TT~M methods, it is better to estimate the cross-correlation rather than ignore it when dealing with targets exhibiting DWNA motion.

Like the CMF, Fig. 4 shows the empirical and theoretical variances for TT+M+X agree when $\sigma_a = \sigma_a = 2$. When $\sigma_a > 2$, both TT+M methods calculate variances lower than the actual variance. The calculated variances for TT+M~X are even lower than those calculated for the TT+M+X yet the actual position variance is worse. Here again, it is better to estimate the process noise (and therefore, the cross-correlation) rather than ignore it for targets exhibiting DWNA motion.

The empirical variances from the various track-to-track methods are compared in Fig. 5. BoB was omitted since it was already shown in Fig. 2 to be much worse. Although it may seem that the CMF should always have the lowest variance of all the methods, it only does when $1.5 < \sigma_a < 2.5$. Since the target's true motion is equivalent to $\sigma_a = 2 \text{ m/s}^2$, this would be the proper "tuning" of the filter to achieve its best results. TT~M and TT+M do as well as or better than CMF when $\sigma_a > 2.5$. The velocity variance becomes much larger for CMF than for TT~M and TT+M when $\sigma_a > 3$. 
As a result, for the DWNA target, the analysis shows estimating the cross-correlation is better than ignoring it. Setting $\sigma_a$ too high appears less erroneous than setting it too low.

B. Constant Velocity (CV) Target

The next type of target considered was one moving with constant velocity. All other aspects of the simulation remained the same. The next four figures (Figs. 6 - 9) follow suit with those for the DWNA target just described.

Since the target is moving with constant velocity, its true process noise is zero. This is reflected in the comparison of the velocity variance for CMF in Fig. 6. As $\sigma_a$ increases, the calculated velocity variance becomes much larger than the actual variance. The actual position variance continues to remain slightly smaller than its calculated variance as $\sigma_a$ increases.

Figure 7 shows the actual variances for the TT–M are much lower than its calculated variances when the target is moving with constant velocity. The calculated variance converges at small $\sigma_a$. This time, notice that although it is still too large, the calculated variance from TT–M–X is closer to the actual variance than those from TT–M+X.

The TT+M case shows interesting differences as well. The position variance for TT+M–X is smaller than for TT+M+X (Fig. 8). The calculated position variance for TT+M–X is in close agreement with the actual variance while the calculated position variance for TT+M+X runs larger than its actual variance. The actual velocity variances converge for these two methods as $\sigma_a$ gets large. Like the position variance, the velocity variance for the TT–M–X is smaller than the variance for TT+M+X. Thus, the TT+M method does better when ignoring the cross-correlation (i.e., treating it as zero) against a constant velocity target.

In fact, notice in Fig. 9 that the TT+M~X method performs better than all the other TTT methods, including CMF, against a constant velocity target. Furthermore, TT+M~X performs better than the other methods regardless of how $\sigma_a$ is set. Additionally, as the figure shows, if $\sigma_a$ is set too high, CMF will actually perform the worst of the TTT methods against a constant velocity target. Thus, ignoring the missing information is better than estimating it when dealing with CV targets.

C. Constant Acceleration Target

The final target motion considered is one that is constantly accelerating. As mentioned, the target continually accelerated with $a_x = a_y = 2 \text{ m/s}^2$ for the entire scenario duration. Since the scenario is $(20 - 1) \times 5 = 135 \text{ s}$, this means the target's speed is $135 \times 2 = 270 \text{ m/s}$ faster than when it started. We realize this is somewhat unrealistic but the acceleration was chosen to emphasize the results from this case. As it turns out, the results are very similar to the constant velocity case just discussed. The TT+M–X method performs the best, i.e., has the smallest variance. So again, for CA targets, treating the missing information as zero is better than estimating it.

However, the CA target case does bring up one key difference from the CV case. For a CA target, the state estimate is heavily negatively biased. This makes sense because the acceleration of the target is not being modeled or estimated in the filter. Fig. 10 compares the actual mean state estimate errors for the TTT methods considered. Notice all estimate errors are negative. However, the TT+M=X method, which produces the smallest actual variance, actually produces the largest bias errors in the state estimates. The CMF method, on the other hand, has the smallest mean state error but one of the largest variances of the methods. As seen earlier, the TT+M+X is unstable when $\sigma_a < 1.25$, but actually yields smallest (least negative) mean state error when $1.25 < \sigma_a < 2.0$. Thus, accelerating targets are problematic for all methods.
This study compared various TTT methods in the case that the process noise used in the sensor trackers was not available at the CFC. The intent was to find out if it is better to estimate this missing information or ignore it. The study computed the actual variances using Monte Carlo analysis. The TTT methods were compared against different target motion types. Three motion types were considered: DWNA, CV, and CA. The results showed that there is no clear winner. Each critically depends on the estimation of the process noise and the target motion types that are of most concern. However, the study showed that it is usually better to overestimate the process noise at the CFC than to underestimate it. This was especially true for the TT+M+X method. The study also showed that although CMF is considered the best way to combine the sensor data, it does not always perform the best under certain conditions such as mismatch of process noise or presumed target motion. While it is known that performance degrades when the filter assumptions are mismatched, it was shown that there are cases when CMF produces larger variance than the other TTT methods. For example, smaller variances result for the TT+M~X method against a CV target and the TT~M methods against a DWNA target when \( \hat{\sigma}_a > \sigma_a \).
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REFERENCES


