Multistatic Tracking Using Bistatic Range - Range Rate Measurements

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Abstract – In this paper, we implement a multidimensional assignment based multistatic tracker and test it on a 3D multitarget tracking scenario that includes crossing targets as well as targets moving in formation. We find that the assignment based multistatic tracker can successfully keep tracking initiated tracks accurately. However, efficient track initiation needs further study.

Keywords: Multistatic tracking, passive radar, multistatic radar, multidimensional assignment.

1 Introduction

Although bistatic radar systems have been of interest to researchers since the early days of radar, it is quoted in [10] that the interest in bistatic radar gets renewed approximately every fifteen years. Recently, there has been a new surge of interest in the studies on bistatic radar systems. In particular, passive bistatic radar systems (PBR) that use non-radar transmitters (transmitters of opportunity) as their illumination source have gained much celebration. The promise of cheap and quiet receivers, aspect angle diversity and improved target tracking accuracy could be considered as the main motivators for this resurge of interest [1].

PBR systems described in the open literature generally utilize cheap antenna arrays that provide poor angular resolution. The main observables for such systems are the bistatic range and bistatic range rate of the target of interest. Therefore, using any such system, multilateration is necessary in order to estimate a target’s position. However, considering a realistic multitarget tracking scenario, multilateration posits a hard data association problem. Nevertheless, the open literature on PBR systems is generally oriented towards the issues of detection while neglecting tracking. There is only few published research that directly address the issue of fusing multiple sets of measurements from separate bistatic receiver-transmitter pairs. The pioneering work on the subject [11] addressed the issue of deriving a unique tracking algorithm structure for multistatic radar systems for single target tracking without considering any specific transmission source. The study also assumed that the angle measurement is available along with the range measurement. In [2] the authors used probability hypothesis density (PHD) filter in a 2D tracking scenario for a passive multistatic radar system (PMR) that utilize multiple FM transmitters and a single receiver. In this contribution, our interest lies mainly in 3D tracking. Although conceptually extendable to the three dimension case, the performance of the PHD filter in such a scenario can be a case for a further study. However, as shown in [5], FM signal waveform properties do not allow accurate bistatic range measurements, thus hindering both association and tracking accuracies. In [3] a multi hypothesis filters (MHT) based multistage tracker was used to track a single target in clutter. The MHT algorithm suggested in [3] utilized first a 1D range tracker to suppress clutter measurements, then 1D range tracks were fused to find likely 2D Cartesian target estimates. Finally, likely 2D Cartesian target estimates were fused to get full 3D tracks. The suggested multistage structure of the tracking algorithm was designed to reduce the measurement to target association load. However, as for the detection ranges considered, the number of possible targets might overload even a multistage tracker. Furthermore, for targets that are close to the receiver, the 2D location estimates formed using an expected target height could vary depending on the true target height; thus reducing the 2D to 3D information flow. The illuminators considered in [3] were digital audio broadcast (DAB) and digital video broadcast-terrestrial (DVB-T) networks. Both DAB and DVB-T have good range resolutions. In [4] a particle filter based algorithm that bypasses traditional two stage (plot extraction, tracking) data to tracker flow, and that tracks directly from the received radar signal was suggested. Unfortunately, the details of the algorithm for the multitarget case were not given due to space limitations.

In this study we consider a relatively difficult multitarget tracking in clutter problem with a multistatic radar system where crossing targets and targets flying in formation are present. Such a scenario requires the multidimensional assignment problem to be solved. We

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implement a tracking algorithm to be used with a network of six bistatic receivers observing a hexagonal region. One primary difference of our approach from the first two studies given above is that our radar system is composed of multiple receivers listening to a single transmitter as opposed to a single receiver listening to multiple transmitters. While increasing the receiver cost and requiring synchronization and communication between receivers and a data processing center, this cellular type approach provides spatial separation for distant targets (targets in different cells), thus reducing number of association hypotheses that the assignment problem needs to solve.

The tracker presented in this study is based on multidimensional assignment algorithm [6-7]. Similar to [7], at each scan tracks are associated with a set of measurements that minimize the global track to measurement association cost. Unassociated measurements are then fed to the track initiator module to test for new tracks. It is assumed that probability of detection is high so that at least three measurements are received from each target in the surveillance region, thereby enabling 3D tracking.

This paper is organized as follows: In section 2 our receiver – transmitter geometry is presented. In section 3 the bistatic measurement model is outlined. In section 4 the tracking algorithm is described. In section 5 our simulation experiment is given and finally in section 6 our results will be summarized.

2 Receiver – Transmitter Geometry

In our set up, the illuminator is placed at the origin of the coordinate system. Encircling the illuminator are the six receivers located at the vertices of a hexagon centered at the origin. The side length of the hexagon is taken to be 20 kilometers. This receiver – transmitter geometry is depicted in Figure 1.

![Figure 1. Transmitter (black triangle), receiver (red square) locations](image)

Although the given specific transmitter-receiver geometry has been assumed, the study, with ease, can be extended to the cases where transmitter is located anywhere in the region encircled by the receivers or even where the transmitter is located outside the receiver formation. The location of the transmitter will affect the amount of information contained in an observation that a receiver-transmitter pair has produced [13]. This will in return affect the tracker’s performance. However, the design of the multidimensional assignment tracker is independent of the receiver-transmitter geometry.

3 Measurement Model

Each receiver measures bistatic range and bistatic range rate of a target corrupted by additive white Gaussian noise. Given the state of a target \( X \) at time \( t \) to be defined by a vector consisting of the target’s position and velocity values as shown in Eq. (1), the bistatic measurement of target’s range and range rate for a receiver \( k \) located at \( R_k \) can be found by using Eq. (2-3) \[3\]

\[
X[t] = [x \, y \, z \, \dot{x} \, \dot{y} \, \dot{z}]^T \quad (1)
\]

\[
r_k = \|p\| + \|p - R_k\| + \epsilon \quad (2)
\]

\[
\dot{r}_k = \left[ \frac{p}{\|p\|} + \frac{p - R_k}{\|p - R_k\|} \right]^T \cdot v + \xi \quad (3)
\]

In Eq. (2-3), \( p \) denotes the position and \( v \) denotes the velocity vector of the target \( X \). \( \epsilon \) and \( \xi \) are measurement noises for range and range rate respectively.

It is assumed that receivers in the constellation report their measurements to a data processing center synchronously and registration errors are not modeled.

4 Tracking Algorithm

The block diagram of the tracking algorithm implemented in this study is shown in Figure 2. As shown in figure, the algorithm comprises two main modules, namely the tracker and the track initiator. In the design of the tracking algorithm, it is assumed that measurements collected from each receiver are reported to a data processing center periodically. Furthermore, communication with receivers is assumed to be synchronized.

At the data processing center, received measurements from each time scan are first populated and collected in a measurement list. The measurement list is then presented to a list of confirmed tracks and generic validation gating [9] is applied to determine likely track-measurement combinations. Each measurement combination\footnote{A valid set of measurements constitutes at least 3 detections. When the detection probability of each receiver is taken to be 0.9 and independent detection among receivers is assumed, a target produces at least 3 detections with a probability of 0.9987.} that has passed the ellipsoidal validation gating is filtered with the state estimator and its track score is calculated. The multidimensional assignment algorithm is next applied to determine the track-
measurement combinations that optimize the global track score. The track list is then updated with the optimal association and the associated measurements are dropped from the measurements list. For the tracks that are not associated with a valid measurement set, a track drop iterator is incremented. If the track drop iterator is incremented for four consecutive scans, then the track is dropped. This completes the operation of the tracker module.

The remaining measurements that have not been associated with any of the confirmed tracks are later collected in the unassociated measurement list, whose elements are fed to the track initiator module. At the track initiator module, the unassociated measurement list is firstly presented to a list of tentative tracks [7] and the tracker module is called back to determine tentative track – measurement associations. The tentative tracks that are updated with a valid set of measurements are then appended to the tracks list and their associated measurements are dropped from the measurements list. The unassociated tentative tracks are dropped from the tentative tracks list.

The final measurement list is used to initiate new tentative tracks. In order to reduce the number of optimizations performed in this step, every measurement that will go through further processing is required to have at least one related measurement (range only validation gating) from the previous two scans\(^2\). Maximum likelihood estimates of hypothesized target states using combinations of range only validation gated measurements from each receiver are then computed. In order to further reduce the number of optimizations, each target state estimate is required to have at least four detections\(^3\). A track score is calculated for each hypothesized target and the multidimensional assignment algorithm is used to extract tentative tracks as shown in [6]. This step completes the operation of the track initiator module.

![Tracking algorithm block diagram](image)

**Figure 2. Tracking algorithm block diagram**

\(^2\) The range only validation gating is applied to a copy of full sets of measurements where the associated measurements are not dropped.

\(^3\) At detection probability of 0.9, the requirement will be valid a probability of 0.9842.

### 4.1 Target motion model

The target state is assumed to evolve according to the discrete white noise acceleration model (DWNAM) [8]. The DWNAM state equation is shown in Eq. (4).

\[
X[t + 1] = FX[t] + \Gamma v(t)
\]  

where,

\[
F = \begin{bmatrix}
1 & T & 0 & 0 & 0 \\
0 & 1 & T & 0 & 0 \\
0 & 0 & 1 & T & 0 \\
0 & 0 & 0 & 1 & T \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad \Gamma = \begin{bmatrix}
\frac{T^2}{2} & 0 & 0 \\
T & 0 & 0 \\
0 & \frac{T^2}{2} & 0 \\
0 & T & 0 \\
0 & 0 & \frac{T^2}{2} \\
0 & 0 & 0 & T
\end{bmatrix}
\]

In Eq. 4, \(T\) represents the sampling period and \(v(t)\) is a three dimensional vector of accelerations. The acceleration in each coordinate is assumed to be normally distributed with zero mean and independent from the accelerations for the other coordinates. In DWNAM model, the acceleration vector is assumed to be constant during each sampling period.

### 4.2 The state estimator

Since the measurement equation is a nonlinear function of the target state, a nonlinear tracking filter has to be used. In this study, a first order extended Kalman filter (EKF) is chosen to be the state estimator.

Let \(Z_{r1r2...r6}\) be a sextuple sets of measurements consisting of a measurement set from each receiver associated with a track\(^4\). The EKF measurement update is performed using the batch vector of measurements formed by stacking the nonempty measurement sets given in \(Z_{r1r2...r6}\).

The Jacobian matrix required to linearize the measurement equations for this batch measurement vector can be found by stacking the Jacobian matrices of measurement equations for each receiver that reports a non-empty measurement set. Let \(X[t + 1|t] = [\hat{x} \hat{\dot{x}} \hat{\dot{\dot{x}}} \hat{\dot{\dot{\dot{x}}}}]^T\) be the predicted state for a given target at scan \(t+1\) and let \(R_k = [r_k r_k r_k]^T\) and \(p = [\hat{x} \hat{\dot{x}} \hat{\dot{\dot{x}}}]^T\) represent the position vectors of receiver \(k\) and the predicted target state; then the Jacobian matrix of range and range rate measurement \(z_k = [r_k \dot{r}_k]^T\) equation is given in Eq. (5-9)

\[
J = \begin{bmatrix}
\frac{dr_k}{dx} & \frac{dr_k}{dy} & \frac{dr_k}{dz} & \frac{dr_k}{dx} & \frac{dr_k}{dy} & \frac{dr_k}{dz} \\
\frac{d\dot{r}_k}{dx} & \frac{d\dot{r}_k}{dy} & \frac{d\dot{r}_k}{dz} & \frac{d\dot{r}_k}{dx} & \frac{d\dot{r}_k}{dy} & \frac{d\dot{r}_k}{dz} \\
\end{bmatrix}
\]  

\(^4\) If not empty, each receiver’s measurement set consists of the vector of its measured range and range rate. When no measurement can pass the validation gate test, an empty set is returned by the receiver therefore representing a misdetection.
\[
\frac{dr_k}{d\Psi} = \frac{\Psi}{\|\cdot\|} \frac{\Psi - r_k}{\|\cdot - R_k\|} \quad (6)
\]
\[
\frac{dr_k}{d\Psi} = 0 \quad (7)
\]
\[
\frac{dr_k}{d\Psi} = \Psi \left( \frac{1}{\|\cdot\|} - \frac{\Psi}{\|\cdot - R_k\|^3} + \frac{1}{\|\cdot - R_k\|^3} \right) \quad (8)
\]
\[
\frac{dr_k}{d\Psi} = \frac{dr_k}{d\Psi} \quad (9)
\]

where \( \Psi = x, y \) and \( z \)

### 4.3 Measurement association

Let \( t \) denote the EKF estimate of a track updated by the sextuple \( Z_{r1r2...r6} \), then the association cost of carrying out this update is defined to be the negative of the dimensionless track score function \([6-7]\) which is given in Eq. (10).

\[
c_{r1r2...r6} = -\log \frac{\Lambda(Z_{r1r2...r6}|t)}{\Lambda(Z_{r1r2...r6}|\emptyset)} \quad (10)
\]

Numerator of the track score function gives the likelihood of the hypothesis where every measurement in the set \( Z_{r1r2...r6} \) is originated from track \( t \). This likelihood function formulized Eq. (11).

\[
\Lambda(Z_{r1r2...r6}|t) = \prod_{k=1}^{6} (1 - P_d)^{1 - u(k)} [P_d p(z_k|t)]^{u(k)}
\]

In Eq. (11), \( z_k \) represents the measurements from set \( Z_{r1r2...r6} \) belonging to receiver \( k \), \( P_d \) represents the probability of detection and \( u \) is an indicator function which returns 1 if \( z_k \) is non empty and zero otherwise. \( p(z_k|t) \) can be calculated using the normality of the measurement residual.

Denominator of the track score function gives likelihood of the hypothesis where every measurement in \( Z_{r1r2...r6} \) is unrelated to the track \( t \). Eq. (12) gives this likelihood and \( \Psi_k \) symbolizes the volume of the field of view of receiver \( k \).

\[
\Lambda(Z_{r1r2...r6}|\emptyset) = \prod_{k=1}^{6} \frac{1}{\Psi_k}^{u(k)}
\]

The goal of the track to measurement association algorithm is to find the track to measurement association with minimum total track score such that:

- Each track is associated with at most one measurement from each receiver.
- Each measurement is associated with at most one track.

For the track initiator module, instead of using the estimate derived by the EKF filter update, the track score is calculated by using the maximum likelihood estimate of the target from the measurement set \( Z_{r1r2...r6} \) after which the measurement association for tentative tracks is carried out.

### 4.4 Multidimensional Assignment Solver

It is well known that the cost minimization described above is an NP hard combinatorial optimization problem. In solving this optimization problem, we follow the search algorithm described in Figure 3.

Although there is no guarantee that the search algorithm will terminate in polynomial time, in our experiments it has been generally observed that the lowest cost association includes lower rank elements from the list of sorted measurement sets, thus this divide and conquer strategy is generally efficient. The time required to solve the multidimensional assignment problem is much lower compared to the time required to do the filtering for all possible measurement sets for each track. Similar findings have been reported in [12].

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>For each track:</td>
</tr>
<tr>
<td></td>
<td>- Sort available measurement sets according to their association costs</td>
</tr>
<tr>
<td>2.</td>
<td>Associate every track with its lowest cost measurement set and calculate the lower bound for association cost</td>
</tr>
<tr>
<td>3.</td>
<td>Check the lower bound association for measurement contentions.</td>
</tr>
<tr>
<td></td>
<td>- if no measurement contention is found return this association,</td>
</tr>
<tr>
<td></td>
<td>- else:</td>
</tr>
<tr>
<td>4.</td>
<td>Find the greedy solution and set the return value to the greedy solution</td>
</tr>
<tr>
<td>5.</td>
<td>Prune any measurement set with worse association cost than the greedy solution from each track’s measurement sets</td>
</tr>
<tr>
<td>6.</td>
<td>Divide the region between lower bound cost and the greedy cost to ( k ) levels</td>
</tr>
<tr>
<td>7.</td>
<td>Starting from the minimum cost level, search for valid measurement set combinations within each level.</td>
</tr>
<tr>
<td></td>
<td>- If a valid measurement set is found:</td>
</tr>
<tr>
<td></td>
<td>+ set it as the return value</td>
</tr>
<tr>
<td></td>
<td>+ prune any measurement set with worse association cost from each track’s measurement sets</td>
</tr>
<tr>
<td></td>
<td>+ cancel searching lower cost levels.</td>
</tr>
</tbody>
</table>

Figure 3. Multidimensional assignment algorithm
5 Experiment

In this study we consider a relatively difficult multitarget tracking problem with a multistatic radar system where crossing targets and targets flying in formation are present. Our simulation experiment models a six target scenario inside an observation cell as shown in Figure 4. The first target in the scenario is the control target which is well separated from the other targets. The control target starts its course at an altitude of 9000 meters and moves linearly throughout its course except two maneuvers where it carries out coordinated turns. The first maneuver is a mild 30° lateral turn and the second maneuver is slightly more aggressive lateral turn accompanied by a climb in altitude. The second and the third targets move linearly at 10000 and 10400 meters respectively. They approach and cross each other with a 400 meter vertical separation. The fourth, fifth and the sixth targets move along the same trajectory with a 400 meter separation at the x axis. In the target scenarios, maneuver accelerations were kept to less than 1 g such that a single tracking filter would be adequate to track targets accurately. To track more agile targets, a multiple model estimator may be used.

Measurements are periodically received at the data processing center at every second. Measurement errors are assumed to be normally distributed with zero mean with standard deviations shown in Table 1. The probability of detection is assumed to be 0.9 for all receivers. Each receiver produces an average of 50 false alarms (Poisson distributed) in every scan. False alarms are created uniformly in between [20000, 60000] meters in range and [-500, +500] meters/second in range rate.

Table 1. Measurement error standard deviations (range error deviation is on top row, range rate error on bottom row)

This set up creates a very hard track initiation problem. Although, accurate track initialization was possible, the computational time required to carry out the optimizations to find the maximum likelihood target position estimate for all possible measurement combinations made it infeasible to run the track initiation algorithm at every scan during the whole scenario in our Monte Carlo trials. Thus, for the results given below, the track initiation part of the tracker was turned off and the tracks were initialized with the maximum likelihood estimate for the right measurement combinations. Maximum likelihood estimator’s Cramer-Rao lower bound (CRLB) for the initial estimate was used as the initial covariance. The CRLB derivation can be found in [13]. The maximum likelihood estimates were updated with the EKF for 5 additional scans before being fed to the tracker module.

Rms position and velocity error averages of all targets are shown Figures 5 and 6. As it can be observed from the figures, the initialization estimate errors decline and arrive at a steady state and increase in the last quarter of the scenario. The increase in error is caused by target maneuvers. At scan 120, the first target initiates its climb with lateral turn while targets 4-6 carry out coordinated turns. The effects of target maneuvers on the rms position error can be observed in more detail in Figures 7-8 where rms position error for each target is plotted individually. It can be observed that the jump in error for target 1 is recovered at the end of the maneuver, while the increase in error is permanent for targets 4-6. This is due to false associations and its effect is much more salient for Case III. Another salient feature is the increase in position as well as velocity error for Target 3 at the 76th scan. As shown in Figure 9, this is caused by an outlier at 64th Monte Carlo trial.

![Figure 4. Simulation Scenario](image)

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 m</td>
<td>40 m</td>
<td>40 m</td>
<td></td>
</tr>
<tr>
<td>1 m/s</td>
<td>2 m/s</td>
<td>1 m/s</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 5. rms position error](image)
The mean rms position and velocity error averages of all targets are tabulated in Table 2. It can be observed that better range measurement leads to better position while better range rate measurement leads to better velocity estimation accuracy. Furthermore, the affect of range rate measurement accuracy is more pronounced than the range measurement accuracy for velocity estimation.

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>42.46 m</td>
<td>21.86 m</td>
<td>15.77 m</td>
</tr>
<tr>
<td>Range Rate</td>
<td>3.13 m/s</td>
<td>4.11 m/s</td>
<td>2.31 m/s</td>
</tr>
</tbody>
</table>

Table 2. Scenario mean rms position and velocity estimate errors

The average duration of each Monte Carlo run is tabulated in Table 6. Reduction of range error standard deviation decreases measurement contentions for Targets 4-6 which significantly reduces the run time of each Monte Carlo trial.

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>652 s</td>
<td>171 s</td>
<td>165 s</td>
</tr>
</tbody>
</table>

Table 6. Average Monte Carlo run duration (seconds)

6 Conclusions

In this paper we implemented a multidimensional assignment based tracking algorithm to track multiple targets in a passive radar network. For tracks that are initialized, it was shown through simulations that the algorithm is capable of tracking targets accurately. However, track initialization was inefficient. In order to initialize tracks efficiently, 2D clustering of measurements
as given in [3] may be utilized. Furthermore, 3D clustering of measurements using three receivers from lookup tables without carrying out maximum likelihood target position estimation may be performed. Also, it shall be pointed out that even rough measurement of elevation and azimuth can provide much reduction in track initiators workload.

For further research, realistic network coverage models that utilize SNR maps and varying probability of detection among receivers may be studied. Moreover, the unresolved measurements may be modeled and utilized.

References


