

calibration factors of synthetic aperture radar (SAR) images are generally obtained by estimation of radar system parameters based on internal calibration and external calibration. In this paper, we propose a simple but efficient method to estimate the calibration factors based on statistical modeling of scattering coefficient. Taking expectation and variance on the linear form of calibration equation, we derive the analytical expressions of such estimator. Modeling the scattering coefficient as Rayleigh, heavy-tailed Rayleigh, log-normal, and Weibull distributions, respectively, we obtain the calibration factor estimators for two kinds of radar receivers: the linear receiver and the square-law receiver. Lastly, Monte Carlo simulation results are provided to demonstrate the efficiency of the proposed calibration factor estimator.

**Keywords:** Synthetic aperture radar (SAR) images, scattering coefficient, calibration factor estimation, Monte Carlo simulation.

1 Introduction

The scattering coefficient of synthetic aperture radar (SAR) images is closely related to the physical and electromagnetic characteristics of surface [1, 2]. We can use this to understand the relation of scattering coefficient to the digital number ( DN ) of SAR images (i.e. the pixel value of SAR images). For urban and mountain regions, the double-bounce reflection happens, so the DN has the highest value. For the forest regions, the trees diffusely scatter a lot of radio waves back to radar, but not as much as the double-bounce reflection, so the DN for the forest usually has the higher value but lower than the urban and mountain regions. For rivers and lakes, most of the radio waves are reflected away from the radar, so the scattering coefficient is the smallest, leading to the smallest value of DN . In a word, the DN depends on the illuminating geometry and the nature of the scattering of the radio waves of the targets, and we can interpret SAR images based on this.

Given the specific terrain surface and some imaging conditions, ideally, the DN obtained should correctly reflect the roughness of the practical terrain surface.

However, some errors exist in the process of imaging such as in sensor sub-system, sensor platform, and signal processing sub-system, and therefore the DN may fail to reflect the scattering coefficient exactly. In this case, how to obtain the true value of scattering coefficient from the DN is a problem about radiometric calibration of SAR images [3]. Generally, this problem is solved according to the calibration equation. It should be noted that the calibration equation is demonstrated to show the linear relation between the scattering coefficient (in dB ) and the log-transformed DN , and the two calibration factors are determined by radar system parameters such as the antenna gain pattern, the transmit power, and the sensor-to-target slant range. Generally, as stated in [3], we can use the internal calibration and external calibration to estimate these radar system parameters, and then obtain the two calibration factors.

In this paper, we propose a simple method to estimate the two calibration factors just based on the statistical modeling of scattering coefficient. Taking the expectation and variance on the linear form of calibration equation, we derive the calibration factor estimator with analytical expressions. According to the estimator, the calibration factors can be easily estimated from the DN as long as the statistical model of scattering coefficient is known to us. As experiments, we model the scattering coefficient as Rayleigh distribution and some commonly used non-Rayleigh distributions such as heavy-tailed Rayleigh, log-normal, and Weibull. For each model, we obtain the estimators for the linear receiver and square-law receiver, respectively. Monte Carlo simulation results demonstrate that the proposed estimator is efficient for estimating the calibration factors.

2 Analytical expression of calibration factor estimator

For the radiometric calibration of SAR images, the linear form of calibration equation is demonstrated as [3]

\[ \sigma_{db}^0 = A + B \cdot \log_{10} DN. \] (1)
Here, \( \sigma^0_{\text{dB}} \) is the scattering coefficient in dB, \( \text{DN} \) is the pixel value of SAR images, \( \log_{10} \cdot \) denotes the common logarithm, and \( A \) and \( B \) are the calibration factors to be estimated. Using the Logarithm Base Change, after some manipulation, we can rewrite (1) as

\[
\log \sigma^0 = C + D \cdot \log \text{DN} ,
\]

where \( C = \frac{A \log 10}{10} \), \( D = \frac{B}{10} \), and \( \log \cdot \) denotes the natural logarithm. It must be stressed that the \( \sigma^0 \) in (2) is the scattering coefficient not in dB (i.e. \( \sigma^0_{\text{dB}} = 10 \cdot \log_{10} \sigma^0 \)).

Taking expectation and variance on both sizes of (2), we have

\[
E \left( \log \sigma^0 \right) = C + D \cdot E \left( \log \text{DN} \right) \]

\[
\text{Var} \left( \log \sigma^0 \right) = D^2 \cdot \text{Var} \left( \log \text{DN} \right) .
\]

From (3), we can easily estimate the \( C \) and \( D \). Then, we can estimate the calibration factors \( A \) and \( B \) by

\[
A = 10 \log 10 \left[ E \left( \log \sigma^0 \right) - E \left( \log \text{DN} \right) \cdot \sqrt{\frac{\text{Var} \left( \log \sigma^0 \right)}{\text{Var} \left( \log \text{DN} \right)}} \right]
\]

\[
B = 10 \sqrt{\frac{\text{Var} \left( \log \sigma^0 \right)}{\text{Var} \left( \log \text{DN} \right)}} .
\]

Here, \( E \left( \log \text{DN} \right) \) and \( \text{Var} \left( \log \text{DN} \right) \) can be empirically estimated from the \( M \) observed samples \( \text{DN}_i \) as follows:

\[
\hat{E} \left( \log \text{DN} \right) = \frac{1}{M} \sum_{i=1}^{M} \log \text{DN}_i ,
\]

\[
\hat{\text{Var}} \left( \log \text{DN} \right) = \frac{1}{M-1} \sum_{i=1}^{M} \left( \log \text{DN}_i - \hat{E} \left( \log \text{DN} \right) \right)^2 .
\]

From (4) and (5), the calibration factors \( A \) and \( B \) can be estimated easily if the expectation and variance of the log-transformed scattering coefficient are known.

### 3 Calibration factor estimator with specific model of scattering coefficient

In this section, we describe the scattering coefficient as Rayleigh model and some non-Rayleigh models including heavy-tailed Rayleigh, log-normal, and Weibull. We focus our attention on how to derive the analytical expressions of the expectation and variance of the log-transformed scattering coefficient. Then, we can readily obtain the estimators for each model.

#### 3.1 Rayleigh model

The classical statistical model of scattering coefficient is, of course, the Rayleigh model, which is based on the assumption that the radar resolution cell contains a large number of scatterers and none of the individual scatterers is significantly larger than the others [1, 2]. The probability density function (pdf) of Rayleigh model at the linear receiver output is given by

\[
f \left( x \right) = \frac{x}{2 \gamma} \exp \left( - \frac{x^2}{4 \gamma} \right) ,
\]

where \( \gamma \) is the scale parameter. Then, we can readily obtain the pdf of log-transformed Rayleigh distribution \( X \) \( (X = \log x) \) by

\[
f_X \left( x \right) = \frac{e^{2x}}{2 \gamma} \exp \left( - \frac{e^{2x}}{4 \gamma} \right) .
\]

From (7), we can obtain the following expectation and variance of the log-transformed Rayleigh distribution

\[
E \left( X \right) = E \left( \log \sigma^0 \right) = -\frac{C_e}{2} + \log 2 + \frac{\log \gamma}{2} .
\]

\[
\text{Var} \left( X \right) = \text{Var} \left( \log \sigma^0 \right) = \frac{\pi^2}{24} .
\]

Here, \( C_e \) denotes the Euler’s constant \( (C_e = 0.5772) \).

Substituting (5) and (8) into (4), after some manipulation, we can estimate the calibration factors for the linear receiver by

\[
\hat{A} = 10 \log 10 \left[ -\frac{C_e}{2} + \log 2 + \frac{\log \gamma}{2} - \frac{\pi \hat{E} \left( \log \text{DN} \right)}{\sqrt{24 \hat{\text{Var}} \left( \log \text{DN} \right)}} \right]
\]

\[
\hat{B} = \frac{10 \pi}{\sqrt{24 \hat{\text{Var}} \left( \log \text{DN} \right)}} .
\]

For the square-law receiver, using the square relation of square-law receiver output to linear receiver output, we can obtain the expectation and variance of the log-transformed Rayleigh distribution directly from (8) as follows:
\[ E(X) = E(\log \sigma^0) = -C_\gamma + 2 \log 2 + \log \gamma \]  
\[ Var(X) = Var(\log \sigma^0) = \frac{\pi^2}{6} \] (10)

Then, substituting (5) and (10) into (4), after some manipulation, we can estimate the calibration factors for the square-law receiver by

\[ \hat{A} = \frac{10}{\log 10} \left[ -C_\gamma + 2 \log 2 + \log \gamma - \frac{\pi \hat{E}(\log \text{DN})}{\sqrt{6\hat{\var}(\log \text{DN})}} \right] \]
\[ \hat{B} = \frac{10\pi}{\sqrt{6\hat{\var}(\log \text{DN})}} \] . (11)

### 3.2 Heavy-tailed Rayleigh model

The heavy-tailed Rayleigh model, which is proposed based on the alpha-stable assumption of the real and imaginary parts of scattered signals, is a generalization of the classical Rayleigh distribution [4]. With its heavier tails than the Rayleigh model, the heavy-tailed Rayleigh model is a better choice for modeling the scattering coefficient of high-resolution radar images of urban scenes and some natural scenes [5, 6]. The pdf of heavy-tailed Rayleigh model at the linear receiver output is given by

\[ f(x) = x \int_0^\infty u \exp(-\gamma u^\alpha) J_0(ux) \, du \] , (12)

where \( 0 < \alpha \leq 2 \) is the characteristic exponent, \( \gamma > 0 \) is the scale parameter, and \( J_0(\cdot) \) is the zero-order Bessel function of the first kind. We can, of course, obtain the pdf of the log-transformed heavy-tailed Rayleigh distribution by

\[ f_X(x) = e^{2x} \int_0^\infty u \exp(-\gamma u^\alpha) J_0(ue^x) \, du \] . (13)

However, due to the lack of the compact analytical pdf, it is not easy to obtain the analytical expressions of the expectation and variance of \( X \) from (13). Here, we solve this problem from the moments of scattering coefficient as follows.

Denoting \( x \) as the scattering coefficient for simplification, we can rewrite the \( p \) th order moment \( E(x^p) \) as \( E(e^{pX}) \). Defining a new random variable \( X \) (\( X = \log x \)), we have

\[ E(x^p) = E(e^{pX}) = E(e^{pX}) \] . (14)

Then, expanding \( e^{px} \) into the power series, we can write \( E(e^{px}) \) as

\[ E(e^{px}) = \sum_{k=0}^\infty E(X^k) \frac{p^k}{k!} \] . (15)

From (14) and (15), we can derive the moments of the log-transformed scattering coefficient by

\[ E(X^k) = \left. \frac{d^k}{dp^k} E(e^{px}) \right|_{p \to 0} \] . (16)

As introduced in [5], the \( p \) th order moment of heavy-tailed Rayleigh distribution for the linear receiver can be written as

\[ E(x^p) = 2^{p+1} \Gamma(p/2+1) \frac{\gamma^p / \alpha \Gamma(-p/\alpha)}{\alpha \Gamma(-p/2)} \] , \( -2 < p < -1/2 \) . (17)

From (16) and (17), after some manipulation, we can obtain the analytical expressions of the expectation and variance of the log-transformed heavy-tailed Rayleigh distribution by

\[ E(X) = E(\log \sigma^0) = C_\gamma \left( \frac{1}{\alpha} - 1 \right) + \frac{\log \gamma}{\alpha} + \log 2 \]  
\[ Var(X) = Var(\log \sigma^0) = \frac{\pi^2}{6\alpha^2} \] . (18)

Substituting (5) and (18) into (4), we can estimate the calibration factors for the linear receiver by

\[ \hat{A} = \frac{10}{\log 10} \left[ C_\gamma \left( \frac{1}{\alpha} - 1 \right) + \frac{\log \gamma}{\alpha} + \log 2 - \frac{\pi \hat{E}(\log \text{DN})}{\alpha \sqrt{6\hat{\var}(\log \text{DN})}} \right] \]
\[ \hat{B} = \frac{10\pi}{\alpha \sqrt{6\hat{\var}(\log \text{DN})}} \] . (19)

For the square-law receiver, similar to the case of Rayleigh model, using the square relation of square-law receiver output to linear receiver output, we can obtain the expectation and variance of the log-transformed heavy-tailed Rayleigh distribution directly from (18) as follows:
\[ E(X) = E(\log \sigma^0) = 2C \left( \frac{1}{\alpha} - 1 \right) + \frac{2 \log \gamma}{\alpha} + 2 \log 2 \]
\[ \text{Var}(X) = \text{Var}(\log \sigma^0) = \frac{2 \pi^2}{3 \alpha^2} \]  

Then, substituting (5) and (20) into (4), we can estimate the calibration factors for the square-law receiver by

\[ \hat{A} = \frac{10}{\log 10} \left[ 2C \left( \frac{1}{\alpha} - 1 \right) + \frac{2 \log \gamma}{\alpha} + 2 \log 2 - \frac{\pi \sqrt{2} \hat{E}(\log \text{DN})}{\sqrt{\hat{\text{Var}}(\log \text{DN})}} \right] \]
\[ \hat{B} = \frac{10 \pi \sqrt{2}}{\alpha \sqrt{3 \hat{\text{Var}}(\log \text{DN})}} \]  

### 3.3 Log-normal model

The log-normal is another model with the capability of describing the heavy tail of scattering coefficient, so it is commonly used to model the high resolution sea clutter and the built-up areas [2, 7]. The pdf of log-normal model at the square-law receiver output is given by

\[ f(x) = \frac{1}{\sqrt{2\pi V} x} \exp \left( -\frac{(\log x - \beta)^2}{2V} \right) \]  

where \( \beta \) is the scale parameter, and \( V \) is the shape parameter. Then, we can readily obtain the pdf of the log-transformed log-normal distribution \( X \) by

\[ f_X(x) = \frac{1}{\sqrt{2\pi V}} \exp \left( -\frac{(x - \beta)^2}{2V} \right) \]  

Obviously, the distribution above is just the Gaussian distribution. So we can obtain the expectation and variance of the log-transformed log-normal distribution by

\[ E(X) = E(\log \sigma^0) = \beta \]
\[ \text{Var}(X) = \text{Var}(\log \sigma^0) = V \]  

Then, substituting (5) and (24) into (4), we can estimate the calibration factors for the square-law receiver by

\[ \hat{A} = \frac{10}{\log 10} \left[ \beta - \sqrt{V \hat{E}(\log \text{DN})} \right] \]
\[ \hat{B} = \frac{10 \sqrt{V}}{\sqrt{\hat{\text{Var}}(\log \text{DN})}} \]  

### 3.4 Weibull model

The Weibull model contains the Rayleigh as a special case, and its modeling performance is approaching the log-normal if appropriate parameters are selected [2, 7]. The pdf of Weibull model for the linear receiver is given by

\[ f(x) = \frac{c x^{c-1}}{b^c} \exp \left( -\frac{x}{b} \right)^c \]  

where \( b \) is the scale parameter, and \( c \) is the shape parameter. Similar to the case of heavy-tailed Rayleigh model, it is not easy to obtain the analytical expressions of the expectation and variance of the log-transformed Weibull distribution from the pdf of the log-transformed Weibull distribution, so we use (16) to solve this problem. The \( p \) th order moment of Weibull distribution for the linear receiver is given by

\[ E(x^p) = b^p T(1 + p/c) \]  

From (16) and (29), after some manipulation, we can obtain the analytical expressions of the expectation and variance of the log-transformed Weibull distribution as follows:
\[ E(X) = E(\log \sigma^0) = -\frac{C_e}{c} + \log b \] (30)

\[ \text{Var}(X) = \text{Var}(\log \sigma^0) = \frac{\pi^2}{6c^2} \]

Then, substituting (5) and (30) into (4), we can estimate the calibration factors for the linear receiver by

\[ \hat{A} = \frac{10}{\log 10} \left[ \log b - \frac{C_e}{c} - \frac{\pi \hat{E}(\log \text{DN})}{c \sqrt{6 \hat{\text{Var}}(\log \text{DN})}} \right] \] (31)

\[ \hat{B} = \frac{10\pi}{c \sqrt{6 \hat{\text{Var}}(\log \text{DN})}} \]

For the square-law receiver, similarly, we can obtain the expectation and variance of the log-transformed Weibull distribution directly from (30) as follows:

\[ E(X) = E(\log \sigma^0) = -\frac{2C_e}{c} + 2 \log b \] (32)

\[ \text{Var}(X) = \text{Var}(\log \sigma^0) = \frac{2\pi^2}{3c^3} \]

Then, substituting (5) and (32) into (4), we can estimate the calibration factors for the square-law receiver by

\[ \hat{A} = \frac{10}{\log 10} \left[ 2 \log b - \frac{2C_e}{c} - \frac{\pi \sqrt{2} \hat{E}(\log \text{DN})}{c \sqrt{3 \hat{\text{Var}}(\log \text{DN})}} \right] \]

\[ \hat{B} = \frac{10\pi \sqrt{2}}{c \sqrt{3 \hat{\text{Var}}(\log \text{DN})}} \] (33)

### 4 Monte Carlo simulation

One of the important problems for the Monte Carlo simulation is how to generate the samples for the scattering coefficient with specific distributions. Using the method presented in [8], we generate the samples with the heavy-tailed Rayleigh distribution. Since the logarithmic transformation of log-normal distribution is Gaussian distribution, we can generate the samples with log-normal distribution directly from the exponential transformation of the corresponding Gaussian samples. Using the method proposed in [9], we can generate the Weibull-distributed samples.

Each Monte Carlo simulation is repeated 100 times independently, and the number of samples is 10000 for each run. The true calibration factors \( A = 5 \) and \( B = 10 \). The average and standard deviation values (in parentheses) of Monte Carlo simulation results are provided at the same time. Modeling the scattering coefficient as Rayleigh, heavy-tailed Rayleigh, log-normal, and Weibull distributions, the calibration factor estimation results are shown in Table 1, Table 2, Table 3, and Table 4, respectively. Obviously, for the two kinds of radar receiver (linear receiver and square-law receiver), the proposed calibration factor estimator can lead to high estimation accuracy no matter what the values are chosen for the model parameters of scattering coefficient.

| Table 1 Calibration factor estimation results of Rayleigh model |
|----------------------------------|-----------------|-----------------|-----------------|-----------------|
| Receiver Type                   | \( \gamma = 1 \) | \( \gamma = 2 \) |
| \( \hat{A} \)                   | 4.9982          | 5.0015          |
| \( \hat{B} \)                   | 9.9801          | 10.0023         |
| \( \hat{A} \)                   | (0.0305)        | (0.0237)        |
| \( \hat{B} \)                   | (0.0930)        | (0.0905)        |

| Table 2 Calibration factor estimation results of heavy-tailed Rayleigh model |
|----------------------------------|-----------------|-----------------|-----------------|-----------------|
| Receiver Type                   | \( \alpha = 0.5 \) | \( \gamma = 1 \) | \( \alpha = 1 \) | \( \gamma = 1.5 \) |
| \( \hat{A} \)                   | 5.0147          | 4.9947          |
| \( \hat{B} \)                   | 10.0085         | 10.0056         |
| \( \hat{A} \)                   | (0.1088)        | (0.0555)        |
| \( \hat{B} \)                   | (0.1123)        | (0.1037)        |

| Table 3 Calibration factor estimation results of log-normal model |
|----------------------------------|-----------------|-----------------|-----------------|-----------------|
| Receiver Type                   | \( \beta = -2 \) | \( V = 1 \) | \( \beta = 2 \) | \( V = 3 \) |
| \( \hat{A} \)                   | 4.9983          | 5.0046          |
| \( \hat{B} \)                   | 10.0006         | 10.0039         |
| \( \hat{A} \)                   | (0.0759)        | (0.0413)        |
| \( \hat{B} \)                   | (0.0811)        | (0.0804)        |

| Table 4 Calibration factor estimation results of Weibull model |
|----------------------------------|-----------------|-----------------|-----------------|-----------------|
| Receiver Type                   | \( \beta = -2 \) | \( V = 1 \) | \( \beta = 2 \) | \( V = 3 \) |
| \( \hat{A} \)                   | 4.9971          | 4.9956          |
| \( \hat{B} \)                   | 9.9937          | 10.0042         |
| \( \hat{A} \)                   | (0.1014)        | (0.0823)        |
| \( \hat{B} \)                   | (0.0637)        | (0.0665)        |
Table 4 Calibration factor estimation results of Weibull model

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5 Conclusions

According to the internal calibration and external calibration, we can estimate the radar system parameters, and then obtain the calibration factors of SAR images. In this paper, we propose a simple method to estimate the calibration factors just based on the statistical modeling of scattering coefficient. Taking the expectation and variance on the linear form of calibration equation, we derive the analytical expressions of such estimator. The only condition for using this estimator is that the statistical model of scattering coefficient is known to us. We model the scattering coefficient as Rayleigh distribution and some commonly used non-Rayleigh distributions, and we focus our attention on how to derive the analytical expressions of the expectation and variance of the log-transformed scattering coefficient. Monte Carlo simulation results demonstrate that the proposed estimator with simple expressions is efficient for estimating the calibration factors from the DN.

In the future research, we should estimate the calibration factors when the scattering coefficient is modeled by other non-Rayleigh distributions such as K model and Ricean model. And, most importantly, we should demonstrate the efficiency of the proposed estimator according to the real SAR image experiments, and finally achieve the radiometric calibration of real images.

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