**Track Management and PMHT**

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**Abstract**—PMHT algorithm, as proposed, promises high performance multi target tracking in clutter with (relatively) modest computational resources. However, when applied to practical target tracking situations, a number of problems need to be overcome. PMHT assumes fixed number of tracks, and furthermore it assumes that all tracks are true tracks. No track quality measure is provided within PMHT to enable false track discrimination. In this paper we add the probability of target existence to PMHT processing, and accommodate a variable number of tracks.

**Keywords:** PMHT, multi target tracking, data association, probability of target existence, false track discrimination, clutter.

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**I. INTRODUCTION**

In many radar, sonar and other target tracking applications measurements may originate from targets, whose existence and trajectories are generally not known *a priori*. These target detections are only present in a scan with some probability of detection, $P_D < 1$. In addition, detections are generated from other random sources, which are usually termed clutter.

In each scan a number of candidate measurements exist for track update. A measurement history is a sequence of measurements, one per each scan. Each such sequence will result in a target trajectory state estimate. The number of measurement histories grows exponentially in time, thus various techniques are used to control their number [1], this submission does not detail them. One way of dealing with complexity, is to reduce the probability density function (pdf) of each track to a single Gaussian. The Probabilistic Multi–Hypothesis Tracker (PMHT) is one such algorithm.

Automatic track initiation and maintenance in the cluttered environment results in existence of both true and false tracks. True track follows a target, and false track does not. A false track can be created when initiating a track using one or more clutter measurements, or a true track may become false due to clutter measurements, or due to an unfavourable target detection sequence, or both. False track discrimination is a necessary procedure which tries to recognize and confirm true tracks, and recognize and terminate false tracks.

PMHT is initially presented in [2]. It is a multi–pass batch algorithm, which uses Expectation Maximization method (EM) [3], [4] to estimate state of tracks in uncertain data association situations. Track state estimates, as well as data association probabilities, are iteratively improved as the algorithm traverses data forward and backward. The algorithm has a great potential, most notably it has computational complexity which grows only linearly in the number of tracks and the number of measurements (although the proportionality constant is rather steep, due to multiple iterations necessary for the algorithm to converge), in certain situations it has excellent multi–target capabilities. PMHT also has weak points. It tends to lose targets more readily then Probabilistic Data Association [5] based algorithms. Recognizing its potential, a number of authors have continued working on PMHT, thus a number of PMHT variants have emerged, some of them presented in [6], [7] and references therein.

Lack of an intrinsic track quality measure is also an issue, preventing a more widespread use of PMHT. Some authors [8] use statistical hypothesis testing outside PMHT to determine whether a track is true or false. Target visibility is an approach published in [9], [10]. All possible combinations of track existences are combined into PMHT. The additional complexity factor is exponential, to be more precise equal to two to the power of the number of potential tracks, effectively rendering it useless in all but the most benign environments.

In this paper we use the data association probabilities already calculated in PMHT to derive an expression for the probability of target existence. The additional computational load is linear and not significant, compared to the base PMHT computational requirements. This removes the false track discrimination obstacle for practical PMHT applications.

Target existence is modeled as a Markov Chain. Two models for target existence have been identified and introduced in [11]. In one, termed Markov Chain One, if the target exists, it is assumed to be always detectable, i.e. its measurements will exist in any given scan with a probability of detection $P_D \leq 1$. The other, termed Markov Chain Two, also allows for the possibility that the target exists, but is temporarily not detectable. For reasons of clarity, in this paper we do not devote much space to the propagation of the target existence, however this is trivially applied to the target existence propagation within PMHT as well.

An additional improvement, essentially decoupled from the probability of target existence, are improved expressions for effective measurements used to update PMHT tracks.

The problem statement and some common notation are presented in Section II. The original PMHT [2] is presented in Section III. In Section IV the equations to update the probabili-
ity of target existence using PMHT are presented. Automatic target tracking with PMHT is discussed in Section V. The concluding remarks are presented in Section VI.

II. PROBLEM STATEMENT

New targets may enter the surveillance area, and old targets may either disappear or leave the surveillance area. For each new target, a new track must be initialized and confirmed as soon as possible, and each track which is, or has become, a false track, should be terminated as soon as possible. Tentative tracks should be resolved into confirmed or terminated as soon as practical.

Without a loss of generality, we assume here a linear system. Target trajectory update at time \( t + 1 \) may be modeled as

\[
x_{t+1} = Fx_t + Gv_t, \tag{1}
\]

where \( v_t \) is a sequence of zero mean, white Gaussian noise samples, with covariance matrix \( Q \). Targets are detected with the probability of detection \( P_D \), which may be different for different targets. If the target is detected, the sensor adds measurement noise, thus the measurement equation becomes

\[
y_t = Hx_t + \mu_t, \tag{2}
\]

where \( \mu_t \) is a sequence of zero mean, white Gaussian noise samples, with covariance matrix \( R \) and uncorrelated with any element of sequence \( v_t \). Although \( R \) may differ from measurement to measurement, we assume constant \( R \) in this paper for reasons of clarity and without loss of generality.

Additionally, clutter measurements may be present at each sampling time, in an unknown number. Without loss of generality, and for reasons of clarity, we assume here uniform clutter measurement density, denoted here by \( \rho \).

In each scan \( t \), a measurement set \( z_t \) of \( n_t \) measurements is received, where \( z_{t,r} \) denotes the \( r \)-th measurement of \( z_t \). Given that there are \( M \) tracks, there is no \textit{a priori} data association indication linking measurement to tracks.

III. PMHT

Here we present the “vanilla” PMHT, as initially presented in [2]. Processing of only one batch of data is considered.

The Expectation Maximization method (EM) [3], [4], is a multi–pass batch algorithm which has been applied to estimate the parameters \( \Theta \) of finite mixture distributions based on “complete” or “non–categorized” data \( Z \). In other words, EM seeks to maximize the probability density function \( p(\Theta | Z) \) by performing the following iterative maximization,

\[
\hat{\Theta}^{(i+1)} = \arg \max_{\Theta} E_{\text{missing data}} \left\{ \ln p(\text{complete data}, \Theta) | Z, \hat{\Theta}^{(i)} \right\}, \tag{3}
\]

where \( E \) denotes expectation, and \( i \) denotes the iteration pass. The “complete data” is the union of the incomplete data (observations), and the “missing data”, in the case of PMHT are the data association pointers. Over multiple batch iterations, PMHT optimizes the data association probabilities and estimates the parameters \( \Theta \), which in this case are the trajectory estimates.

State of each track \( m \) consists of

- the state estimate, denoted here by \( x_{t,m}^{(i)} \), where \( i \) denotes the iteration pass, \( t \) denotes the time sequence (scan) index, and
- the probability \( \pi_{t,m}^{(i)} \) of each measurement at time \( t \) being the detection of track \( m \),

where \( t = 0, \ldots, T, \ m = 1, \ldots, M \). The constraint on values of \( \pi_{t,m}^{(i)} \) is

\[
\sum_{m=1}^{M} \pi_{t,m}^{(i)} = 1 \tag{4}
\]

The state of each track \( m \) is initialized by values for iteration \( 0 \), chosen appropriately.

Each iteration pass \( i + 1 \) starts from time \( t = 0 \) to \( t = T \) (the forward pass). The probability that measurement \( r \) is the detection of track \( m \) at time \( t \) is calculated as

\[
\omega_{m,t,r}^{(i+1)} = \frac{\pi_{t,m}^{(i)} N(z_{t,r}; Hx_{t,m}^{(i)}, R)}{\sum_{s=1}^{M} \pi_{t,s}^{(i)} N(z_{t,r}; Hx_{t,s}^{(i)}, R)}, \tag{5}
\]

where \( N(z; m, R) \) denotes the normal (Gaussian) pdf of variable \( z \), with mean \( m \) and covariance \( R \). Please note here that the probabilities \( \omega_{m,t,r}^{(i+1)} \) are assumed independent within the track \( r \) and not mutually exclusive, i.e. \( \sum_{r=1}^{T} \omega_{m,t,r}^{(i+1)} \leq 1 \). In other words, PMHT relaxes the mutual exclusivity constraint usually enforced by other target tracking algorithms. Updated values of \( \pi_{t,m}^{(i+1)} \) are given by

\[
\pi_{t,m}^{(i+1)} = \frac{1}{n_t} \sum_{r=1}^{n_t} \omega_{m,t,r}^{(i+1)} \tag{6}
\]

Equivalent measurement for track \( m \) at time \( t \) is defined by its mean

\[
z_{t,m}^{(i+1)} = \frac{\sum_{r=1}^{n_t} \omega_{m,t,r}^{(i+1)} z_{t,r}}{\sum_{r=1}^{n_t} \omega_{m,t,r}^{(i+1)}} \tag{7}
\]

and covariance

\[
\hat{R}_{t,m}^{(i+1)} = \frac{R}{\sum_{r=1}^{n_t} \omega_{m,t,r}^{(i+1)}} \tag{8}
\]

Track \( m \) is updated using the measurement sequence \( z_{t,m}^{(i+1)} \), \( \hat{R}_{t,m}^{(i+1)} \), \( t = 1, \ldots, T \), applied to Kalman filter in the forward direction from, \( t = 1, \ldots, T \). The forward Kalman filtering is initialized with the mean and covariance at time 0

\[
\hat{y}_{0|0} = \bar{x}_{0,m}, \quad P_{0|0} = P_{0,m} \tag{9}
\]

with the forward recursion defined by

\[
P_{t+1|t} = FP_{t|t}F^T + GQG^T
\]

\[
K_{t+1} = P_{t+1|t}H^T \left( HP_{t+1|t}H^T + \hat{R}_{t+1|m} \right)^{-1}
\]

\[
P_{t+1|t+1} = (I - K_{t+1}H) P_{t+1|t}
\]

\[
\hat{y}_{t+1|t+1} = F\hat{y}_{t|t} + K_{t+1} \left( z_{t+1}^{(i+1)} - HF\hat{y}_{t|t} \right)
\]

\[
x_{t,m}^{(i+1)} = \hat{y}_{T,T}
\]
where $I$ denotes the identity matrix. The smoothing Kalman filtering operation in the backward direction, $t = T - 1, \ldots, 0$, is given by

$$
x_{t,m}^{(i+1)} = \hat{y}_{t|t} + P_{t|t} F_{t+1|t} x_{t+1,m}^{(i)} - F \hat{y}_{t|t}.
$$

After the EM iterations have converged, the final smoothing Kalman filtering operation also updates the estimated covariance matrices by

$$
P_{t,m} = P_{t|T} = P_{t|t} + \left( P_{t|t} F_{t+1|t} P_{t+1|t}^{-1} \right) T.
$$

### IV. PMHT WITH TARGET EXISTENCE

In the presence of substantial clutter, in a large surveillance area, it is advantageous to have prior probabilities that track $m$ originates a measurement $(\pi_{t,m}^{(i)}$ in PMHT), depend also on the measurement. It is not “reasonable” that this probability should be the same for measurements near the predicted track position, and for the “far away” measurements. In a manner of Linear Multitarget [12], these probabilities are as

$$
\omega_{m,t,r}^{(i+1)} = \frac{P_{(i+1)} m,t,r}{\sum_{s=1}^{n_t} P_{m,t,s}^{(i)}}
$$

where

- $\chi_{t|t-1,m}^{(i)}$ is the predicted target at time $m$ at time $t$ and iteration $(i)$,
- $P_{m,t,r}$ is the measurement $r$, track $m$ innovations pdf equal to $\mathcal{N}(z_{t,r} ; z_{t,m}^{(i)}, S_{t,m}^{(i)})$, where
- $z_{t,m}^{(i)} = H x_{t-1,m}^{(i)}$ is the initial predicted measurement position of track $m$ at time $t$, and
- $S_{t,m}^{(i)} = H P_{t-1,m}^{(i)} H^T + R$ is the error covariance matrix of $z_{t,m}^{(i)}$,
- $p_{m,t,r}^{(i)}$, $i > 0$ is the measurement $r$ pdf given target $m$ trajectory state $x_{t,m}^{(i)}$ equal to $\mathcal{N}(z_{t,r} ; H x_{t,m}^{(i)}, R)$

In effect, prior probability that one measurement originates from track $m$ at time $t$, $P_{x_{t|m}}^{(i)}$, is distributed among the measurements. In the PMHT forward pass, the probability that measurement $r$ is the detection of track $m$ at time $t$ is iteratively calculated as

$$
\omega_{m,t,r}^{(i+1)} = \frac{\pi_{m,t,r}^{(i)} \prod_{t=1}^{M} \left( 1 - \pi_{m,t,r}^{(i)} \right) p_{m,t,r}^{(i)}}{\sum_{u=1}^{M} \pi_{u,t,r}^{(i)} \prod_{t=1}^{M} \left( 1 - \pi_{u,t,r}^{(i)} \right) + \sum_{u=1}^{M} \pi_{u,t,r}^{(i)} p_{m,t,r}^{(i)}}
$$

Then, in the manner of [13], we massage the data association probabilities $\omega_{m,t,r}^{(i+1)}$ calculated by equation (14) to obtain mutually exclusive data association probabilities within each track, for each iteration $(i > 0)$. The probability that none of the measurements is the track $m$ detection, given that there can be up to only one detection from (a possible) target followed by track $m$, at time $t$ is given by

$$
P_{m,t,0}^{(i+1)} = C_{m,t}^{-1} \prod_{s=1}^{n_t} \left( 1 - \omega_{m,t,s}^{(i+1)} \right).
$$

In the same manner, the conditional probability that measurement $r = 1, \ldots, n_t$ is the target $m$ detection, is given by

$$
P_{m,t,r}^{(i+1)} = C_{m,t}^{-1} \omega_{m,t,r}^{(i+1)} \prod_{s=1}^{n_t} \left( 1 - \omega_{m,t,s}^{(i+1)} \right)
$$

Constant $C_{m,t}^{-1}$ is determined by utilizing

$$
\sum_{r=0}^{n_t} P_{m,t,r}^{(i+1)} = 1
$$

finally delivering data association probabilities for track $m$ at time $t$

$$
P_{m,t,0}^{(i+1)} = \frac{1}{1 + \sum_{u=1}^{n_t} \omega_{m,t,u}^{(i+1)}}
$$

$$
P_{m,t,r}^{(i+1)} = \frac{\omega_{m,t,r}^{(i+1)}}{1 + \sum_{u=1}^{n_t} \omega_{m,t,u}^{(i+1)}}
$$

Events that measurement $r$ is the detection of target $m$, with data association probabilities $P_{m,t,r}^{(i+1)}$, $r > 0$, imply
target existence. Event that none of the measurements is the detection of target \( t \), with the data association probability of \( P_{\chi \chi}(i+1) \) is split in target existence and non-detection, and non–target existence, [14]. Given the a priori probability of target existence for track \( m \) at time \( t \), iteration \((i + 1)\), is \( \chi \chi(i+1,m) \). A posteriori probability of target existence update is given by

\[
\chi \chi(i+1,m) = P_{\chi \chi}(i+1) \frac{(1 - P_D) \chi \chi(i+1,m) + \sum_{s=1}^{n} P_{\chi \chi}(i+1,s)}{1 - P_D \chi \chi(i+1,m)} + 1 - P_{\chi \chi}(i+1) \tag{19}
\]

Predicted probability of target existence, \( \chi \chi(i+1,m) \) in equation (19) is obtained by propagating the probability of target existence \( \chi \chi(i-1,m) \) as Markov Chain [11].

Each new track is given an initial probability of target existence, and the simplest choice is using a constant value for all new tracks.

Using the a posteriori probability of target existence, we can calculate data association probabilities conditioned on target existence

\[
P_{\chi \chi}(i+1) = \frac{P_{\chi \chi}(i+1)}{P_{\chi \chi}(i+1,m) \chi \chi(i+1,m)} \tag{20}
\]

The proper way to implement the forward recursion, or to estimate \( \chi \chi(i+1,m) \), is to use Probabilistic Data Association [5] using data association probabilities calculated by equation (20). Alternatively, in the spirit of original PMHT, if we define the equivalent measurement and its measurement noise covariance matrix, originally defined as in equations (7) and (8), as

\[
\chi \chi(i+1) = \frac{1}{1 - \beta(i+1)} \sum_{r=1}^{n} \beta(i+1) \chi \chi(i+1,m) \chi \chi(i+1,m) \chi \chi(i+1,m)
\]

and then implement the Kalman filter as in equation (10), we will obtain correct forward state estimation \( \chi \chi(i+1,m) \), with somewhat optimistic forward state estimation error covariance matrix \( P_{\chi \chi}(i+1,m) \).

V. AUTOMATIC TARGET TRACKING WITH PMHT

Automatic target tracking usually implies track initiation using sensor measurements, as well as the false track discrimination procedure where false tracks are recognized and terminated, and true tracks are recognized and confirmed. Therefore, the total number of existing tracks vary from one scan to the other.

The original PMHT, as proposed in [2], proposed fixed number of tracks, denoted here by \( M \). It also did not provide a track quality measure, to be used in the false track discrimination procedure.

However, the original PMHT did not preclude having a variable number of tracks in each time \( t \), \( M = M(t) \). Having a variable number of tracks in each time \( t \) of the batch will just complicate the indexing in Section III. Each track which is initialized in the middle of the batch, or terminated in the middle of the batch, (or both), would have different “equivalent” batch starting and ending times. For these tracks the batch recursion would be altered in an obvious manner, not detailed here for reasons of brevity and clarity.

Probability of target existence is often used in a simple manner to facilitate the false track discrimination [11]. Each new track is given a tentative status, and an initial probability of target existence. If the probability of target existence of a tentative track \( m \) reaches a confirmation threshold, track \( m \) gets confirmed. If the probability of target existence of a track \( m \) reaches a termination threshold, track \( m \) gets terminated.

We can apply the same policy here. Probability of target existence is updated in each PMHT batch iteration, as detailed in Section IV. Confirmation is (obviously) best performed once the PMHT iterations have converged. (At least) three possible ways to deal with the track termination can be envisaged, enabling the designer to chose between false track discrimination efficiency and the required computational resources. Terminating the track once the PMHT iterations have converged will obviously produce the best results, at the expense of highest computational resource requirement. Terminating the track and removing it completely from the batch in the first iteration that the probability of target existence breached the termination threshold is most computationally efficient, although producing the worst false track discrimination outcome. The third option is to terminate the track, but not removing it from the batch (just updating the end time of the track as discussed above) in the first iteration where the probability of target existence breached the termination threshold has both false track discrimination performance and the computational requirements between the two extremes described above. The numbers involved depend on the clutter measurement density, which translates in the average number of false tracks initiated at each scan.

VI. CONCLUSIONS

In this paper we extend PMHT algorithm for multiple target tracking in clutter. Automatic target tracking implies initializing tracks using radar measurements. It implies the existence of both true tracks, and the false tracks. New tracks are constantly initialized, and false tracks are detected and terminated, therefore a variable number of tracks must be accommodated. Automatic target tracking is facilitated by calculating the probability of target existence, as well as
using a variable number of tracks in PMHT batch processing. The probability of target existence is updated in each PMHT recursion.

The probability of target existence can be used as the track quality measure to confirm true tracks, and to terminate false tracks (false track discrimination). Three possible ways to deal with the terminated tracks are presented, enabling the designer to choose between false track discrimination efficiency and the required computational resources.

The probability of target existence also improves upon PMHT calculations, as the calculated probabilities of false track data associations include the low probability of target existence.

**REFERENCES**


