Sensor Fusion Enhancement via Optimized Stochastic Resonance at Local Sensors

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Abstract—This paper considers the decentralized fusion problem involving local sensor detection as well as the fusion of decisions transmitted over non-ideal transmission channels in a wireless sensor network. Prime emphasis is given to the enhancement of several fusion rules using a recently developed stochastic resonance methodology applied at the local sensors. Further, it is shown that the optimal form of the stochastic resonance probability mass density for the decentralized sensor fusion problem retains the same form as that previously developed for the single sensor case.

Keywords: Sensor fusion, decentralized detection, Rayleigh fading channels, stochastic resonance

I. INTRODUCTION

Advances in wireless communications and networking have led to considerable research interest in wireless sensor networking. A wireless sensor network (WSN) often contains a large number of sensors with power and communication bandwidth restrictions among the sensors as well as the fusion center. Therefore, communication is usually limited to that of simple binary decisions. Each sensor node processes the local observation data set, makes a local decision regarding the presence or absence of a signal, and passes the decision to the fusion center via a wireless channel. For systems with high signal-to-noise ratio (SNR) and/or effective channel error correction coding, the channels between local sensors and the fusion center can often be assumed lossless. However, in applications in which these conditions are not applicable, such assumptions can be seriously misleading.

Accounting for non-ideal transmission channels between local sensors and the fusion center, channel aware signal processing for the distributed detection problem has been developed in [1]–[3]. The optimal local decision rule was shown to be a monotone likelihood ratio partition of its observation space, provided the observations were conditionally independent across the sensors.

The impact of the non-ideal transmission channels to the fusion rule design has also been addressed [4]–[6]. In [4], optimum tests and corresponding thresholds were derived, but required global system knowledge. The system model adopted in [5] included the Rayleigh fading channel and based upon this model, an optimal likelihood ratio (LR)-based fusion algorithm was derived. This test, however, requires the maximum amount of system information, including instantaneous channel state information (CSI) and local sensor detection performance indices. Although the maximum ratio combiner (MRC) and the Chair-Varshney [7] fusion rules were shown to be near optimal for the cases of low-channel and high-channel SNR, respectively, the equal gain combiner (EGC) method was shown to outperform both of these fusion rules for practical SNR values. In [6], the likelihood ratio based on channel statistics (LRT-CS) was derived and shown to perform well as compared to the optimal LR fusion rule. The test eliminates the need for instantaneous CSI, but still requires knowledge of the channel statistics as well as the performance indices of the local sensors.

Most channel aware local decision rule designs assume that the designer has some knowledge of the channel characteristics, either instantaneous CSI or channel statistics. Therefore, the local decision rules need to be synchronously updated with the change of channel characteristics. However, in some cases, the local sensor may not have the ability to change its detector corresponding to changes of channel characteristics, i.e., the local detector is fixed. In this paper, by adopting a recently derived optimization procedure developed within the stochastic resonance (SR) detection methodology for the Neyman-Pearson [8]–[13] as well as the Bayesian [12]–[14] framework, we apply SR to the local sensors to improve the performance of WSN with fixed local detectors. SR is a phenomenon that occurs in some non-linear systems where the signals can be enhanced by adding suitable noise under certain conditions. The SR effect was first introduced by Benzi et al. in [15] and has been observed and applied in numerous nonlinear systems [16]–[20]. In the signal detection area, SR has also been successfully applied to improve the signal detectability.

For the single sensor case, it has been shown that the optimal SR process to be added to the data input at the detector has a pdf consisting of two Kronecker delta functions each occurring with probability λ and (1 − λ), respectively, for the Neyman-Pearson (NP) framework. For the Bayesian case, it has been shown to be a single delta function with unit probability; i.e., a constant. Here, we show that these pdf forms also apply to the distributed sensor fusion case.

The paper is organized as follows. Section II contains the
problem description. Section III addresses the enhancement procedure via stochastic resonance while performance results are presented in Section IV. Finally, we present conclusions and future considerations in Section V.

II. STATEMENT OF THE PROBLEM

In this paper, we consider the problem of testing two hypotheses, denoted by \( H_0 \) and \( H_1 \). A total number of \( K \) sensors are used to collect observations \( X_k \), for \( k = 1, \cdots, K \). We assume throughout this paper that the observations are conditionally independent, i.e.,

\[
p(X_1, \cdots, X_K|H_i) = \prod_{k=1}^{K} p(X_k|H_i), \quad i = 0, 1. \quad (1)
\]

Upon observing \( X_k \), each local sensor makes a binary decision using a fixed non-parametric, suboptimal detector,

\[
U_k = \gamma_k(X_k) \quad k = 1, \cdots, K.
\]

The decisions \( U_k \) are sent to a fusion center through parallel transmission channels characterized by

\[
p(Y_1, \cdots, Y_K|U_1, \cdots, U_K) = \prod_{k=1}^{K} p(Y_k|U_k). \quad (2)
\]

Thus, from (2), the channels are orthogonal to each other, which can be achieved through, for example, partitioning time, frequency or combinations thereof. The fusion center takes the channel output \( \{Y_1, \cdots, Y_K\} \) and makes a final decision \( U_0 \in \{H_0, H_1\} \).

\[
U_0 = \gamma_0(Y_1, \cdots, Y_K). \quad (3)
\]

As shown in Fig. 1, prior to processing at the local sensors, we may add independent SR noises to the local observations \( \{X_1, \cdots, X_K\} \) and obtain a new set of observations \( \{Z_1, \cdots, Z_K\} \) where

\[
Z_k = X_k + \nu_k, \quad k = 1, \cdots, K, \quad (4)
\]

and SR noise \( \nu_k \) is either an independent random signal with probability density function (pdf) \( p_{\nu}(\cdot) \) or a nonrandom signal. Thus, the local detector will be applied to the new observations to obtain the local decisions,

\[
U_k = \gamma_k(Z_k) \quad k = 1, \cdots, K.
\]

Our goal is to find the form of optimal SR noise pdf \( p_{\nu}(\cdot) \) such that the best performance is obtained at the fusion center.

III. FORM OF OPTIMAL SR NOISE PDF

After adding independent SR noise to the local observations, we have, for \( k = 1, \cdots, K \), the pdf for the new observations,

\[
p_{Z_k}(Z_k) = p_{X_k}(X_k) * p_{\nu_k}(\nu_k) = \int p_{\nu_k}(\nu_k)p_{X_k}(Z_k - \nu_k)d\nu_k
\]

Therefore, the local detection characteristics are

\[
P_{D,k}^{Z} = P(U_k = 1|H_1) = \int \gamma_k(Z_k)p_{Z_k}(Z_k|H_1)dZ_k
\]

\[
= \int p_{\nu_k}(\nu_k) \left( \int \gamma_k(Z_k)p_{X_k}(Z_k - \nu_k|H_1)dZ_k \right) d\nu_k
\]

\[
= E_{\nu_k}(F_{1,\gamma_k}(\nu_k)) \quad (5)
\]

where

\[
F_{i,\gamma_k}(\nu_k) = \int \gamma_k(Z_k)p_{X_k}(Z_k - \nu_k|H_i)dZ_k, \quad i = 0, 1.
\]

Similarly, we have

\[
P_{F,k}^{Z} = P(U_k = 1|H_0) = E_{\nu_k}(F_{0,\gamma_k}(\nu_k)). \quad (6)
\]

In the following, we adopt a person-by-person optimization (PBPO) approach to find the form of optimal SR noise pdf,
i.e., we optimize the SR noise for the jth sensor given fixed SR noise at all other sensors and a given fusion rule. The PBPO approach is almost invariably used for finding the local optimum in the distributed detection systems [2], [3], [21].

A. Neyman-Pearson Detection

Under the NP criterion, the overall probability of detection at the fusion center can be expressed as

$$ P_D = \int_y \gamma_0(y)p(y|H_1)dy $$

$$ = \int_y \gamma_0(y) \prod_{k=1}^{K} [p(Y_k|U_k = 1)P_{D,k}^Z + p(Y_k|U_k = -1)(1 - P_{D,k}^Z)] dy $$

Similarly, we can also expand the probability of false alarm at the fusion center with respect to the SR noise at the jth local sensor such that

$$ P_{F,J} = E_{\nu_j}(G_1(\nu_j)) $$

where, for $i = 0, 1$,

$$ G_i(\nu_j) = \int_y \gamma_0(y) [p(Y_j|U_j = 1) - p(Y_j|U_j = -1)] F_{1,\gamma_e}(\nu_j) $$

$$ \prod_{k=1, k\neq j}^{K} \left[ p(Y_k|U_k = 1) - p(Y_k|U_k = -1) \right] E_{\nu_k}(F_{1,\gamma_k}(\nu_k)) dy $$

Similarly, we can also expand the probability of false alarm at the fusion center with respect to the SR noise at the jth local sensor which yields

$$ P_{F,A} = E_{\nu_j}(G_0(\nu_j)). $$

Therefore, following the same discussion as in [9], we can easily show the following result for the form of optimal SR noise at the fusion center

$$ p_\nu^{NP}(\nu) = \lambda \delta(\nu - c_1) + (1 - \lambda) \delta(\nu - c_2) $$

where $0 \leq \lambda \leq 1$, $\delta(\cdot)$ is the kronecker delta function and $c_1, c_2$ are two constants which can be determined as in [12].

B. Bayesian Detection

Under the Bayesian criterion, we assume the prior probability for hypotheses $H_0$ and $H_1$ are $\pi_0$ and $\pi_1$, respectively. Thus, the overall probability of error at the fusion center can be written as

$$ P_e = \pi_0 P(U_0 = 1|H_0) + \pi_1 P(U_0 = 0|H_1) $$

$$ = \pi_1 + \int_y \gamma_0(y) \pi_0 p(y|H_0) - \pi_1 p(y|H_1) dy $$

$$ = \pi_1 + \pi_0 P_{F,A} - \pi_1 P_D. $$

As in Section III-A, we can further expand the error probability $P_e$ with respect to the SR noise at the jth local sensor,

$$ P_e = \pi_1 + \pi_0 E_{\nu_j}(G_0(\nu_j)) - \pi_1 E_{\nu_j}(G_1(\nu_j)) $$

$$ = \pi_1 + E_{\nu_j}(J(\nu_j)) $$

where

$$ J(\nu_j) = \pi_0 G_0(\nu_j) - \pi_1 G_1(\nu_j). $$

Therefore, following the same discussion as in [14], we can easily show

Theorem 2: Under the Bayesian criterion, the form of optimal SR noise at the jth local sensor such that the error probability at the fusion center is minimized can be expressed as

$$ p_\nu^{Bay}(\nu) = \delta(\nu - c_0) $$

where

$$ c_0 = \arg_{\nu_j} \min J(\nu_j) $$

Given Theorem 1 and Theorem 2, an iterative PBPO algorithm similar to that in [3] can be proposed to find the SR noise at the local sensors that are at least locally optimum.

Iterative PBPO algorithm:

1) Initialize the SR noises at all local sensors $\nu_k$, $k = 1, 2, \ldots, K$ and set $r = 1$ (iteration index);
2) For each local sensor, obtain the person-by-person optimal SR noise $\nu_j$ of jth local sensor given the fixed SR noises at all other local sensors and a given fusion rule;
3) Compute the performance measure at the fusion center, e.g., $(P_D, P_{F,A})$ pair for NP framework and $P_e$ for Bayesian framework, and compare it with the performance measure obtain in last iteration($r$-1). If the difference is less than a prescribed value, stop. Otherwise, set $r = r + 1$ and go to Step 2.

IV. A DETECTION EXAMPLE

In this section, through a detection example, we demonstrate the performance advantage by applying the SR methodology at the local sensors. We consider the detection of a known signal in independent noises with $K$ sensors. Thus the two hypotheses under test are, for $k = 1, \ldots, K$, $i = 1, \ldots, N$

$$ H_0 : \ X_k[i] = w_k[i], $$

$$ H_1 : \ X_k[i] = A + w_k[i] $$
where $A > 0$ is a known dc signal, and the additive noises $w_k[i]$ are independently and identically distributed (i.i.d.) noise samples with a symmetric Gaussian mixture noise pdf

$$p_w(w) = \frac{1}{2}N(w; -\mu_w, \sigma_w^2) + \frac{1}{2}N(w; \mu_w, \sigma_w^2)$$  \hspace{1cm} (15)

where

$$N(w; \mu_w, \sigma_w^2) = \frac{1}{\sqrt{2\pi\sigma_w}} \exp \left[ -\frac{(w - u_w)^2}{2\sigma_w^2} \right].$$

In the following, we set $A = 1$, $u_w = 3$, $\sigma_w^2 = 1$.

Each local sensor employs a fixed suboptimal detector with respect to the local observations,

$$U_k = \gamma_k(X_k) = \begin{cases} 1 & T(X_k) \geq t \\ -1 & T(X_k) < t \end{cases}$$  \hspace{1cm} (16)

where

$$T(x) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{1}{2} + \frac{1}{2} \text{sgn}(x[i]) \right)$$  \hspace{1cm} (17)

and $t$ is a constant. The non-parametric sign detector is used here since it makes no assumptions regarding the underlying non-Gaussian data statistics.

All the local decisions are transmitted through parallel Rayleigh fading channels to the fusion center. Thus we have, for $k = 1, \ldots, K$,

$$Y_k = h_k U_k + n_k$$

where $h_k$ is the fading channel gain and $n_k$ is zero-mean Gaussian random variable with variance $\sigma^2$. We assume the Rayleigh fading channel has unit power, i.e., $E[h_k^2] = 1$. At the fusion center, we consider three fusion rules; namely, the optimal LRT [5], the likelihood ratio test with channel statistics (LRT-CS) [6], and the equal gain combiner (EGC) [5].

1) Optimal LR-based Fusion Rule: This test assumes knowledge of the instantaneous CSI for the Rayleigh fading channel, $h_k$, as well as the local sensor performance metrics $P_{D,k}$ and $P_{F,A,k}$. It was derived in [5] and is expressed as

$$\sum_{k=1}^{K} \log \left[ \frac{P_{D,A,k} e^{-\frac{(y_k - h_k)^2}{2\sigma^2}} + (1-P_{D,A,k}) e^{-\frac{(y_k + h_k)^2}{2\sigma^2}}}{P_{F,A,k} e^{-\frac{(y_k - h_k)^2}{2\sigma^2}} + (1-P_{F,A,k}) e^{-\frac{(y_k + h_k)^2}{2\sigma^2}}} \right] .$$

2) LRT-CS: This test eliminates the need for instantaneous CSI, but still requires knowledge of the channel statistics as well as the performance metrics of the local sensors. It was derived in [6] and is expressed as

$$\Lambda_r(y) = \sum_{k=1}^{K} \log \left( \frac{1 + [P_{D,k} - Q(aY_k)] \sqrt{2\pi} a Y_k e^{-\frac{(aY_k)^2}{2}}} {1 + [P_{F,A,k} - Q(aY_k)] \sqrt{2\pi} a Y_k e^{-\frac{(aY_k)^2}{2}}} \right)$$

where $a = 1/(\sqrt{1 + 2\sigma^2})$ and

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt.$$

3) Equal Gain Combiner (EGC): This test requires minimal information. It is expressed as [5]

$$\Lambda_3(y) = \frac{1}{K} \sum_{k=1}^{K} Y_k.$$}

We first consider the detection problem under the Neyman-Pearson (NP) framework. We set $K = 8$, $N = 30$, the i.i.d. channel SNR to 5dB and generate the receiver operating characteristics (ROC) curves for all three fusion rules with or without SR noises at local sensors by Monte Carlo simulation with 300,000 trials. The SR noises we add to the local sensors are the optimal SR noise with pdf in the form of (10). In this case, we also fix thresholds of the fusion rules. As we can see from Fig. 2, after adding optimal SR noise to the local sensors, the performances of all three fusion rules are much better than those without SR noises. The performance of optimal LR-based fusion rule still has the best performance among the three fusion rules, which coincides with the results in [6].

We then consider the detection problem under the Bayesian framework. We set $K = 10$, $N = 1$, the i.i.d. channel SNR to 0dB and plot the error probability at the fusion center as a function of SR noise parameter $c$. The threshold that achieves minimum error probability for the fusion rules, optimal-LRT and LRT-CS, is the ratio of the prior probabilities, as is widely known. However, the threshold for the EGC fusion rule is adjusted for each $c$, to obtain minimum $P_e$. We also show in Fig. 3, the performance of EGC, when its threshold is fixed at zero for all $c$. As expected, $P_e$ for EGC with fixed threshold is the highest.

As stated in Theorem 2, the optimal SR noise at the local sensors under the Bayesian framework is a constant which satisfies (14). Here the parameter $c$ is the constant added to the local observations and the optimal point $c_0$ can be easily found from the plot. As shown in Fig. 3, after adding optimal SR noise, the error probability of all three fusion rules are much less than those without SR noise ($c=0$). We can also
SNR=0dB. There are ten sensors.

Fig. 3. Probability of error for various fusion rules with i.i.d. channel

see that the optimal values of parameter $c$ are different among three fusion rules due to the fact that the optimal SR noise depends on the channel characteristics and the fusion rule (cf. eq. (8),(12)). All three fusion rules use different sets of channel information, e.g., full channel information for the optimal LR-based fusion rule, partial channel information for LRT-CS and no channel information for EGC.

V. CONCLUSIONS

In this paper, we have considered the decentralized sensor fusion problem involving local sensor detection as well as the fusion of decisions transmitted over non-ideal transmission channels in a wireless sensor network. Prime emphasis is given to the enhancement of several fusion rules using a recently developed optimized stochastic resonance methodology applied at the local sensors. Further, it is shown that the optimal form of the stochastic resonance probability mass density for the decentralized sensor fusion problem retains the same form as that previously developed for the single sensor case. Performance results are obtained using both the Neyman-Pearson and Bayesian frameworks. The results demonstrate the enhancement obtained after insertion of SR at the local sensors. For the Bayesian case, the parameter associated with the stochastic resonance was shown to depend upon the specific fusion rule. Future work will address the application of the SR process at both the local sensors as well as the fusion center.

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