Automatic Ship Positioning and Radar Biases Correction Using the Hausdorff Distance

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Abstract - *This paper describes a novel technique to obtain radar biases estimates that can effectively reduce mismatches in track association algorithms. This is accomplished by matching ship-borne radar images to geo-referenced satellite images. The matching is performed through the minimization of the averaged partial Hausdorff distance between data points in each image. The minimization rapidly yields robust latitude and longitude position estimates, as well as ship heading and radar biases. The accuracy of the measurements is improved by feeding them into a Kalman filter, which also yields estimates for the ship's velocity. The method can be employed for automatic radar calibration of bearing and range biases, while it also serves as an alternative effective position sensor for GPS-denied environments.*

Keywords: Radar biases, Satellite images, Hausdorff distance, Kalman filtering, track-to-track association.

1 Introduction

The integration of satellite and radar images is becoming a common tool in modern navigation consoles, which may be used for purposes other than ship guidance [1].

A novel technique based on the Extended Kalman Filter was proposed in [2] to estimate a ship's position from electronic charts and radar images. It does not require GPS, gyro heading nor speed information from the ship's log unit. The method, however, relies on the operator to detect and select the corresponding points between the electronic chart and the radar image. This is a major disadvantage, since she or he may disregard other crucial functions, introduce significant errors and may be affected by fatigue.

Other methods, such as those described in [3-7], do not perform satisfactorily for the problem stated since objects can be totally or partially occluded, or they can have important distortions due to the polar nature of the measurements (radar scans). These restrictions are lifted in correspondence techniques that are based on the Hausdorff distance (HD) and extensions such as the partial Hausdorff distance [8, 9].

In this paper, an approach is proposed to estimate the ship's location and speed, as well as the radar biases necessary to correct range and bearing measurements. The approach is based on the minimization of the HD for automatic map-matching of radar and geo-referenced satellite imagery, and may be particularly relevant for track-to-track association and other tracking procedures [10].

At the heart of the process is the comparison of images obtained from "first hits" of the ship-borne radar with geo-referenced satellite images. The latter may be pre-processed by an edge extractor and transformed into Voronoi matrices, which will simplify posterior computations of the HD. The comparison of the images is performed in terms of the averaged partial HD, which is a criteria to measure how dissimilar are the shapes of two sets of points in a metric space or sets of pixels in the case of digital images. If the radar and the satellite images do not match, the radar image is translated, rotated and scaled until a best fit with the satellite image is found. Once a set of translation, rotation and scaling values minimizing the dissimilarity between the radar and satellite images is found, these values are employed to generate a vector of ship position, heading and radar biases measurements. These measurements are filtered using a Kalman Filter (KF) designed in terms of the ship's dynamic model. The position, heading and radar bias estimates are sent to the ship's navigation console or fed back to the matching stage in order to initialize a new iteration of the matching process, so an appropriate reference satellite image can be selected, and an initial radar image transformation is generated.

2 The Hausdorff Distance

The computation of the HD is a technique to measure the degree of similarity among different objects. Consider two sets of points, one being the model and the other the real image, the HD between them is small when every point in one of the sets is near to some point in the other. Given sets $A = \{a_1, a_2, \dots, a_p\}$ and $B = \{a_1, a_2, \dots, a_q\}$, the HD between A and B is defined as:

$$H(A, B) = \max(h(A, B), h(B, A)) \quad (1)$$

where
$$h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\| \quad (2)$$

and $h(A, B)$ is the directed HD between sets A and B . The algorithm sorts the points in A according to their distance to the nearest point in B and selects the farthest as the result. For instance, if $h^* = h(A, B)$, then every point in A is at most at a distance h^* of a point in B , and the pair of points with distance h^* are the ones where the largest discrepancy between the two sets occurs.

Figure 1 shows a geometric representation of the HD as applied to pattern recognition. Here, sets A and B are the model and real image, respectively. By rotating and translating the model, a satisfactory matching is obtained as the one that minimizes $H(A, B)$.

Figure 1: Pattern matching using the Hausdorff distance.

In practical applications, comparing only two sets of data is not enough, since there are multiple candidates B_i that can be similar to the model A , even if the reference pattern is clearly defined.

In order to reduce the number of calculations required to compute the HD, expressions (1) and (2) are restated in terms of the Voronoi surfaces of sets A and B . Let $d_p(x) = \min_{p \in P} \|x - p\|$, then the HD can be written as:

$$H(A, B) = \max \left(\max_{a \in A} d_B(a), \max_{b \in B} d_A(b) \right) \quad (3)$$

The mapping $x \rightarrow d_p(x)$, $x \in \mathbb{R}^2$ defines the so called Voronoi surface of set P and has also been referred to as distance transform; see, for example, [11]. For practical purposes, rasterized approximations of the Voronoi surfaces are calculated using the distance transform. Such computed values are then stored in a matrix called the Voronoi matrix [12].

The usefulness of the Voronoi matrix is in that if an affine transformation is applied to one of the sets, then the new HD between the two sets can be calculated by looking at the appropriate entries of the Voronoi matrix calculated for the sets before the transformation was applied. Thus the Voronoi matrix needs to be computed only once. A detailed description of the algorithm is found in [8].

Some modifications to the previous algorithm are necessary for applications where sets A and B are not identical, as would typically occur in the presence of occlusions, measurement noise and image distortions.

The latter is particularly valid for radar images where the measured set is obtained in polar co-ordinates, so offsets in range will cause a shrinkage or enlargement of the objects. Sometimes these differences can be also introduced in the intermediate stages such as edge detection, expansion, rotation, translation and others. To reduce their impact, some further steps are introduced as explained next.

2.1 Partial Hausdorff Distance

The above mentioned sources of error will generate some false-positive matches with a distance significantly larger than the one of any true-positive match. Taking advantage of the fact that the HD computation procedure determines the distance of the farthest point in B to A by ranking the distances of all points in B to points in A , a way of reducing erroneous matches is to employ the so-called partial Hausdorff distance. This is done by selecting the K^{th} distance in the ranking, instead of the largest one [8]. More specifically, the K^{th} directed partial HD is defined recursively for $K = q, q-1, q-2, \dots, 2, 1$ as:

$$h_K(B, A) = \max_{b \in B^K} d_A(b) \quad (4)$$

where

$$B^K = B^{K+1} - \{b_{K+1}^*\} \text{ and } b_K^* = \arg \max_{b \in B^K} d_A(b) \quad (5)$$

with initial values $B^{q+1} = B$ and $b_{q+1}^* = \{\emptyset\}$. Here,

$h_q(B, A) = h(B, A)$ and $h_q(B, A) \geq h_{q-1}(B, A) \geq \dots \geq h_1(B, A)$ since $B^q \supset B^{q-1} \supset \dots \supset B^1$. Hence, this definition implies that there are K measurement points in B within a distance $h_K(B, A)$ from A (the K^{th} partial HD). Since more than one image transformation may result in similar values for $h_K(B, A)$, an effective criteria for a successful matching is to minimize the average of partial Hausdorff distances smaller or equal to $h_K(B, A)$ for some chosen K . This average is also known as *modified HD* and is given by:

$$\bar{h}_K(A, B) = \frac{1}{K} \sum_{i=1}^K h_i(A, B) \quad (6)$$

By considering $\bar{h}_K(B, A)$ instead of $h_K(B, A)$, it is ensured that more measurement points closely resemble the model. For practical purposes, it is convenient to define the ratio of model points employed in the calculation of the average as $\lambda = K/q$. Then K may be selected in terms of the ratio λ simply as $K = \lambda q$, with $1/q \leq \lambda \leq 1$. The value of λ is found empirically as the one that minimizes the matching error for a set of image and model pairs.

2.2 The Matching Strategy

The search for the optimal solution is based on the application of translation, rotation and scaling transformations to the images in such a way as to

minimize (6), thus yielding a set of transformations that maximizes the similarity between A and B . In order to further reduce the processing time and facilitate the matching process, edges in the images are extracted using the Canny edge detector [13]. A reliable and rapidly converging approach to minimize (6) was implemented in terms of standard gradient methods.

3 Robustness and Accuracy of the Matching Method

In this section the robustness and accuracy of matching method based on the averaged partial HD is tested in terms of simulated data. The simulations consider a 500×500 pixels reference image containing a 116×116 pixels square in its centre. The simulated data representing the contour obtained by the ship-borne radar is generated by adding noise to a given percentage of samples of the original reference square. They are also rotated 10° counter-clockwise, and translated 150 pixels to the left and 150 pixels up. Two different parameters are adjusted for the different scenarios of measured data: the percentage of outliers or measurements affected by noise and the average magnitude of the noise. The noise affecting each measurement sample, if any, is assumed to be zero-mean Gaussian with standard deviation equal to the scenario's noise level.

The robustness of matching method based on the averaged partial HD is apparent from this results which show practically no variation in the position or heading error for different percentage values of noisy samples and noise levels. The quality of the match may be appreciated from figures 2 and 3, which show the initial measurements and their final position after matching, respectively.

Figure 2: Initial misalignment between simulated data (dots) and the reference data (line)

Figure 3: Final match of simulated data (dots) and the reference data (line)

Figure 4: Convergence of the averaged partial Hausdorff distances for the different scenarios with percentages of noisy samples NP and noise levels NL

Figure 4 shows the convergence of the averaged partial HD to a minimum in an almost linear fashion for all the different scenarios. The final value of the averaged partial HD increases proportionally to the noise level, however, it is worth noting that this value remains zero for the simulations in which only 20% of the samples are corrupted by noise. This is because the ratio λ for calculation of the partial HD was set at 0.7, which means that only 70% of the total number of samples that best match the reference set are used. The remaining samples that include those corrupted by noise are automatically discarded from the calculation of the averaged partial HD. Figure 5 clearly shows that the regular HD curves have

largest values beyond the threshold. While scenarios with noise percentages below 30% yield final $\bar{h}_k(A, B)$ equal to zero, scenarios with noise percentages above 30% yield final $\bar{h}_k(A, B)$ which are non-zero and are proportional to the standard deviation of the noise for a fixed threshold λ . Simulation results also confirm that the accuracy is high, as expected according to the theoretical result in [14], which states that the matching error due to spatial sampling is at most one rasterization unit (see claim 3 in [14]) for the noise-free situation. This result may also be extended to the zero-mean Gaussian noise situation.

Figure 5: Sorted Hausdorff distances for the simulation with NP=100% and NL=10

4 Radar and Satellite Matching

The aim of this application is to obtain location and speed estimates for a cruising ship, and to calibrate a maritime radar by eliminating biases in bearing and range measurements. Satellite images are employed as reference models to which the navigation radar images must be matched.

In order to ensure a reasonably good estimation accuracy, a large number of correspondence points covering 360° and stretching along the full distance range should be sought. In general, this is possible as radar scans typically result in numerous sparse points. A detailed analysis about the observability for different correspondence scenarios is given in [2].

The transformation model used to match sets A (radar image) and B (satellite image) is based on a reference frame given in polar co-ordinates (due to the nature of radar scans), thus the actual transformations produce non-rigid displacements of the objects in the image, which distort their shape. For a given point in the radar image, the measurement model is

$$\begin{aligned} c_{iX} &= (R_i + \xi_R) \cos(\theta_i + \xi_\theta) + X + n_X \\ c_{iY} &= (R_i + \xi_R) \sin(\theta_i + \xi_\theta) + Y + n_Y \end{aligned} \quad (7)$$

where R_i and θ_i are the polar co-ordinates of the point c_i whose value is given in UTM co-ordinates X and Y , and where n_X and n_Y are assumed to be zero-mean Gaussian noise arising from errors in measuring correspondences in the radar scan image due to clutter, occlusions or broad radar pulses, among other.

Variables ξ_R and ξ_θ , are common to all points selected for matching and account for the measurement errors induced by biases in range and bearing, respectively. The orientation variable ξ_θ describes the rotation that suffers the radar image due to disturbances, such as misalignments in the antenna position sensors (encoders, synchros, etc.), antenna boresight and the azimuth error generated by the local level reference frame.

4.1 Choice of Kalman Filter Parameters

The well known standard Kalman filter [15] is implemented to smooth the measurements computed by means of the matching process and to estimate the vessel's speed and course. The ship motion is described in terms of a simple dynamic model which assumes that only the propeller thrust, rudder angle and disturbances, represented by Gaussian noises, affect the ship location and heading (see [2] for a complete description of the model). The ship model considers a seven component state vector $x = [X, Y, V_X, V_Y, \theta', \omega, \xi_R]$, where X and Y are the geographical location co-ordinates of the ship in UTM units, V_X , V_Y are the velocities along axes X and Y , respectively, θ' is the heading angle measured clockwise with respect to the Y axis and satisfying $\theta' = \theta + \xi_\theta$ (θ being the bias-free heading, and ξ_θ the bearing bias). The variable ω is the rate of change of the course direction. The variable ξ_R describes the radar

range bias and is assumed here that $\frac{d \xi_R}{dt} = 0$. Although

this variable cannot be considered constant in the long term due to changes in the radar calibration, in the short term (time required to carry out a full calibration) its change can be considered negligible. The ship is driven by means of the propellers' thrust, u_v , and the rudder steering force u_ω . The propeller thrust is projected onto the inertial co-ordinate system through the heading angle θ' . The model considers also disturbances $w = (w_v, w_\omega)$ added to longitudinal and turning velocities to account for external non-deterministic forces mainly due to wind, currents and waves. To keep the technique simple, it is reasonable to assume that these random variables can be described by zero-mean Gaussian probability distributions.

On the other hand, the measurement model considers an observation vector $z = [X_m, Y_m, \theta_m, \xi_{Rm}] =$

$[X + n_X, Y + n_Y, \theta' + n_\theta, \xi_R + n_\xi]$, where n_X , n_Y , n_θ and n_ξ are zero-mean Gaussian noises added to account for errors occurring in the matching process described in the previous section. The process and measurement noise covariances are given by matrices $R = \text{cov}(n_k)$, and $Q = \text{cov}(w_k)$, respectively. The choice of filter parameters Q , R and the initial error covariance matrix $P(0/0)$ are crucial for a good performance of the filter. Some well-known approaches to make this selection rigorous can be found in [16] and [17]. In this paper, the practical approach suggested in [18] is followed for the initial estimate of the state estimation error covariance matrix, setting large values in its diagonal ($P(0|0) = 10^3 \times I_{7 \times 7}$). This has shown to behave satisfactorily in our case, ensuring a rapid convergence of the filter to reasonable estimates. The covariance matrices Q and R are assumed diagonal and constant. The values for the process noise described by the matrix Q is upper bounded by the maneuvering capability of the ship and thus must be chose accordingly. For instance, a change in course of $0.001 [\text{rad}/s^2]$ is typical for a ship maneuver, so disturbances should not exceed this value. The thrust force generated by changes in the propeller revolutions also binds the longitudinal acceleration. For a coastguard vessel, $0.01 [m/s^2]$ would be a common value. The latter can be also used to account for dynamic variations in the radar range bias. Hence,

$$Q = \begin{bmatrix} \sigma_{wX}^2 & 0 & 0 & 0 \\ 0 & \sigma_{wY}^2 & 0 & 0 \\ 0 & 0 & \sigma_{w\theta}^2 & 0 \\ 0 & 0 & 0 & \sigma_{w\xi}^2 \end{bmatrix} = \begin{bmatrix} 0.01^2 & 0 & 0 & 0 \\ 0 & 0.01^2 & 0 & 0 \\ 0 & 0 & 0.001^2 & 0 \\ 0 & 0 & 0 & 0.01^2 \end{bmatrix} \quad (8)$$

There is no simple method to select adequate values for the measurement noise matrix R , since it involves modeling the various sources of error in the radar-satellite matching procedure. One approach is to use the computed Hausdorff distances to determine the noise in X - Y position measurements and range bias, i.e. $\sigma_{nX} = \sigma_{nY} = \sigma_{n\xi} \approx 50.0 [m]$. The noise in angular measurements is determined by the radar resolution. A value of $\sigma_{n\theta} \approx 0.02 [\text{rad}]$ is used here.

Finally, the matrix R is given by

$$R = \begin{bmatrix} \sigma_{nX}^2 & 0 & 0 & 0 \\ 0 & \sigma_{nY}^2 & 0 & 0 \\ 0 & 0 & \sigma_{n\theta}^2 & 0 \\ 0 & 0 & 0 & \sigma_{n\xi}^2 \end{bmatrix} = \begin{bmatrix} 50.0^2 & 0 & 0 & 0 \\ 0 & 50.0^2 & 0 & 0 \\ 0 & 0 & 0.02^2 & 0 \\ 0 & 0 & 0 & 50^2 \end{bmatrix} \quad (9)$$

5 Experimental Results

Tests were carried out with radar data collected from a navigation console in order to demonstrate the performance of the HD-based matching technique. The data corresponds to a sequence of radar scans taken from a patrol boat in the Concepcion Bay, Chile ($36^\circ 42'S$ and $73^\circ 05'W$) sailing east at variable speed (from 12 knots down to 2 knots at constant 90° heading course). Heading and log data was also recorded in order to analyze and compare the results. The sequence consists of 54 radar scans, taking a total of 135 seconds. Data was acquired during navigation for posterior off-line processing. Figures 6 and 7 show the satellite and radar image, respectively.

After the application of the edge detector, the superposition of both images clearly shows a mismatch as illustrated in Figure 8. The application of the proposed matching method with $\lambda = 0.7$ (see equation 4) to find the transformation that minimizes (6) results in a tight match as shown in figure 9. In fact, the initial value of (6) in this case is computed to be 14.82 [pixels] and attains a minimum of 1.42 [pixels].

The matching procedure is carried out for every radar scan every 2.5 seconds, and the resulting position and radar biases obtained are fed into a KF to obtain estimates of the ship's location, heading and speed. These position estimates are shown in figure 10 for the complete sequence, with units in meters referred to an arbitrary UTM coordinate (Zone 18S : $[X_0, Y_0] = [680736.8, -4068896]$). The estimates for speed and heading are shown in figures 11 and 12. As expected, significant differences appear between these values and the data registered from the log unit and gyroscope. A possible reason for this result is the presence of sea currents and winds that creates a difference between the heading and the actual course (or direction of the velocity vector at the centre of gravity), generally referred as the drifting angle. Radar biases for range and bearing are shown in figures 13 and 14.

Figure 6: Satellite image.

Figure 7: Radar image

Notice that discretization levels are not negligible and can be clearly seen in figures 10 and 13. Satellite images used in this work have pixel widths of 14.5 meters and the radar bearing resolution is 0.5° . Better accuracy would require higher resolution that in turn would increase the computational burden.

Regarding processing time, the technique has proved to be adequate for a real time implementation. In spite of a slow initial computation of matching variables (approximately 30 seconds), the following scans take no more than 4 seconds in a commercial PC with an Intel Pentium 4 processor running at 2.4GHz. This is over the 2.5 seconds limit imposed by the scanning period, but an especially design architecture could overcome this limitation.

Figure 8: Initial misalignment of satellite (continuous line) and radar (dots) images

Figure 9: Matching of the two sets.

Figure 10: Position measurements and estimates.

Figure 11: Ship speed magnitude.

Figure 12: Ship heading.

Figure 13: Range bias estimate.

Figure 14: Bearing bias estimate.

It is worth pointing out some issues about accuracy at high speeds. In the case of non-maneuvering vessels moving at a constant velocity and employing navigation radars with scan periods of less than 2.5 seconds (typical), the accuracy of the proposed approach should not deteriorate substantially. If a ship keeps a straight course, the Kalman filter will adequately estimate its position because the model employed to describe the ship's motion considers first and second order terms (velocities and accelerations) which are bounded, as in the real ship. A different situation arises when maneuvers are made. In this case, the accuracy may deteriorate. However, two approaches can be used to tackle this problem. One of them consists in using the log unit's rudder and yaw information (gyro) to feed the filter, i.e. the model for the Kalman filter should include these manipulated variables (see [2] for a detailed discussion). The second approach consists in augmenting the order of the ship's dynamical model to a third order model. This should reduce errors caused by maneuvers, however, rapid changes would not be compensated as effectively as with the first approach due to the filter dynamics. In our application, this problem is not crucial since maneuvers are not frequent. If they occur, the data can be ignored during this event, based on the sudden variations detected from the attitude and speed sensors.

Another issue that has an effect on the accuracy is the coastline structure and its physical characteristics, which may give rise to a stiff estimation problem, i.e. possible estimation errors due to few useful measurements coming from a limited angular sector. This case is discussed in [2], where it is demonstrated that some observability conditions must be met for the method to work satisfactorily. The characteristics of the coastline could in some cases affect the radar echoes, however this can be handled by using electronic chart information that would not only alert on the existence of cliffs or mud estuaries, but also coastline variations caused by tides or presence/absence of vessels in wharfs. In such a scenario, the HD-based matching method could automatically reject scan samples presenting large distances beyond some predetermined level. Furthermore, a reliability factor of each scan sample of the coastline could be introduced into the system to ignore certain echo zones, for instance, depending on the morphology and physical properties of the coastline stored in the hydrographic charts; see [2]. Reflections from buoys or beacons could be useful as long as they do not show significant drifting due to currents or tides. However, it is arguable whether satellite images would detect these objects. Information about their existence and location could probably be extracted from hydrographic charts present in most current maritime navigation systems.

6 Conclusions

A method to estimate the position, heading and velocity of a ship, as well as its radar biases has been described. The approach relies on matching radar images to geo-referenced satellite images, thus freeing the vessel's dependence on other positioning systems, such as GPS, or less accurate triangulation techniques that employ beacons or landmarks. This can be particularly useful when GPS instrumentation is not available or denied [1]. The technique can also provide a reliable source of speed in areas with strong sea currents where the log unit will yield distorted measurements.

Central to the matching process is the identification of a set of image transformations that minimize the average Hausdorff distance. This process yields position and radar bias values that are passed to a KF in order to obtain better estimates of the position, heading and velocity of the ship. Experimental results obtained using real ship navigation data demonstrate the effectiveness of the approach as it yields rapidly converging accurate estimates. Moreover, the approach is amenable for real-time implementation.

Another advantage of the method is related to maintenance. Radar maintenance is periodically required to correct range and bearing biases, but experience shows that this necessity is rarely observed. Calibration of elements such as position encoders/synchros, antenna beam boresight and gyrocompasses are crucial for the performance of the navigation system, especially when multiple sensors, either internal or external, demand a data fusion procedure. This technique provides an automatic calibration procedure that reduces these sources of error.

A very simple model for ship dynamics has been used for the KF, so aspects such as sway (lateral movement) or rotational moment of inertia have been neglected. Better results should be attainable by introducing these type of variables. A comprehensive treatment of ship dynamical models can be found in [19].

Future work considers testing other estimation strategies based, for example, on particle filters, and incorporating the proposed approach into an automated ship guidance system.

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