Gaussian Mixture Cardinalized PHD Filter for Ground Moving Target Tracking

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Abstract - The cardinalized probability hypothesis density (CPHD) filter is a recursive Bayesian algorithm for estimating multiple target states with varying target number in clutter. In particular, the Gaussian mixture variant (GMCPHD) for linear, Gaussian systems is a candidate for real time multi target tracking. The present work addresses the following three issues: (i) we show the equivalence between the GMCPHD filter and the standard Multi Hypothesis Tracker (MHT) in the case of single targets; (ii) using a Gaussian sum approach, we extend the GMCPHD filter by employing digital road maps for road constraint targets. The utilization of such external information leads to more precise tracks and faster and more reliable target number estimates; (iii) we model the effect of Doppler blindness by a target state dependent detection probability, leading to more stable target number estimation in the case of low Doppler targets.

Keywords: Probability hypothesis density filter, PHD, multi-target tracking, cardinalized probability hypothesis density filter, CPHD, road targets, ground moving target indicator radar (GMTI)

I. INTRODUCTION

The Probability Hypothesis Density filter for tracking multiple targets in clutter [1] has received a lot of attention recently (see, e.g. [2]–[7]), for it avoids the explicit enumeration of all possible multi target–multi detection assignments that leads to the so-called combinatorial disaster. In numerical application (e.g. particle filters) it indeed beats the curse of dimensionality because the dimension of the PHD problem is equal to the dimension of the single target state.

An important step towards practical applicability was made by the Gaussian mixture version of the PHD [6], [8] for linear Gaussian target models. A disadvantage of the PHD, however, is due to the Poisson assumption for the target number (cardinality) distribution that often leads to strong fluctuations in the target number estimators and to the missed detection problem [5]. This problem has been cured, recently, by the so-called Cardinalized PHD, CPHD [9], [10]. A more intuitive “physical state” derivation of the PHD and CPHD filter has been presented in [11]. For the CPHD filter, too, recently a Gaussian mixture variant has been developed, the GMCPHD filter [12], [13].

In the present paper we apply the GMCPHD filter to the problem of tracking groups of ground moving targets. In particular, we focus on the issue of integrating digital road maps into the algorithm and the problem of modeling the clutter notch of GMTI radar sensors (GMTI: ground moving target indicator) for low Doppler targets.

The paper is organized as follows: In Section II we recapitulate the Gaussian mixture cardinalized probability hypothesis density (GMCPHD) filter equation [9]–[12]. In the situation of at most one target, we show in Section III the equivalence to the single target multi hypothesis tracking (MHT) filter [14], [15] with sequential likelihood ratio test for track extraction. The GMCPHD filter is extended by introducing digital road maps in Section IV and by modeling the Doppler blindness (clutter notch) in Section V. After briefly discussing some implementation issues in Section VI, we present simulation results for groups of road targets in Section VIII.

II. GMCPHD FILTER

In the Gaussian mixture variant of the CPHD, the PHD at scan $k$, $v_{k|k}(x)$, is described as the weighted sum of $J_{k|k}$ normal distributions:

$$v_{k|k}(x) = \sum_{j=1}^{J_{k|k}} w_{k|j} \mathcal{N}(x; m_{k|j}, P_{k|j})$$

with $m_{k|j}$ and $P_{k|j}$ being the mean and covariance matrix of component $j$. The estimated target number is given by the integral of $v(x)$:

$$n_{k|k} = \int v_{k|k}(x) \, dx = \sum_{j=1}^{J_{k|k}} w_{k|j}.$$  \hspace{1cm} (2)

In the CPHD, parallel to the PHD, the cardinality distribution $p_{k|k}(n) = p(n|Z^k)$ is estimated iteratively. Equivalently to (2), the target number can also be calculated by:

$$n_{k|k} = \sum_{n=1}^{\infty} n \cdot p_{k|k}(n).$$  \hspace{1cm} (3)

The prediction and update step for each individual component is performed exactly as in Kalman or MHT filtering supplement by so-called birth and death processes.
Prediction: given the PHD for scan \( k-1 \), the probability of target “death”, \( d \), the target dynamics matrix \( F_k \), and process noise covariance \( D_k \), the predicted components are:

\[
\begin{align*}
\mathbf{w}^{(j)}_{k|k-1} &= (1-d)\mathbf{w}_{k-1|k-1} \\
\mathbf{m}^{(j)}_{k|k-1} &= F_k \mathbf{m}^{(j)}_{k-1|k-1} \\
\mathbf{P}^{(j)}_{k|k-1} &= F_k \mathbf{P}^{(j)}_{k-1|k-1} F_k^T + D_k
\end{align*}
\]

Additional components are introduced by a birth model which describes potentially new targets appearing in the field of view of the sensor. A simple realization of a birth model is to introduce at each scan one test component with a certain weight \( w_{\text{birth}} \), positioned in the middle of the field of view with zero velocity, and a large covariance \( \mathbf{P}_{\text{birth}} \) covering the whole field of view of the sensor. The number of components after prediction, denoted by \( J_{k|k-1} \), is equal to \( J_{k-1|k-1} + n \) plus the number of new born components.

The prediction equation for the cardinality distribution can be written in terms of a “transfer matrix” \( M \):

\[
P_k(n, n') = \sum_{m=0}^{\infty} p_{k-1|k-1}(m) M(n, n')
\]

\[
M(n, n') = \sum_{i=0}^{\min(n, n')} \binom{n'}{i} \cdot \binom{n}{n-i} \cdot \binom{n}{1} \cdot \binom{n-1}{d} \cdot \binom{n-1}{d+1} \cdot \ldots \cdot \binom{n-1}{d+m-1}
\]

The present context of Gaussian mixtures can be preserved in the case of linear measurements with Gaussian measurement errors \( \mathbf{w}_k \) with (potentially detection dependent) covariance \( \mathbf{R}_k(s) \): for a missed detection, the updated component mean and covariance are equal to the predicted ones. Assigning the true identity of a new target and updating the parameters.

The new target detection likelihood function \( l_k(s|x) \) for a missed detection, the updated component mean and covariance are equal to the predicted ones. Assigning the true identity of a new target and updating the parameters.

\[
\begin{align*}
L(\mathbf{Z}_k|n) &= \sum_{j=0}^{\min(m, n)} \beta_k^{(j)} \frac{n!}{(n-j)!} \left(1 - p_d\right)^{n-j} \\
\sigma_j(\{L^{(1)}_k, \ldots, L^{(m)}_k\})
\end{align*}
\]

with

\[
\begin{align*}
\alpha_k^{(j)} &= \sum_{n=0}^{\infty} \frac{n!}{(n-j)!} p_{k|k-1}(n) (1 - p_d)^{n-j} \\
\beta_k^{(j)} &= p_c (m-j) \frac{(m_k-j)!}{m_k!} \lambda^{-j}
\end{align*}
\]

with the single detection likelihood function \( l_k(s|x) \). \( p_c(m) \) denotes the probability for \( m \) false alarms, where a homogeneous false alarm (clutter) density is assumed for simplicity.

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\end{align*}
\]

Inserting the Gaussian mixture (1) into (9), we obtain for each original component \( m_k+1 \) new components, one from the missed detection part, \((1-P_d)\), denoted by \( s = 0 \), and one for each detection \( z^{(s)}_k \), \( s = 1, \ldots, m_k \). After updating, the total number of components therefore is \( J_{k|k} = (m_k+1)J_{k-1|k-1} \).

In general, the probability of detection, \( P_d \), may depend on the target state, which could be modeled by a component dependent \( P_d^{(j)} \). (See Section V for a more refined treatment.)

In that case, we also need the weighted average

\[
P_{d,k-1} = \frac{1}{n_k|k-1} \sum_{j=1}^{J_{k-1|k-1}} w_k^{(j)} P_d^{(j)}
\]

which replaces \( P_d \) in (15) and (16), see [11].

For a missed detection, the updated component mean and covariance are equal to the predicted ones. Assigning the true detection \( s \) to component \( j \), denoted by \( a_k^{(j)} = s \), the update follows the usual Kalman filter equations:

\[
\begin{align*}
\mathbf{m}_{k|k}^{(j,s)} &= \mathbf{m}_{k|k-1}^{(j)} + \mathbf{W}_{k}^{(j,s)} (z_k^{(s)} - H_k \mathbf{m}_{k|k-1}^{(j)}) \\
\mathbf{P}_{k|k}^{(j,s)} &= [1 - \mathbf{W}_{k}^{(j,s)} H_k] \mathbf{P}_{k|k-1}^{(j)}
\end{align*}
\]

with

\[
\begin{align*}
\mathbf{W}_{k}^{(j,s)} &= \mathbf{P}_{k|k-1}^{(j)} H_k^T (\mathbf{S}_{k|k}^{(s)})^{-1} \\
\mathbf{S}_{k|k}^{(s)} &= \mathbf{R}_k + H_k \mathbf{P}_{k|k-1}^{(j)} H_k^T
\end{align*}
\]

The component weights are updated by \( w_{k|k}^{(j,s)} = G_{k}^{(j,s)} w_{k|k-1}^{(j)} \) where the updating factors are given by:

\[
G_{k}^{(j,s=0)} = \frac{L(\mathbf{Z}_k|n) - L(\mathbf{Z}_k)}{L(\mathbf{Z}_k)} (1 - P_d^{(j)})
\]

\[
G_{k}^{(j,s>0)} = \frac{P_d^{(j)} L(\mathbf{Z}_k|a_k^{(j)} = s)}{L(\mathbf{Z}_k)} N(z_k^{(s)}; z_k^{(j,s)}, \mathbf{S}_{k|k}^{(s)})
\]
with clutter density $\lambda$, predicted detection $z^{(j)}_k = H_k m^{(j)}_{k|k-1}$, and innovation covariance $S^{(j,s)}_k$, see (23).

Remark: the above update equations for weights, estimates, and covariances are obtained by using (1) and (18) and applying the product formula for normal densities:

$$N(x; X, Y)N(y; z, Z) = N(x; a, A)N(y; b, B)$$ with $a = Xz, \ b = z + W(x - Xz), \ A = Y + ZXZ^\top, \ B = (1 - WX)Z$, (25)

$$= 1/n_{k|k-1} \int P_{d}(x) v_{k|k-1}(x) f^{(s)}_k(z^{(s)}_k|x) dx$$ (29)

$$= 1/n_{k|k-1} \sum_{j=1}^{J_{k|k-1}} P_{d}^{(j)} v_{k|k-1}^{(j)} N(z^{(s)}_k; z^{(j)}_k, S^{(j,s)}_k)$$ (30)

The function $\sigma_j(A)$ in the likelihood ratios above, finally, is defined as the sum over all possible products of elements of the set $\mathcal{A}$ with $j$ different factors:

$$\sigma_j(\{y_i, \ldots, y_m\}) = \sum_{1 \leq i_1 < \cdots < i_j \leq m} y_{i_1} y_{i_2} \cdots y_{i_j}$$ (31)

and $\sigma_0 = 1$.

III. COMPARISON TO MHT

As mentioned above, besides birth and death processes, each Gaussian component is predicted and updated in the same way as in MHT. Each component has a unique predecessor, and the series of such predecessors form the history of a track. The exponentially increasing number of components, i. e. tracks, can be reduced by standard pruning and merging techniques borrowed from MHT. What is different, however, is the calculation of the multi target likelihood ratios and, hence, the components’ weights. In the multi target MHT, all possible associations between detections and tracks are enumerated and for each multi target–multi detection association, the likelihood function has to be calculated. The number of associations grows exponentially with the number of tracks $J_{k|k}$, and the number of detections $n_k$. In contrast, in GMCPHD the likelihood ratios are calculated by sums of products of single detection likelihoods where the length of the sums is limited by the maximum number of targets and the number of the detections. It is, hence, polynomial in target and detection numbers.

A. Track extraction

We compare the track extraction process in MHT and GMCPHD, respectively, in the case of single (or well separated) targets, e. g. $n \in \{0, 1\}$. In GMCPHD a track may be declared extracted if the target probability $p_{b|k}(1) = n_{k|k}$, see (3), exceeds a certain threshold: $n_{k|k} > n_c$. This criterion is equivalent to a threshold on the likelihood ratio given target number one and zero:

$$R_k \equiv \frac{p(Z_k^b|1)}{p(Z_k^b|0)} = \frac{\pi(0) p(1|Z_k^b)}{\pi(1) p(0|Z_k^b)} = \frac{\pi(0)}{\pi(1)} 1/n_{k|k}$$ (32)

where $\pi(n) = p_{b|0}(n)$ are the prior probabilities for $n$ targets. Assuming, for simplicity, equal priors $\pi(0) = \pi(1)$, the criterion reads: $R_k > R_0 = n_c/(1 - n_c)$. Applying (10) iteratively, the likelihood ratio can be written as:

$$R_k = \prod_{i=1}^{k} \frac{L(Z_i|1)}{L(Z_i|0)}$$ (33)

In MHT too, the above likelihood ratio serves as the criterion that a track is extracted [14], [15]. For single target MHT, the likelihood ratio can be shown to be equal to the sum of the unnormalized hypothesis weights $\tilde{w}^{(j)}_k$, provided the initial hypothesis weights sum up to one [16]:

$$\tilde{R}_k = \sum_{j=1}^{J_{k|k}} \tilde{w}^{(j)}_k \quad \text{with} \quad \tilde{R}_0 = 1.$$ (34)

In our comparison of MHT and GMCPHD track extraction, we neglect birth and death processes, although they could be incorporated into the MHT as well. Since only the updated quantities are needed, we shorten the notation and replace the double index “$k|k$” by “$k$” etc. For simplicity, and as usual in the tracking literature, we also assume homogeneous, Poisson distributed false alarm numbers with density $\lambda$. In the single target case, $(p_{b|k}(n) = 0, n > 1)$, all coefficients $\alpha^{(j)}_k$, see (16), vanish for $j > 1$, and the GMCPHD filter equations are strongly simplified:

$$\alpha^{(0)}_k = 1 - P_{d,k-1} n_{k|k}; \quad \alpha^{(1)}_k = n_{k|k}$$ (35)

$$\beta^{(0)}_k = p_c(m); \quad \beta^{(1)}_k = \frac{p_c(m) }{\lambda V}$$ (36)

$$L(Z_k|a_k^{(j)} = 0) = p_c(m)$$ (37)

$$L(Z_k|a_k^{(j)} = s) = \frac{p_c(m) }{s \lambda} N(z_k^{(s)}; H_k x_k, S_k^{(j,s)})$$ (38)

$$L(Z_k) = [1 - (P_{d,k-1} - Q_k) n_{k|k-1}] p_c(m)$$ (39)

$$L(Z_k|n = 0) = p_c(m)$$ (40)

$$L(Z_k|n = 1) = [1 - P_{d,k-1} + Q_k] p_c(m)$$ (41)

$$Q_k = \frac{1}{\lambda} \sum_{s=1}^{m_k} L^{(s)}_k$$ (42)

with the single detection likelihood $L^{(s)}_k$ given in (29) and the size of the field of view of the sensor $V$. Inserting the above into (24) and (10) we obtain the updated component weights and expected target number:

$$w^{(j,0)}_k = \frac{1}{1 - P_{d,k-1}} \frac{w^{(j)}_{k-1}}{1 - (P_{d,k-1} - Q_k) n_{k|k-1}} \quad \text{w}^{(j,0)}_k = \frac{1}{1 - P_{d,k-1} - Q_k}$$ (43)

$$w^{(j,s)}_k = P_{d}^{(j)} N(z_k^{(s)}; z_k^{(j)}, S_k^{(j,s)}) \frac{1}{1 - (P_{d,k-1} - Q_k) n_{k|k-1}} \quad \text{w}^{(j,s)}_k = \frac{1}{1 - P_{d,k-1} - Q_k}$$ (44)

$$n_k = n_{k|k-1} - (P_{d,k-1} - Q_k) n_{k|k-1}$$ (45)
We note that without any detection ($Q_k = 0$) the updated probability of target existence becomes:

$$p_k(1) = n_k = n_{k-1} \frac{1 - P_{d,k-1}}{1 - P_{d,k-1} n_{k-1}}$$  \hspace{1cm} (46)

avoiding the PHD’s missed detection problem [5]. Inserting (40) and (41), into (33), the likelihood ratio finally reads:

$$R_k = \prod_{l=1}^{k} [1 - P_{d,l-1} + Q_l]$$  \hspace{1cm} (47)

In MHT, the corresponding updated weights are iteratively calculated by:

$$\tilde{w}^{(j)}_{k} = (1 - P_{d}^{(j)}) \tilde{w}^{(j)}_{k-1}$$  \hspace{1cm} (48)

$$\tilde{w}^{(j,s)}_{k} = \frac{P_{d}^{(j)}}{\lambda} \mathcal{N}(z_k^{(s)}; z_k^{(j)}, S_k^{(j,s)}) \tilde{w}^{(j)}_{k-1}$$  \hspace{1cm} (49)

Apparently, the weight updating factors are equal to those in GMCPHD, see (43) and (44), besides a scan dependent, but component independent, factor. If we start, at $k = 0$, with equal relative component weights in MHT and GMCPHD, respectively, they will remain equal scan by scan besides a constant:

$$\tilde{w}^{(j)}_{k} = w_{k}^{(j)} \sum_{j} \tilde{w}^{(j)}_{k-1} = w_{k}^{(j)} \frac{R_k}{n_k}$$  \hspace{1cm} (50)

In other words, component mean, covariances, and relative weights all are identical in single target MHT and GMCPHD. How about likelihood ratios? Inserting the update equations (48) and (49) into (34), and using (50), the MHT likelihood ratio can be written as:

$$\tilde{R}_k = \sum_{j=1}^{J_{k-1}} (1 - P_{d}^{(j)}) \tilde{w}^{(j)}_{k-1}$$  \hspace{1cm} (51)

$$+ \frac{1}{\lambda} \sum_{j=1}^{J_{k-1}} P_{d}^{(j)} \sum_{s=1}^{m_k} \mathcal{N}(z_k^{(s)}; z_k^{(j)}, S_k^{(j,s)}) \tilde{w}^{(j)}_{k-1}$$

$$= \tilde{R}_{k-1} [1 - P_{d,k-1} + Q_k]$$  \hspace{1cm} (52)

with $Q_k$ given in (42). Apparently, $\tilde{R}_k$ obeys the same recursion relation as $R_k$, see (47). Since, by construction (34), $\tilde{R}_0 = 1$ and, by definition (32), $R_0 = 1$, it is clear that both likelihood ratios are identical scan by scan.

To summarize: neglecting birth and death processes (which could be incorporated into a MHT algorithm, too), single target MHT and GMCPHD for $n \leq 1$, including track extraction by sequential likelihood ratio testing, are identical algorithms. Mixture component reduction by pruning and merging can be performed in the same way within GMCPHD and MHT.

IV. ROAD MAP INTEGRATION

Ground traffic usually is bound to roads. Even military vehicles, in particular convoys, prefer to use roads whenever possible while off-road traffic is the exception. There are different approaches in the literature on how to integrate road map constraints into a Bayesian tracking algorithm. A central difficulty, in particular in the case of curved roads, lies in the fact that the target motion on road is highly nonlinear in cartesian or sensor coordinates. A flexible approach, suited for winding roads, is given in [17]. There, the target dynamics is modeled in road coordinates, i.e. the target state is described by a two-dimensional vector $\mathbf{x} = (l, \dot{l})^T$ parametrized by the arc length $l$ on road (or mileage), and the speed $\dot{l}$. In most areas, road information is provided in relatively high precision in digital vector maps. Typically, a curved road is approximated by a set of linear road segments $i = 1, \ldots, N_{seg}$, defined by their starting position, $s_i$, normalized direction $t_i$, length $l_i$, and, potentially, certain attributes like road type etc. These linear segments allow for a linear mapping between road and cartesian coordinates. Each road segment $i$ has an uncertainty due to discretization and map errors. These uncertainties are modeled by a Gaussian noise term in the mapping from road to cartesian coordinates, quantified by a covariance $R_i$.

$$p(\mathbf{x} | \mathbf{x}, i) = \mathcal{N}(\mathbf{x}; t_i \mathbf{g}_{g-r}, \mathbf{s}_i)$$  \hspace{1cm} (53)

using the affine transformation: $t_i \mathbf{g}_{g-r} \mathbf{x} + \mathbf{s}_i$ with

$$\mathbf{g}_{g-r} = \begin{pmatrix} t_i & 0 \\ 0 & t_i \end{pmatrix}, \quad \mathbf{s}_i = \begin{pmatrix} s_i - l_i t_i \\ 0 \end{pmatrix}$$  \hspace{1cm} (54)

As in Section II, the predicted PHD in road coordinates is written as a Gaussian mixture:

$$v_{k|k-1}(\mathbf{x}) = \sum_{j=1}^{J_{k|k-1}} w_{k|k-1}^{(j)} p_j(\mathbf{x} | \mathbf{Z}^{k-1})$$  \hspace{1cm} (55)

$$p_j(\mathbf{x} | \mathbf{Z}^{k-1}) = \mathcal{N}(\mathbf{x}; \mathbf{m}_{k|k-1}^{(j)}, \mathbf{P}_{k|k-1}^{(j)})$$  \hspace{1cm} (56)

Each component $j$ of the predicted PHD in road can be decomposed into a Gaussian mixture over road segments:

$$p_j(\mathbf{x} | \mathbf{Z}^{k-1}) \approx \sum_{i=1}^{N_{seg}} f_{k|k-1}^{(i,j)} \mathcal{N}(\mathbf{x}; \mathbf{m}_{k|k-1}^{(i,j)}, \mathbf{P}_{k|k-1}^{(i,j)})$$  \hspace{1cm} (57)

where the normalized weights $f_{k|k-1}^{(i,j)}$ correspond to the probability for a target living on road segment $i$. The calculation of the road segment dependent weight, mean, and covariance is given in [17]. Typically, $f_{k|k-1}^{(i,j)}$ is sharply peaked about a segment $i_{\text{max}}$ (depending on $j$) and negligible for segments far away from $i_{\text{max}}$. Using (53) and (57), the density in cartesian coordinates, too, is described by a Gaussian mixture over road segments.

$$p_j(\mathbf{x} | \mathbf{Z}^{k-1}) = \sum_{i=1}^{N_{seg}} f_{k|k-1}^{(i,j)} \mathcal{N}(\mathbf{x}; \mathbf{m}_{k|k-1}^{(i,j)}, \mathbf{P}_{k|k-1}^{(i,j)})$$  \hspace{1cm} (58)

$m_{k|k-1}^{(i,j)}$ and $P_{k|k-1}^{(i,j)}$ are calculated from their counterparts in road coordinates by Kalman filter like equations, see [17]. In the update step, each component $(i, j)$ is assigned to each detection $s$ and updated according to (20), (21), and (24). The weight update factor depends on road segment $i$, PHD.
component \( j \), and detection \( s \), and is denoted by \( g_k(i, j, s) \). The new normalized road segment weights are given by:

\[
f_k^{(i, j, s)} = \frac{g_k^{(i, j, s)}}{G_k} f_k^{(i, j)}
\]

(59)

\[
G_k = \sum_{i, j, s} g_k^{(i, j, s)} f_k^{(i, j)}
\]

(60)

The sum \( G_k^{(i, j, s)} \) plays the role of the update factor for the component weight (cmp. (24)): \( u_k^{(i, j, s)} = G_k^{(i, j, s)} w_k^{(i, j)} \), which are, of course, not normalized. For the calculation of \( g_k(i, j, s) \), the weighted single detection likelihood functions are required, see (30):

\[
f_k = \frac{1}{n_{k-1}} \sum_{j=1}^{J_{k-1}} P_d^{(j)} w_k^{(i, j)} \cdots \sum_{i=1}^{N_{seg}} \frac{f_k^{(i, j)} N(z_k^{(i, j)}; z_k^{(i, j)}, S_k^{(i, j)})}{N_{seg}}
\]

(61)

The transformation segment wise back to road coordinates is a simple projection onto each road segment (see [17]). The result, again, is a Gaussian mixture over road segments. This mixture, finally, can well be approximated by a single normal distribution in road coordinates using second order moment matching.

\[
v_{k|k}(\tilde{x}) \approx \sum_{j=1}^{J_{k-1}} w_{k|k}^{(j)} N(\tilde{x}; \bar{m}_{k|k}^{(j)}, \bar{P}_{k|k}^{(j)})
\]

(62)

which now can be predicted to the next time step.

The algorithm is summarized in Fig. 1: each PHD component itself is decomposed into a Gaussian mixture over road segments (a). For each segment, the mapping to cartesian coordinates is linear, preserving the Gaussian mixture shape of the density (b). The filter update (c) leads to new estimates and new road segment weights, their weighted average is the update factor for the component weight. Each PHD component is mapped back to road segments (d), and before the next prediction step (f), the Gaussian mixture over road segments is approximated by a single Gaussian in continuous road coordinates (e). Other treatments of the Gaussian mixture, avoiding this approximation, are possible, but for realistic model parameters, we have found the second order moment matching approximation to be in excellent agreement with more refined methods [17]. The price of the road segment approach is an additional computational load proportional to the number of relevant road segments which is of the order of 3-6 for typical parameter values.

V. DOPPLER BLINDNESS AND TERRAIN OBSCURATION

As mentioned in Section II, the detection probability \( P_d \) in general, is a function of target state. In particular for GMTI radar, due to the clutter notch of the sensor, \( P_d \) sharply drops to zero when the relative range rate between target and sensor,

\[ n_c \equiv |\dot{r}] = \frac{\dot{\tilde{r}} \cdot (r - r_s)}{|r - r_s|} \]

(63)

falls below a certain threshold, the minimum detectable velocity, \( v_m \). In addition, ground targets often become invisible due to terrain obscuration, e. g. by hills, which also depends on the sensor–target geometry. Within the Gaussian mixture framework, such a state dependent \( P_d(x) \) can be approximated by a component dependent \( P_d^{(j)} \) obtained by averaging \( P_d(x) \) over the component distribution, or simply by the value of \( P_d \) at the estimated target state:

\[
P_d^{(j)} \approx \int P_d(x) N(x; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}) \, dx
\]

(64)

or

\[
P_d \approx P_d(m_{k|k-1}^{(i)})
\]

(65)

While (64), in general, is difficult to calculate, (65) is a rather crude approximation when the target state covariances increases. In the case of terrain obscuration, the situation is easier for road targets where \( P_d(x) \) may well be approximated by its value in the center of each road segment, \( P_d((s_i + s_{i+1})/2) \) \( (i = 1, \ldots, N_{seg}) \).

For modeling the clutter notch, there is an alternative approach [18], [19] that preserves the Gaussian mixture structure of the PHD. The target state dependent detection probability is approximated by:

\[
P_d(x) = P_d \left[ 1 - c_m N(n_c(x); 0, Q_m) \right].
\]

(66)

with \( c_m = v_m \sqrt{\frac{\pi}{\log 2}} \) and \( Q_m = \frac{v_m^2}{2 \log 2} \). Far away from the clutter notch, \( P_d(x) \) approaches the saturated value \( P_d \), decreases by a factor of two for \( n_c = v_m \), and vanishes exactly at \( n_c = 0 \). By linearizing \( n_c(x) \) around its predicted value.
Physically, the missed detection hypothesis means, besides the 
already see that each new component now is split into two. That 
If we insert (67) into the filter update equation (9), we immedi-
ately, to a finite number of terms. Since we do not know the number 
In particular, the estimated target number is always positive.

Typically, we chose the maximum component number as 
10 times \( N \). To reduce the number of components, we 
merging components with overlapping maxima. In constrast 
to [6], [8], we do not assume to have prior knowledge on 

the location of targets sources and drains. For the detection 
of new targets we introduce a “test component” with small 
weight, location in the center of the field of view, zero speed 
and a large covariance given by the size of the field of view 
and the maximum velocity. For road targets, it is positioned 
somewhere on the road with covariance given by the length of 
the road and the maximum speed. This test component suffices 
for a quick detection of one or more new appearing target(s).

For plotting the history of a track, the actual track \((j,s)\) is 
connected with its predecessor \((j)\). In the case of merged 
tracks, the track with the largest weight is appointed prede-
cessor.

\[
\mathbf{x}_k = (x, y, \dot{x}, \dot{y})^\top.
\]

In road coordinates, the target state is parametrized by the arc 
length and velocity on road, as described above: \(\mathbf{x}_k = (l, \dot{l})^\top\). For the filtering, the target dynamic is modeled by a linear Markov process:
\[
\mathbf{x}_{k+1} = F_{k+1|k} \mathbf{x}_k + G_{k+1|k} \mathbf{v}_{k+1}
\]

We follow the realization given in [16], where the matrices 
\(F_{k+1|k}\) and \(G_{k+1|k}\) are given by
\[
F_{k+1|k} = \begin{pmatrix} 1 & t_{k+1|k} \\ 0 & e^{-t_{k+1|k}/\theta_t} \end{pmatrix}, \quad G_{k+1|k} = \begin{pmatrix} 0 \\ \Sigma_{k+1|k} \end{pmatrix}
\]

with time difference \(t_{k+1|k} = t_{k+1} - t_k\) and \(\Sigma_{k+1|k} = \nu_t \sqrt{1 - e^{-2t_{k+1|k}/\theta_t}}\). For normal, bias free process noise \(\nu_{k+1}\), it can be shown that the modeled target velocity is ergodic and given by
\[
E[\dot{l}_{k+1}] = 0, \quad \operatorname{Var}[\hat{l}_m] = \sigma_{\nu_t}^2 \exp \left[ \frac{t_k - t_m}{\theta_t} \right]
\]

For off-road targets, the corresponding four-dimensional version of (78) is used. The parameters \(\nu_t\) and \(\theta_t\) have to be chosen according to the expected dynamics of the targets under consideration. \(\nu_t\) limits the velocity standard deviation with \(\nu_t = 20 m/s\) being a typical value for road targets. The so-called maneuver correlation time \(\theta_t\) describes the agility of the targets and typically is chosen between 30 and 90 s. The matching of the parameter values with the true target behavior influences the quality of the track estimates. The measurement, too, is assumed to be a linear function of the target state:
\[
\mathbf{z}_k = H_k \mathbf{x}_k + \mathbf{w}_k,
\]

with Gaussian, bias free measurement noise \(\mathbf{w}_k\) with covari-
ance \(\mathbf{R}_k\). In the case of azimuth \(\varphi_k\) and range \(r_k\) measurements with independent noise variances \(\sigma_{\varphi}\) and \(\sigma_r\), the measure-
ments can be transformed to cartesian \((x, y)\) coordinates by a rotation about the azimuth angle. Then, the measurement

\[
\mathbf{z}_k = H_k \mathbf{x}_k + \mathbf{w}_k,
\]

(0)
matrix is strongly simplified, but the noise covariance becomes target state dependent:

$$H_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad R_k \approx Q_k D Q_k^T \quad (81)$$

$$Q_k = \begin{pmatrix} \cos \varphi_k & \sin \varphi_k \\ -\sin \varphi_k & \cos \varphi_k \end{pmatrix}, \quad D = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \tau^2 \sigma_\phi^2 \end{pmatrix} \quad (82)$$

For simplicity, we consider a static sensor platform. The Clutter notch of the sensor is modeled by a step function: $P_d = 0$ for radial velocity $|\dot{r}_k| < v_m$, and $P_d = \text{const.} \neq 0$, otherwise.

**VIII. SIMULATION RESULTS**

In the following we present first results of the implementation of the GMCPHD algorithm for ground moving targets, as described above. Fig. 2 depicts a snapshot of tracks of two group targets, with four single targets each. The groups move at constant speed of 12 m/s and enter the crossing just one after the other. Although the tracks of individual components sometime disappear and reappear, the groups are continuously tracked, and the correct target number is estimated, even without using road-maps.

![Fig. 2. Snapshot of tracks of two groups with four targets each; crossing roads. (Axis scale in meters.)](image)

Fig. 3 shows the gain in track precision and smoothness if road-map information is used for tracking, again for 8 targets, $P_d = 0.8$, and 20 false alarms on average. Typical cardinality estimates are provided in Fig. 4 for the same parameters.

The sum of weights (2) and the average of the cardinality distribution (3) provide numerically identical results, which serves as a code check. The maximum of the cardinality distribution is an integer fluctuating between 7 and the true value 8. Target number estimation in the case of changing (visible) target numbers is shown in Fig. 5, sampled over 100 Monte Carlo runs. In this scenario, there are three targets on the same road as in Fig. 3. First they are stopping and, hence, undetectable, then one target after the other starts moving along the road. After 60 scans, i.e. 10 min. one after the other stops, and 34 scans later they start to move on, again. The observable target number, therefore, changes between zero and three. The CPHD estimates follow the observable target number, however track extraction is faster, typically by 5-7 scans, if road-map information is taken into account. The effect of Doppler blindness is considered in the snapshots, Fig. 6, where the tracks of a 7-target group are plotted at different scan numbers. First, the group moves mostly in radial direction to the sensor, (positions a) and b)), implying a high $P_d$. The target number is correctly estimated and even the structure of the group is resolved. Later, at positions c) and d), the group rather moves in cross-range direction, implying a low $P_d$. Taking, the clutter notch into account, the target number estimate is still in good agreement with the true target number,

![Fig. 3. Scenario and tracks for a group of 8 road targets. Above: without, below: with road-map information.](image)

![Fig. 4. Estimated target number for a group of 8 targets on road.](image)
although the individual targets cannot be resolved anymore.

![Diagram](image6.png)

Fig. 6. Overlay of four snapshots of tracks of a group with seven targets (see text).

IX. CONCLUSION

We have applied the Gaussian mixture version of the Cardinalized Probability Hypothesis Density (GMCPHD) filter to the problem of tracking ground moving targets. A Gaussian sum approach is used to integrate digital road-maps into the Bayesian algorithm. The approach takes road-map and discretization error explicitly into account. Using road-map information, the tracks are not only smoother, but the tracker reacts faster to changing target numbers. The problem of Doppler blindness of GMTI radar sensors is treated by modeling the target state dependent detection probability. It leads to more stable tracks and a better target number estimation for low Doppler targets. Finally, we have proven that in the single target case ($N \in \{0, 1\}$), the GMCPHD filter is equivalent to the multihypothesis tracker including a sequential likelihood test for track extraction. In general, the GMCPHD filter, therefore, can be considered as an extension of single target MHT to the multitarget situation.

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