Abstract—Energy based detection measures sensor received signal strength (RSS) transmitted from a target. In this paper, we propose a new approach for estimating a moving target trajectory over a sensor field via energy based detections as an alternative to trilateration positioning or nonlinear estimation. In 2D case, possible target locations described by a RSS ratio from two sensors are approximated using a set of Gaussian random variables which are referred to as location measurements. At each data collection time, several sets of such measurements can be found from RSS ratios which are due to multiple sensor detections. A track splitting filter is used to perform either measurement fusion and target state estimation using these measurements.

The RSS ratio data mapping via Gaussian density approximation plays a key role in the proposed target tracking method and is robust in the sense that it can tolerate larger RSS noise and using additional sensor detections to improve tracking performance over trilateration based techniques. The effectiveness of the propose method is demonstrated via an example of tracking a moving target over a sensor network of small acoustic sensors.

Keywords: RSS, Data Mapping, Target Tracking, Gaussian Density Approximation, Nonlinear filtering.

I. INTRODUCTION

Integrated sensing processing using multiple sensor detections is one of the most popular target detection strategies for sensor networks of small sensors, which are usually constrained by cost, size and information processing ability, etc [1], [2]. Among signal detection techniques, the energy based signal detection, which is referring the RSS (or RSS indicator) from a target to a signal propagation model to find parameters of the target, can easily be made by low cost sensors such as acoustic sensors. As observed in [3], it has a relaxed requirement for synchronization of sensors as opposite to those which measure time difference of arrival (TDOA) or time of arrival (TOA). In the literature, application examples may be found in localizing mobile users in wireless communications systems [4], [5], sensor network localizations [6], [7] and target tracking in distributed sensor networks [3], [8], [9], etc. In principle, finding target location from the RSS collected from multiple sensors is a common idea of these applications.

In this paper, a moving target tracking over a network of small sensors in a 2D Cartesian coordinate system is considered. A set of motes are distributed with known locations to cover the whole surveillance area and they can transmit the detected RSS to a fusion center via wireless communication links. A target detection will be declared by a sensor if the RSS exceeds a threshold. In this case, we say that the sensor is activated. At each data collection time, a set of RSSs from activated sensors are expected to be received by fusion center. Our problem is to estimate the trajectory of the underlying target based on RSSs received by the fusion center.

With negligible RSS noise, approaches based on trilateration can provide exact solution of closed form to the problem [3], [5], [8], where at each time, the location “measurement” of the target is computed from the RSSs of multiple sensors and the target trajectory can be estimated based on the target location measurement sequence. The methodology of these approaches are quite similar to those with TOA and TDOA signals as seen in the literature [10]–[12].

However, trilateration based methods usually treat the underlying location state as unknown constant in the localization process and are therefore sensitive to data noise. In fact, with RSS of low SNR, these methods can either generate significant error or fail to have a real solution as pointed in [13]. In addition, deterministic localization scheme does not offer filtering any unbiased noise out and improving localization accuracy by using more data.

In the presence of RSS noise, optimization procedures as in [3], [5]–[7], which minimize the distance error based on the least square criterion, can be an option to improve localization accuracy in the trilateration framework given that there will be enough data. The underlying target location may be estimated under the maximum likelihood estimation criterion. However, as strong nonlinearity is present between the RSS detection and the target location, nonlinear filters such as particle filters have to be considered.

In this paper, we propose a new method to deal with the underlying target tracking problem in the presence of noisy energy based detections of a sensor network. A target detection, which is characterized by a RSS ratio transmitted from two sensors, defines a nonlinear and non-Gaussian density of target location. We approximate the possible target locations by a set of Gaussian densities with assumption that one of them represents the true target location density. Therefore, at each data collection time (scan), a set of RSS ratios which arise due to the detection of target by multiple sensors are mapped.
into a set of location measurement subsets. In each location measurement subset, we assume that only one measurement is from the target while the other measurements are from clutter. It turns out that the underlying problem is in fact the problem of target tracking in clutter. Our approach convert the system observability problem into system measurement uncertainty problem and enable the nonlinear, partially observed tracking problem to be addressed using linear filters with data association.

At each scan, location measurement subsets are fused using an integrated track splitting (ITS) tracker. This results in a most likely target location measurement which is modeled as the target true location corrupted with a zero-mean Gaussian noise. A Kalman filter is then used to recursively update the target true location. Given that an empirical model can be obtained as described in [4], we will consider target detection from relative RSS in this work.

A. RSS Model

Assuming a set of $M$ sensors with known locations $s_1, \ldots, s_M$ are connected wirelessly over the area of interest. At each data collection time, if a target is detected by an individual sensor, the sensor RSS will be transmit to fusion center. We assume that the target state of interest is a vector consisting of position and velocity components in both $x$ and $y$ coordinates.

The RSS at the $i$th sensor is proportional to the inverse of quadratic distance between target and sensor. Given that an empirical model can be obtained as described in [4], we will use an electromagnetic signal decay model proposed in [9] for this work to demonstrate the principle of our target tracking idea.

The RSS at the $i$th out of $M$ sensors follows an isotropic signal power attenuation model

$$E_i = \frac{E_0}{1 + \alpha r_i^n} \quad i = 1, 2, \ldots, M$$  \hspace{1cm} (1)

where $\alpha$ is an adjustable constant and $n$ is the signal decay exponent which takes values between 2 and 3 (for this work, we use $n = 2$), $r$ is the relative distance between the target detected and the sensor. Denoting by $x_k$ the target state at time $k$, and $H$ a transition matrix, the relative distance may be expressed as

$$r_i = \| Hx_k - s_i \|$$  \hspace{1cm} (2)

where $s_i$ is the location vector of the $i$th sensor. $E_0$ in (1) is the signal strength transmitted from target, i.e., the signal power received from the target at zero distance $r_i = 0$.

Taking noise into account, the RSS at the $i$th sensor may be modeled as

$$Y_i = E_i + v_{i,k}$$  \hspace{1cm} (3)

where the noise term $v_{i,k}$ is approximated as a zero-mean Gaussian with known variance $\sigma_i^2$, i.e., $v_{i,k} \sim N(0, \sigma_i^2)$, which may consist of contributions from

1) unexpected sources, such as other objects, electronic interferences;
2) quantization errors and device calibration error;
3) thermal-noise, etc.

It turns out that we may use signal to noise ratio (SNR) to measure the uncertainty of received signal, which is defined as

$$\text{SNR} = 10 \log_{10} \left( \frac{E_i}{\sigma_i} \right), \quad i = 1, 2, \ldots, N$$  \hspace{1cm} (4)

A threshold $\text{SNR}_0$ is set such that a target detection is declared by the sensor if the value of RSS exceeds it. Given $\text{SNR}_0$, the sensor maximum detection range can be calculated using (1) and (4). Interested reader may refer to [9] for more detail.

B. Work with Relative RSS

By considering a target detection as the ratio of RSSs collected from two activated sensors, we are able to deal only with the relative RSS and eliminate the unknown target transmitted signal strength $E_0$. It can also remove the impact due to non stationary signal strength from a target. Therefore, as in [3], [8], we will consider target detection from relative RSS in this work.

Assuming at time $k$, $m \leq M$ sensors are activated by a target of state $x_k$, and we denote the set of independent RSS ratios from $m$ sensor detections as $\Omega_{k,m}$. It is observed in [3], only $|\Omega_{k,m}| = m - 1$ of total $m(m - 1)/2$ ratios of RSSs are independent. In general, we need three independent ratios of the RSSs to completely localize a target in 2D case in the absence of noise.

Using Equation (1), we may express the RSS ratio received from the $i$th and $j$th sensors as

$$\mathcal{L}_h = \frac{Y_i}{Y_j} = \frac{1 + \alpha_1 r_i^2}{1 + \alpha_2 r_j^2}$$  \hspace{1cm} (5)

where the relative distances are given by (2).

In the absence of noise, each RSS ratio $\mathcal{L}_h$, $h = 1, \ldots, m - 1$ in Equation (5) defines a curve, on which a target source will yield a constant RSS ratio received by both sensors. Figure

\footnote{Although in most case, two independent RSS ratios suffice for 2D localization problem, solution ambiguity may occur and can be resolved using an additional RSS ratio.}
The basic idea is described below:

- In view of the fact that each RSS ratio defines a compact space for possible target location state, we can sample the space in a finite set of Gaussian random variables such that the target location state will be covered with certain probability.
- Treat the set of random variables as a set of noisy location measurements, while assuming only one of them is from the target.

In the presence of noise (i.e., RSS with low SNR), the value of $\frac{Y_i}{Y_j}$ is bounded by $\left[\frac{E_i - \sigma}{E_i + \sigma}, \frac{E_j + \sigma}{E_j - \sigma}\right]$, where $E_i$ is the truth of RSS measured by the $i$th sensor and both $E_i$ and $\sigma$ are defined in (3). Since $E_i$ is unknown, a conservative bound is used by replacing $E_i$ with $Y_i$, so that we have

$$\frac{Y_i}{Y_j} \in \left[\frac{Y_i - \sigma}{Y_j + \sigma}, \frac{Y_i + \sigma}{Y_j - \sigma}\right]$$

which states that possible target location states are included in the region enclosed by two finite curves $\frac{Y_i - \sigma}{Y_j + \sigma}$ and $\frac{Y_i + \sigma}{Y_j - \sigma}$.

The Gaussian density approximation technique uses a set of Gaussian random variables to represent the region defined by $\left[\frac{Y_i - \sigma}{Y_j + \sigma}, \frac{Y_i + \sigma}{Y_j - \sigma}\right]$. Each of these random variables refers to a sampling point. Therefore, possible location region of a target from a noisy RSS ratio measurement is statistically captured by a set of Gaussian densities. Of course, the more sampling points are used in the region, the better the data mapping approximation will be.

![Fig. 2](image)

**Target space approximation via Gaussian sum. Curve A is defined by** $\frac{Y_i - \sigma}{Y_j + \sigma}$ and **Curve B is defined by** $\frac{Y_i + \sigma}{Y_j - \sigma}$.
ment”, i.e.,

\[ z_{k,i} = H x_k + \nu_{k,i} \quad \nu_{k,i} \sim \mathcal{N}(0, c_i) \]  

(8)

while other measurements are from clutter.

As a consequence of using the mapped measurements, the nonlinear estimation problem can be solved using the techniques of linear state estimation with data association. In this work, we chose to use an ITS tracker.

B. Tracking using mapped data

At each time \( k \), a set of \( m - 1 \) independent RSS ratios \( \mathcal{L}_h \in \Omega_{k,m}, h \in m - 1 \) are found from the RSS received by \( m \) sensors. In the fusion center, they are mapped into \( m - 1 \) sets of location measurements \( Z_k = \{ Z_1, \cdots, Z_{m-1} \} \) using specified sampling parameters (such as the number of sampling points \( N \)). The target tracking problem is then formulated as finding the conditional posterior density \( p(x_k|Z_{1:k}) \).

We solve this problem in two steps below.

1) At each data collection time \( k \), find an equivalent target location measurement, denoted as \( \hat{Z}_k \), by probabilistically combining the data set \( Z_k = \{ Z_1, \cdots, Z_{m-1} \} \) using an ITS algorithm. Combining data is performed in a recursive form as follows.

\[ p(Z_k|Z_{1:m-2}) = \frac{1}{\delta} p(Z_{m-1}|Z_k)p(Z_k|Z_{1:m-2}) \]  

(9)

where \( \delta \) is a normalization factor.

The most likely target location measurement, denoted as \( \hat{Z}_k \), is the estimated mean value of (9).

2) Run a Kalman filter to estimate the underlying target state using the most likely target measurement sequence, i.e., \( p(x_k|\hat{Z}_1, \cdots, \hat{Z}_k) \).

Alternatively, we can perform standard Bayesian recursion for all location measurements under ITS algorithm. However, in our experience this would yield a bit more computational complexity while a similar track accuracy is observed.

C. ITS algorithm

Described in [15], [16], the ITS algorithm is a multiple hypotheses type tracking filter and it follows all measurement histories by its component propagation. A component is the statistics of a particular validated measurement history. In other words, a component is a special track that follows a single measurement path along time. Track state estimate is obtained by summarizing its components in the form of Gaussian mixture under a probabilistic framework of track existence. Appropriate hypotheses management techniques are used to keep the number of components under control. In our simulation, a mixture reduction algorithm [17] is used to keep the number of components within 10. Reader may refer to [15] for more detail of this algorithm.

The ITS tracker is well fitted in the data fusion problem in the sense that it can carry a non Gaussian density \( p(Z|Z_1) \) using its components in the form of Gaussian mixture and recursively update the posterior density \( p(Z|Z_{1:m-1}) \).

That, we use an ITS tracker estimate target location measurements at each data collection time from a set of (mapped) non Gaussian distributed location measurements.

IV. SIMULATION AND RESULTS

To verify the proposed approach, we simulate a scenario of tracking a moving target over a sensor network of acoustic sensors and compare the performances of proposed algorithm in several situations. \( M = 36 \) motes (with acoustic sensor on board) are uniformly deployed over a 250 m\(^2\) surveillance area. All acoustic sensors are assumed to have been calibrated and their locations are exactly known to fusion center.

Data is generated based on the model (1) and (3), where we assume the signal strength from target is of level 10, \( \alpha_i = 1, \forall i \in M \) and \( n = 2 \). A threshold for minimum SNR is chosen such that target will not be detected by a sensor when the SNR of the sensor RSS is below the threshold (which yields maximum detection range \( r_{\text{max}} = 50 \) m for all sensors in this scenario).

Target motion follows a simple model

\[ x_{k+1} = F x_k + w_k \]  

(10)

where \( x_k \) is the target state, a vector with four components that includes target position and velocity in a 2D Cartesian coordinate system.

\[ F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]

and \( T = 5 \) (sec) is a fixed interval between data collections. The system process noise is assumed to be Gaussian, i.e., \( w_k \sim \mathcal{N}(0, Q_k) \)

\[ Q_k = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} & 0 & 0 \\ \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \frac{T^3}{3} & \frac{T^2}{2} \\ 0 & 0 & \frac{T^3}{3} & T \end{bmatrix} q, \]

Fig. 3. Moving target trajectory on the field of sensor network.
As shown in Figure 3, target start moving from location 
[50, 110](m) at velocity [0.8, 0.8](m/s). It changes its velocity to [0.8, -0.8] and [-1, 0.8] at scans \( k = 25 \) and \( k = 40 \) respectively.

At each data collection time \( k \), fusion center may receive RSSs from up to 7 activated sensors and works out the set of \( m - 1 \) independent RSS ratios \( \Omega_{k,m} \). If only one sensor detected RSS is received at \( k \), there will be no measurement at scan \( k \).

There are quite a few ways to practically mapping RSS data into location data. In our simulation, a simple Gaussian sum density approximation procedure as described below is used to obtain the set of \( N \) Gaussian densities for each RSS ratio \( L_h \in \Omega_{k,m}, \) where both \( N = 20 \) and \( N = 50 \) are considered.

1. Base on each \( L_h = \frac{Y_h}{\sigma_h} \) and the locations of the \( i \)th and \( j \)th sensors, we compute the corresponding curve and uniformly take \( N \) sampling points \( \{z_1, \cdots, z_N\} \) as the mean values of \( N \) Gaussian densities.

2. Repeat Step 1 for the RSS ratios \( L'_h = \frac{Y_h + \sigma_h}{\sigma_h} \), \( L''_h = \frac{Y_h - \sigma_h}{\sigma_h} \), and we then obtain \( \{z'_1, \cdots, z'_N\} \) and \( \{z''_1, \cdots, z''_N\} \) respectively.

3. For each sampling point \( z_i \), \( i = 1, \cdots, N \), compute the distances \( d_1 = ||z_i - z_{i-1}||, d_2 = ||z_i - z'_i|| \) and \( d_3 = ||z_i - z''_i|| \). From distances \( d_1, d_2, d_3 \), find the variance of the \( i \)th density. This Step is repeated for all \( N \) densities.

The above Steps are performed for all \( m - 1 \) independent RSS ratios at each time \( k \).

In the simulation, two noise levels \( \sigma_1 = 0.0001 \) and \( \sigma_i = 0.018, \forall i \in M \) are considered, which yield the averaged SNRs for RSS received by fusion center 52.13 (dB) and 7.05 (dB) respectively. In each case, 100 runs are performed.

The performance of tracking is measured using root mean squared (RMS) error criterion. Table IV presents the time averaged overall performance comparison. In the table, the term complexity reflect the computational cost of the algorithm and its value is the CPU time normalized by the slowest CPU time in the case \( N = 50 \) and \( SNR = 7dB \). The values of time averaged RMS errors indicate that estimation error becomes smaller when the SNR of RSS is higher.

<table>
<thead>
<tr>
<th>Sampling Pts</th>
<th>SNR(dB)</th>
<th>RMSE-P</th>
<th>RMSE-V</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 20 )</td>
<td>52.13</td>
<td>1.2656</td>
<td>0.2602</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>7.05</td>
<td>4.1673</td>
<td>0.7938</td>
<td>0.3064</td>
</tr>
<tr>
<td>( N = 50 )</td>
<td>52.13</td>
<td>0.4820</td>
<td>0.1406</td>
<td>0.8406</td>
</tr>
<tr>
<td></td>
<td>7.05</td>
<td>4.0798</td>
<td>1.1012</td>
<td>1</td>
</tr>
</tbody>
</table>

The RMS error performance versus time is shown in Figures 4 and 5. Given that these results are consistent with our intuition, it is observed when higher SNR is present, using more sampling points in data mapping can reduce the estimation error since the "measurement" resolution cell tends to be smaller. However, this is not the case when SNR of RSS is lower since the error due to signal noise dominates the volume of the "measurement" covariance.

Figure 6 displays both the estimated target trajectory and mapped location data accumulated from 60 scans. Similar performance has been observed in our simulation for this target tracking scenario when using an ITS tracker for all time rather than using the two steps strategy as outlined in Section III. However, the computational load in the former case is almost doubled to that in latter case when 10 components are allowed to be maintained in the ITS algorithm.

V. CONCLUSION

In the presence of RSS noise, trilateration based methods, which use a pair of RSS ratios to localize a static target, give no guarantee for the underlying target to be observed as there...
could be no solution. Therefore, a single RSS ratio becomes a realistic window from which the responsible target may be observed. However, we face a nonlinear, partially observed system estimation problem.

In this paper, target tracking in a distributed sensor network with energy based detections is considered. A data mapping scheme is proposed in this work. It uses Gaussian density approximation technique to approximate a nonlinear, non-Gaussian density which describes possible target location measurements. As a consequence, the system observability problem is converted into system measurement uncertainty problem and we can address the nonlinear, partially observed tracking problem using linear filters with data association. In the present simulation example, we implement an ITS tracker to fuse noisy data from multiple sensors to obtain the most likely (or equivalent) target location measurement at each scan. A Kalman filter is applied to estimate the target state based on the sequence of the most likely target location measurements. Simulation results demonstrate the effectiveness of the proposed method.

The proposed method is suitable for other applications such as those localization problem [5], [13] in the frameworks of circular positioning and hyperbolic positioning in the presence of measurement noise.

One of the research challenges is how to map sensor detections (RSS ratios) which are contributed by multiple target sources and therefore tracking multiple targets. If we cannot differentiate multiple sources from time domain, additional information will be necessary to resolve the RSS ambiguity which arises due to the presence of multiple sources and we will continue to look at this problem in our future research.

REFERENCES