STDF Model Based Maritime Situation Assessments

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Abstract – The State Transition Data Fusion (STDF) Model is an extension of the dominant sensor fusion paradigm to provide a unification of both sensor and higher-level fusion. Maritime Domain Awareness (MDA) is the problem of situation awareness in the maritime domain. This paper outlines an application of the STDF model to perform automated situation assessments for an aspect of MDA.

Keywords: Sensor Fusion, Higher-Level Fusion, Data Fusion, JDL Model, Object Assessment, Situation Assessment, Maritime Domain Awareness, State Transition Data Fusion.

1 Introduction

The Joint Directors of Laboratories (JDL) model remains the most dominant model of data fusion (e.g. [1],[2]). For a few reasons Lambert ([3]) has evolved a deconstruction of the JDL model for data fusion in which: the machine specific elements are removed; “level 0” is subsumed within “level 1”; and “level 4” is subsumed within “level 1”, “level 2” and “level 3” as appropriate.

Figure 1 exposes the resultant deconstructed JDL model, where:

- object assessments are stored representations of objects;
- situation assessments are stored representations of relations between objects; and
- impact assessments are stored representations of effects of relations between objects.

Both the JDL model and its deconstructed form celebrate the difference between object, situation and impact fusion, but at the expense of demonstrating their unity. In response Lambert ([4]) proposed the State Transition Data Fusion (STDF) model. Like the JDL model, the STDF model is a functional model. However, the STDF model is a less abstract model than the JDL model and seeks to demonstrate a unifying framework across the three levels of the deconstructed JDL model.

2 Maritime Domain Awareness

Maritime Domain Awareness (MDA) is the problem of situation awareness in the maritime domain. One advantage of the deconstructed JDL model is that it highlights the correspondence between the fusion process and situation awareness, as defined by Endsley ([5]). In particular, situation awareness is data fusion performed by people, while data fusion is like situation awareness performed by machines. Data fusion is therefore highly applicable to all aspects of MDA.

One of the simpler MDA challenges is to identify commercial maritime traffic operating on sea lanes from among a myriad of water and air craft. A complex North Atlantis scenario has been developed in partnership, with Canadian partners from DRDC Valcartier implementing “level 1” ground truth. The scenario includes: six nations with alliances and hostilities, destroyers, frigates, mine vessels, patrol boats, submarines, merchant ships, whaling and counter whaling vessels, coast guard vessels, tourist vessels, commercial aircraft, surveillance aircraft, military strike aircraft, and military and civilian helicopters. The author has identified the significant sea lanes, which are illustrated in Figure 2.
The Canadian participants also have implemented sensor models and a “level 1” fusion process that introduces measurement errors into object assessments of the scenario. This paper discusses the challenge of generating situation assessments of the merchant shipping in the scenario, based upon the object assessments received.

3 The STDF Model

Under the STDF model, Env : Time Step → P(States) denotes the environment of interest, understood in terms of states and transitions between those states. Time is monitored through the structure of time steps <Time Step; ≤> where ≤ is an unbounded, discrete, linear ordering with x < y = df x ≤ y & ¬(x = y). Consequently, for each time step k there is a unique successor time step k+1 defined by z = k+1 = df k < z & ∀m (k < m ⇒ z ≤ m). The set States abstractly denotes the set of all possible states of the world. At any time step k ∈ Time Step, the world is composed of a number of states from P(States), for power set P(x) = {u | u ⊆ x}. Under the STDF model, the environment of interest Env is understood in terms of state transition identities. Each identity s : Time Step → (States ⊆ Λ) is a state transition identity, where s(k) = ⊥ indicates that s does not exist at time step k. The environment of interest Env is then understood in terms of some set of state transition identities J ⊆ {s : Time Step → (States ⊆ Λ)}, so that Env = {<k, s(k)> | s ∈ J & ¬(s(k) = ⊥)}. The uppermost portion of Figure 3 illustrates a state transition identity s.

f : Time Step → (DB × S × D × R × A × P × O × I × U) is a state transition data fusion process under the STDF model, where S, D, R, A, P, O, I and U are the sets of possible formal theories for sensation, detection, registration, association, state prediction, observation prediction, initiation, and update respectively, and DB is a database, with f(k+1) = <DB(k+1), S(k+1), D(k+1), R(k+1), A(k+1), P(k+1), O(k+1), I(k+1), U(k+1)>.

At time step k+1 the machine employs sensation theory S(k+1) to sense a set of states s(k+1) = {s(k+1) | i ∈ N′(#s(k+1))} in the environment Env(k+1) through its sensors and transfers its sensed data e(k+1) = {e(k+1) | i ∈ N′(#e(k+1))} to an observation process1. The observation process involves a detection process that uses detection theory D(k+1) to identify detections d(k+1) = {d(k+1) | m ∈ N′(#d(k+1))} (including false alarms) from the sensed data e(k+1), such that (D(k+1) ∪ e(k+1)) ⊆ d(k+1) & ∀m (d(k+1) ∪ e(k+1)) ⊆ o(k+1) for ∀o(k+1) ∈ o(k+1); and then an association process. The association process first draws upon a prediction process, which: accesses some previous representations ˇs(k) = {ˇs(k) | i ∈ N′(#s(k))} ⊆ DB(k) of states at time k; uses a state prediction process with theory P(k+1) to posit predicted states ˇs(k+1) = {ˇs(k+1) | i . j ∈ N′(# ˇs(k+1))} with possibly multiple predictions from each state ˇs(k) [k] such that (P(k+1) ∪ ˇs(k)) ⊆ ˇs(k+1) & then applies these to an observation prediction process employing theory O(k+1) to posit predicted observations ˇe(k+1) = {ˇe(k+1) | i . j ∈ N′(# ˇe(k+1))} for which (O(k+1) ∪ ˇe(k+1)) ⊆ ˇe(k+1) & ˇe(k+1) & then transfers control to an explanation process. If the observation o(k+1) is matched to a predicted observation

1 N′(n) = {1, ..., n} for n ∈ N – {0}.

Figure 3. The State Transition Data Fusion Model
\( \hat{d}_i^j(k+1|k) \), then an update process uses update theory \( U(k+1) \) to update the representation \( \hat{s}^i(k|k) \) of the state \( s^i(k) \) with the explained state \( \hat{s}^i(k+1|k+1) \) for state \( s^i(k+1) \) where \( (U(k+1) \cup \{ \hat{s}^i_j(k+1|k) \} \cup \{ o^i'(k+1) \}) \neq \hat{s}^i(k+1|k+1) \) (or possibly \( \hat{s}^i_i(k+1|k+1) \), \( \hat{s}^i_j(k+1|k+1) \), ...), if multiple hypothesis updates are to be countered). If no observation \( o^i'(k+1) \) is matched to a predicted observation \( \hat{d}_i^j(k+1|k) \), then an update process uses update theory \( U(k+1) \) to potentially update the representation \( \hat{s}^i(k|k) \) of the state \( s^i(k) \) with the explained state \( \hat{s}^i(k+1|k+1) = \bot \). If the observation \( o^i'(k+1) \) fails to match a predicted observation \( \hat{d}_i^j(k+1|k) \), then an initiation process applies initiation theory \( I(k+1) \) to insert a representation of a new state with state information \( \hat{s}^i(k+1|k+1) \), for \( r > \# \hat{s}^i(k|k) \), such that \( (I(k+1) \cup \{ o^i'(k+1) \}) \neq \hat{s}^i(k+1|k+1) \). Through a data fusion process involving prediction, observation and explanation, the related states of the world \( \{ s(t) | t \in Time_Step & t \leq k \} \) are taken to form a state transition identity that is represented in the machine as a set of explained states \( \{ \hat{s}^i(t) | t \in Time_Step & t \leq k \} \).

The STDF model can be applied to all three levels of the deconstructed JDL model ([4]). For object assessments, States comprises objects with measurable properties. An object instance at time \( k \) is a state of the world \( s(k) \) understood as a state vector \( u(k) \) of measured values. An object \( \{ s(t) | t \in Time_Step & t \leq k \} \) at time \( k \) is understood as a set of transitioning object instances and so each object at time \( k \) is understood as a set of state vectors \( u(k) = \{ u(t) | t \in Time_Step & t \leq k \} \). For situation assessments, States comprises situations expressed as sets of sets of statements about the world. A situation instance at time \( k \) is a state of the world \( s(k) \) understood as a state of affairs \( \Sigma(k) \), comprising a set of statements about the world in some formal language. A situation \( \{ s(t) | t \in Time_Step & t \leq k \} \) at time \( k \) is understood as a set of transitioning situation instances and so each situation at time \( k \) is understood as a set of states of affairs \( \Sigma(k) = \{ \Sigma(t) | t \in Time_Step & t \leq k \} \). For impact assessments, States comprises scenarios expressed as sets of situations. A scenario instance at time \( k \) is a state of the world \( s(k) \) understood as a scenario state \( \delta(k) = \{ \delta(n) | n \in Time & n \leq k \} \), for monotonic look ahead time \( \delta(k) > k^2 \). A scenario \( \{ s(t) | t \in Time & t \leq k \} \) at time \( k \) is understood as a set of transitioning scenario instances and so each scenario at time \( k \) is understood as the set of scenario states \( \delta(k) = \{ S(t) | t \in Time & t \leq k \} = \{ \{ \Sigma(n) | n \in Time & n \leq \delta(n) \} | t \in Time & t \leq k \} \).

4 Situation Assessments for MDA

This paper focuses on situation assessments for merchant ships in the maritime domain of Figure 2. Each subsection of this section successively applies each of the elements of Figure 3.

4.1 Detection

Following [4], for situation assessments the detection process in Figure 3 is the object assessment process. In the merchant shipping problem, each object \( i \) at time step \( k \) is a state \( s^i(k) \), which as a state vector \( u^i(k) \), is ideally explained through a state estimate \( \hat{u}^i(k|k) \) expressed as a column vector of the form \( \langle x, y, v_x, v_y \rangle \), where \( x \) and \( y \) are Cartesian coordinates relative to the sensor origin, and \( v_x \) and \( v_y \) are the velocity components respectively in the \( x \) and \( y \) directions. Uncertainty is expressed by a 4\times4 covariance matrix \( P^i(k|k) \) so that under the multivariate normal distribution assumptions of the object model outlined below, \( \hat{u}^i(k|k) \) is the expected value \( \mu = \langle x, y, v_x, v_y \rangle \) and the probability \( p(<x, y, v_x, v_y>) \) of any column vector \( \langle x, y, v_x, v_y \rangle \) is given by the probability density function

\[
\begin{align*}
f(x) &= \frac{1}{(2\pi)^{n/2}|P|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T P^{-1} (x-\mu)\right). 
\end{align*}
\]

A typical Kalman filter model of the environment is given by \( \hat{u}^i(k+1) = F(k+1) \hat{u}^i(k) + G(k+1) + g(k+1) \) for object \( i \), where: \( \hat{u}^i(k) \) and \( u^i(k+1) \) are the unknown \( n \)-dimensional state vectors at time steps \( k \) and \( k+1 \) respectively; \( F(k+1) \) is a given behavioral model of the object at time step \( k \) expressed as an \( n \times n \) state transition matrix; \( g(k+1) \) is process noise expressed as an \( n \)-dimensional zero mean, white, Gaussian noise vector; and \( \hat{u}^i(k+1) \) is the known deterministic \( n \)-dimensional input change between time steps \( k \) and \( k+1 \). The observation of object \( i \) at time step \( k+1 \) is modeled by \( \hat{y}(k+1) = H(k+1) \hat{u}(k+1) + y(k+1) \), where: \( H(k+1) \) is a 4\times4 measurement matrix; and \( y(k+1) \) is the 4-dimensional zero mean, white, normally distributed noise vector with known 4\times4 measurement noise covariance matrix. The details of the Canadian “level 1” solution, which is a detection theory \( D(k+1) \) from the “level 2” perspective, lie beyond the scope of this paper.

4.2 Registration

The object assessment process produces an object assessment report detailing a set of observation vectors \( \langle i, t, \tau, \hat{u}^i(k+1|k+1), P^i(k+1|k+1), src_id, src_loc \rangle \) where: \( t \) is the detection timestamp; \( \tau \in \{ \text{surface, air, subsurface, unknown} \} \) is the environment type; \( src_id \) identifies the sensor; and \( src_loc \in [-\pi/2, \pi/2] \times [-\pi, \pi] \) identifies the location of the source in latitude and longitude radians. At the situation assessment level, the interest in each object relates to its position, speed and course. The semantic registration process therefore translates the observation vector information into statements about an object’s position, speed and course.

Given state vector \( \hat{u}^i(k+1|k+1) = \langle x, y, v_x, v_y \rangle \), the distance of the object from the sensor is \( d_{k+1}^i = \sqrt{(x^2 + y^2)} \); the speed is \( v_{k+1}^i = \sqrt{(v_x^2 + v_y^2)} \); and the course \( c_{k+1}^i = (\pi/2 - \alpha) \) if \( v_x \geq 0 \) & \( v_y \geq 0 \); \( (\pi/2 + \alpha) \) if \( v_x \geq 0 \) & \( v_y < 0 \); \( -(\pi/2 - \alpha) \) if \( v_x \leq 0 \) & \( v_y \geq 0 \); \( -(\pi/2 + \alpha) \) if \( v_x < 0 \) & \( v_y \leq 0 \).
\[ \text{and } v_y < 0; \ (\frac{3\pi}{2} - \alpha) \text{ if } v_y < 0 \text{ and } v_x < 0; \ (\frac{3\pi}{2} + \alpha) \text{ if } v_x < 0 \text{ and } v_y < 0, \text{ for } \alpha = \arctan \left( \frac{v_y}{v_x} \right). \]

For src\_loc = <\theta, \varphi>, the position \( \mathbf{c}_{k+1} \) of the object is the great circle translation of src\_loc by distance \( d_{k+1} \) and angle \( \varphi \) (e.g. [6]). This information is expressed propositionally by \( \Phi_k(k+1) = (i, t, \mathbf{c}_{k+1}) \text{ and speed}(i, t, \mathbf{v}_{k+1}) \text{ and course}(i, t, \mathbf{c}_{k+1}). \)

The object assessment information comes with a degree of uncertainty expressed by the covariance matrix \( \Sigma(k+1) \). The probability of \( \text{at}(i, t, \mathbf{v}) \), for any position \( \mathbf{v} \), depends on \( \langle x, y \rangle \) value of the assessed state vector \( \mathbf{u}(k+1|k+1) \), and so \( \text{at}(i, t, \mathbf{v}) \)

\[
\frac{1}{2\pi} \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{pmatrix} \exp \left( -\frac{1}{2} \begin{pmatrix} x - \mu_x \\ y - \mu_y \end{pmatrix}^T \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{pmatrix}^{-1} \begin{pmatrix} x - \mu_x \\ y - \mu_y \end{pmatrix} \right),
\]

where \( \langle x, y \rangle \) is the Cartesian translation of \( \mathbf{v} \). \( \Sigma(\mathbf{v}) \) is the upper partition of covariance matrix \( \Sigma = \left[ \begin{array}{cc} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{array} \right] \) and \( \Sigma_{xx} = \Sigma - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx} \). The probability of speed\((i, t, v)\) for any speed \( v \) depends on the \( \langle x, y \rangle \) value of the assessed state vector \( \mathbf{u}(k+1|k+1) \), with the value of \( p(\langle x, y \rangle) \) given by

\[
\frac{1}{2\pi} \begin{pmatrix} \sigma^2_x & \sigma^2_y \\ \sigma^2_y & \sigma^2_x \end{pmatrix} \exp \left( -\frac{1}{2} \begin{pmatrix} x - \mu_x \\ y - \mu_y \end{pmatrix}^T \begin{pmatrix} \sigma^2_x & \sigma^2_{xy} \\ \sigma^2_{yx} & \sigma^2_y \end{pmatrix}^{-1} \begin{pmatrix} x - \mu_x \\ y - \mu_y \end{pmatrix} \right),
\]

The probability of course\((i, t, c)\) for any course \( c \) also depends on \( \langle x, y \rangle \) value of the assessed state vector \( \mathbf{u}(k+1|k+1) \), and can be determined analogously case wise. Conditional distributions can also be recovered, such as \( p(\langle x, y \rangle > \langle x, y \rangle | \text{at}(i, t, \mathbf{v})) \), thereby identifying conditional propositional distributions such as \( p(\text{speed}(i, t, v) | \text{at}(i, t, \mathbf{v})) \). In this way the probabilities associated with the marginal, conditional and joint propositional distributions are known. Numerical integration of multivariate normal distributions can be calculated with moderate accuracy in real time ([7]) through techniques such as Gauss-Hermite quadrature (e.g. [8]).

Formally, \( \langle \mathcal{C}, \mathcal{P}, p_e \rangle \) is a probability space in which \( \mathcal{C} \) is the sample space of all joint observational propositions; \( \mathcal{P} = \mathcal{P}(\mathcal{C}) \) is the set of observational propositions for power set \( \mathcal{P} \); and \( p_e : \mathcal{P} \to [0, 1] \) is a probability measure. For each time step \( k \), random variable \( \Phi_k(k) : \mathcal{C} \to \{0, 1\} \) is a random variable that identifies the truth of an observational proposition about object \( i \) at time step \( k \). Using the theory \( \mathcal{R}(k+1) \) outlined here, the semantic registration process stores the expected joint proposition and its probability \( p_e(\Phi_k(k+1)) = p_e(\text{at}(i, t, \mathbf{c}_{k+1}) \text{ and speed}(i, t_{k+1}, \mathbf{v}_{k+1}) \text{ and course}(i, t_{k+1}, \mathbf{c}_{k+1})) \), and inference rules for computing the remaining joint probability values for propositions other than the expected proposition.

### 4.3 Representation

Section 4.1 noted that the object assessment state equation often assumes a linear model \( y(k+1) = F(k+1) y(k) + g(k+1) + n(k+1) \) in which uncertainty is assumed to accord with a multivariate normal distribution that is determined from expected value (mean) and covariance parameters. The re-conceptualisation from States as sets of measurable state vectors for object assessments to States as sets of symbolically based states of affairs for situation assessments, means that the state equation for object assessments is re-conceptualised to a state equation \( \Sigma(k+1) \subseteq \{ \sigma | (F(k+1) \cup \Sigma(k) \cup \Pi(k+1) \cup \Delta(k+1)) \} \). For situation assessments, where: \( \Sigma(k) \) is the unknown state space of objects at time step \( k \), expressed as a formal theory; \( F(k+1) \) is a given functional model of the object at time step \( k \) expressed as a formal theory; \( \Pi(k+1) \) is uncertainty expressed as a probabilistic assignment of truth to atomic formulae; \( \Delta(k+1) \) is the known input change between time steps \( k \) and \( k+1 \), expressed as a formal theory; \( \Sigma(k+1) \) is the unknown state space of objects at time step \( k+1 \), expressed as a formal theory; and \( \Delta \) is a formal theory of meaning (see [10]).

In general the formal theory of uncertainty \( \Pi(k+1) \) cannot be assumed to be a multivariate normal distribution, and so situation assessment prediction requires an alternative to the traditional expected state equation \( \Sigma(k) \subseteq \{ \sigma | (F(k+1) \cup \Sigma(k) \cup \Pi(k+1) \cup \Delta(k+1)) \} \). It is possible to express a probabilistic assignment of truth to atomic formulae; \( \Delta(k+1) \) is the known input change between time steps \( k \) and \( k+1 \), expressed as a formal theory; \( \Sigma(k+1) \) is the unknown state space of objects at time step \( k+1 \), expressed as a formal theory; and \( \Delta \) is a formal theory of meaning (see [10]).

For the merchant shipping problem, the state of affairs of interest are among the simplest possible, (a) because each contains only one vessel, and (b) because each features numerically based propositions. More complex states of affairs arise in other aspects of the North Atlantic scenario that (a) describe relations between many vessels, and (b) contain no numerical information. These are not discussed in this paper, though the approach taken can be extended to include them. For the merchant shipping problem, the state of affairs for object \( i \) at step \( k \) is represented by \( \Sigma(k) \).

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3 Here, as in [9], the term “formal theory” does not assume a logically closed set of formal language sentences.
Let sea_lane(n, b, e) denote that sea lane number n begins at location b and ends at location e. Sea lanes are conceived of as directed, so that

∀n ∀b ∀e (sea_lane(n, b, e) \iff sea_lane(-n, e, b)).

The scenario of Figure 2 therefore has 22 sea lanes, hereafter labeled from -11 to 11. For an object and a given sea lane, interest centres on: (i) whether the object is on the sea lane; (ii) how far the object has traveled long the sea lane; (iii) where the object is across the sea lane; and (iv) what speed the object is traveling at.

To facilitate an implementation, definite possible worlds can be created for these issues of interest at a desired level of resolution. Concerning (iii), allowing sea worlds can be created for these issues of interest at a

\[
\text{maximum speed of a merchant ship is taken to be } s(n)_{\text{max}}/r(n)_{s}. \text{ A modal } \text{theory for sea lane, interest centres on: (i) whether the object is on sea lane; (iii) where the object is across the sea lane, and (iv) what speed the object is traveling at.}
\]

Concerning (ii), allowing a resolution of r(n) along the central line through b and e for sea_lane(n, b, e) of length distance(b, e) / r(n). Concerning (iv), allowing a speed value of

\[
v_k = s_k(r(n)) \text{ for each } k \in \{1, \ldots, N(n)\} \text{ and each } i \in \{1, \ldots, n\}. \text{ The set of possible worlds at time } k \text{ is then defined by }
\]

\[
\mathcal{W} = \{ \text{on_lane}(i, t_k, n), \text{along_lane}(i, t_k, n, \alpha) \}
\]

across_lane(i, t_k, n, \alpha), speed(i, t_k, s_k) \}

k ∈ Time_Step & t_k ∈ Time & \alpha ∈ ATM(n) &
\]

X_k ∈ XTC(n) & s_k ∈ SC(n) & n ∈ [-11, ..., 11] \}

∪ \{ -on_lane(i, t_k, n, \alpha(i, t_k, \ell)) \}

k ∈ Time_Step \}

where t_k and \ell are parametric in the definition of possible worlds but assigned when used, and \#\mathcal{W} = 1 + \sum_{n=-11}^{11} (2 \cdot r(n) \cdot r(n) \cdot r(n)). As the number of possible worlds is finite, they can be enumerated. Hereafter each possible world for each object i at time step k has an identifying index and is denoted by W_i(k). Concerning the truth function \text{v, v}(\alpha, W_i(k)) \in \{0, 1\} in accordance with the usual modal logic semantics \forall \alpha \in P \text{ and } \forall W_i(k) \in \mathcal{W}. \text{ Accessibility relation } R \text{ is used to represent transitions, with } R(W_i^j(t), W_i^k(t)) \text{ admitted for reflexivity and } R(W_i^j(k), W_i^j(t)) \text{ admitted only if } i \text{ can transition from } W_i^j(k) \text{ at time step } k \text{ to } W_i^j(t) \text{ at time step } t \text{ (see section 4.4).}

To superimpose a probability distribution, let \mathcal{W}, E, \mu \text{ be a probability space in which: } \mathcal{W} \text{ is the sample space of all possible worlds for object } i; E = P(\mathcal{W}) \text{ is the set of possible events for object } i; \text{ and } \mu : E \rightarrow [0, 1] \text{ is a probability measure. For each k, let random variable } W_i(k) : \mathcal{W} \rightarrow [0, 1] \text{ identify the truth of possible worlds at time k. Situations can be understood in terms of partial worlds. Each partial world } \omega(k) \text{ at time k then corresponds to an event in } E \text{ and so } \mu(\omega(k)) = \sum_{W_i(k) \in \omega(k)} \mu(W_i(k)). A state of affairs } \Sigma(k) \text{ at time k}

\[
\text{includes the meaningful consequences of a partial world, i.e. } \Sigma(k) \subseteq \{ \sigma \mid \omega(k) \in \mathcal{W} \}. \text{ Let } p = \mu \cup \mu. \text{ At time step k, the conditional distribution } p(W_i^j(k) | W_i^k(k)) = p(W_i^j(k) \mid \{\Phi(1), \ldots, \Phi(k)\}) \text{ over all possible worlds for given observations of } i \text{ up to time step k, is stored and so can be used to compute } p(\alpha(k)) \text{ for each partial world } \alpha(k), \text{ and therefore } p(\Sigma(k)) \text{ for any state of affairs } \Sigma(k). \text{ The expected state of affairs } \Sigma^i(k) \text{ at time k for object i can be determined from the maximum posterior probability } p(W_i^j(k) \mid \{\Phi(1), \ldots, \Phi(k)\}).
\]

4.4 State Prediction

The association process needs to reconcile the new observational propositions with previously known states of affairs in the environment. It first uses theory F(k+1) to predict the future states of affairs (section 4.4), and then theory O(k+1) to identify what would be observed should those predicted states obtain (section 4.5).

As the conditional probability of each state of affairs of interest can be derived from the conditional distribution across possible worlds, it is sufficient to concentrate on the latter. With a conditional independence assumption and p(W_i^j(k) | k) denoting p(W_i^j(k) \mid \{\Phi(1), \ldots, \Phi(k)\})), p(W_i^j(k+1) | \{\Phi(1), \ldots, \Phi(k)\}) = p(W_i^j(k+1) \mid \{\Phi(1), \ldots, \Phi(k)\}) \text{ for each partial world } \alpha(k), \text{ and therefore } p(\Sigma(k)) \text{ for any state of affairs } \Sigma(k). \text{ The expected state of affairs } \Sigma^i(k) \text{ at time k for object i can be determined from the maximum posterior probability } p(W_i^j(k) \mid \{\Phi(1), \ldots, \Phi(k)\}).
associated with a sea lane, region and speed, independently of time. A contiguity relation \( \text{ctgs}(W_{m_i}, W_{j}) \) can then be defined to identify spatially contiguous world sets, while the probability of transition between spatially contiguous world sets is specified by \( p(W_{j} | W_{m_i}) \). I then define relation \( R_h \) by \( R_h(W_{m_i}, W_{j}) = \text{ctgs}(W_{m_i}, W_{j}) \) & \( p(W_{j} | W_{m_i}) > 0 \). \( R_h \) identifies the most basic transitions (accessibilities) and the conditional probability of each. The transitive effect of \( R_h \) is then given by \( R_h(<W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}>, t, p) \) which identifies: a path \( <W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}> \) of successively contiguous possible world sets with positive transitional probability; the minimum time \( t \) required to transition that path; and the probability \( p \) of that path being transitioned. As each possible world definition includes a region and an average speed, a heuristic search can determine both paths and the minimum time \( t \) for transitioning that path. \( p \) is the product of conditional probabilities. For each path \( <W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}> \) selected by \( R_h \) with \( W_{m_1} = W_{i_1} \) and \( W_{j} = W_{i_k} \), the probability \( p \) in \( R_h(<W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}>, t, p) \) is 

\[
p = \sum_{c=1}^{q} \prod_{c=1}^{q} p(W_{i_c} | W_{m_1}) \] and so by conditional independence, 

\[
p(\{ W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j} \}) = p \cdot p(W_{m_1}) \cdot \sum_{s} \delta[p(\exists \beta_2, ..., \beta_{k-1}: \exists R_h(<W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}>, t, p))] \] 

and so 

\[
p(W_{j} | W_{m_1}) = \sum_{s} \delta[p(\exists \beta_2, ..., \beta_{k-1}: \exists R_h(<W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}>, t, p))] \] 

For times \( t^k_k \) and \( t^k_{k+1} \) associated with time steps \( k \) and \( k+1 \) respectively, \( p(W_{j}(k+1) | W_{m_1}(k)) \) is therefore defined by 

\[
p(W_{j}(k+1) | W_{m_1}(k)) = \sum_{s} \delta[p(\exists \beta_2, ..., \beta_{k-1}: \exists R_h(<W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}>, t, p))] \] 

As all the world set information is known a priori, these computations can be pre-computed except for the simple \( t \leq t^k_{k+1} - t^k_k \) test.

The accessibility relation is then formally defined by 

\[
R'(W_{m_1}(k), W_{j}(k+1)) = \exists \beta \exists \beta_2 ... \exists \beta_{k-1} (R_h(<W_{m_1}, W_{i_2}, ..., W_{i_{k-1}}, W_{j}>, t, p) \land t \leq t^k_{k+1} - t^k_k) \] 

with the additional stationary requirement \( \forall W_{m_1}(p(W_{j}(k+1) | W_{m_1}(k)) > 0) \) to ensure reflexivity. This facilitates predictive probabilistic modal reasoning. Applying section 4.3, for any proposition \( \alpha \in P \), \( p(\alpha) = \sum_{W_{m_1} \in \{\forall(V(R(W, V) = v), U = 1)\}} p(W) \) and so for any \( \beta \in P \), \( p(\beta) = \sum_{W_{m_1} \in \{\forall(V(R(W, V) = v), U = 1)\}} p(W) \). But \( p(\beta) = \prod_{U \in |V| R}(v(\beta, U) = 1) \) iff \( \forall V (R(W, V) = v(\beta, V) = 1) \)

\[
p(\beta) = \sum_{W_{m_1} \in \{\forall(V(R(W, V) = v), U = 1)\}} p(W) \] 

4.5 Observation Prediction

Section 4.1 reflected that the observation of object i at time step k+1 is often modeled by an m-dimensional vector \( q'(k+1) = H(k+1) u'(k+1) + \nu(k+1) \). Re-conceptualising States as states of affairs for situation assessments, re-conceptualises the m-dimensional observation vector to a theory 

\[
\Phi(k+1) \subseteq [\sigma | (H(k+1) \cup \Sigma(k) \cup (\lambda(k+1)) h_u, \sigma)] \] 

for observation theory \( H(k+1) \) and uncertainty theory \( \lambda(k+1) \). Consequently a formal theory \( H(k+1) \) is required to identify expected observation proposition \( \Phi'(k+1) \) for object i at time step k+1.

For the merchant shipping problem, sea lane options potentially generate multiple observation propositions, and so indexed predicted observation propositions \( \Phi'_i(k+1) \) may be generated. Let \( D = \{ W_{j}(k+1) | p(W_{j}(k+1) | \{\Phi'(1), ..., \Phi'(k)\}) > \tau \} \) for some threshold \( \tau \). \( D \) identifies the dominant possible worlds in the distribution. If \#D = 0, then a new D is formed with a lower threshold. The closer the possible worlds distribution approximates a uniform distribution, the harder it becomes to nominate an expected value. If \#D > 1 then there is a single dominant world \( W_{j}(k+1) = \{\text{on_lane}(i, t, n), \text{along_lane}(i, t, n, a), \text{across_lane}(i, t, n, x), \text{speed}(i, t, s)\} \). The midpoints of the intervals a and x are used to define position \( l \); the midpoint of interval s defines v, and the angle from position \( l \) to the endpoint of sea lane n provides course c, where \( \Phi'(k+1) \) = \{\text{atti}, t_{k+1} \}, speed(i, t_{k+1}, v), course(i, t_{k+1}, c)\}. For \#D > 1 there are several dominant worlds. For each world \( W_{j}(k+1) \in D \), an equivalence class denoted \( \tilde{W}_{j}(k+1) \), is defined by:

\[
\tilde{W}_{j}(k+1) \in D = \{ W_{j}(k+1) \}; \\
\tilde{W}_{j+1}(k+1) = \tilde{W}_{j}(k+1) \cup \{ W_{m_1}(k+1) | W_{j}(k+1) \in \tilde{W}_{j}(k+1) \cup \text{ctgs}(W_{j}(k+1), W_{m_1}(k+1)) \}; \\
\tilde{W}_{j}(k+1) = \frac{\beta}{\int_{q=0}^{\infty} \tilde{W}_{j+1}(k+1) \} = \tilde{W}_{j}(k+1) \}.
\]
Concerning (a), I evaluate the probability distribution has associated q at time \( k+1 \) with i at time \( k+1 \) (given that two pieces of information: (a) the observation distribution object q is in fact the predicted observation proposition whether the reported observed proposition course(i, t, i at time step k). Association theory partitioning of contiguous worlds on the same sea lane in evaluates the probability that

\[
\hat{D} = \{ \hat{W}_i^k(k+1) \mid W_i^k(k+1) \in D \}
\]

then defines a partitioning of contiguous worlds on the same sea lane in D. If \([\text{along}_\text{lane}(i, t, n, [a_n, a_i]), \text{across}_\text{lane}(i, t, n, [a_n, x_n]), \text{speed}(i, t, [a_n, s_n])] \subseteq W_i^k(k+1)\) for each

\[
W_i^k(k+1) \in \hat{W}_i^k(k+1),
\]

then I define

\[
\widehat{a}_i^k = \frac{\sum_{q=1}^{\# \hat{W}_i^k(k+1)} \left[ \frac{a_{q,b} + a_{q,c}}{2} \right] p(W_i^k(k+1))}{\sum_{q=1}^{\# \hat{W}_i^k(k+1)} p(W_i^k(k+1))}
\]

with \( \widehat{a}_i^k \) and \( \widehat{a}_j^k \) defined analogously. The values \( \widehat{a}_i^k \) and \( \widehat{a}_j^k \) identity the expected along, across and speed values respectively for each sub-distribution \( \hat{W}_i^k(k+1) \in \hat{D} \), and so by defining \( \vec{c}_i^k \) from \( \widehat{a}_i^k \) and \( \vec{c}_j^k \) as \( \widehat{a}_i^k \) and \( \widehat{a}_j^k \) as the angle from \( \vec{c}_i^k \) to the end point of sea lane n, each \( \hat{W}_i^k(k+1) \in \hat{D} \) generates an expected observation proposition \( \hat{\Phi}_i^k(k+1|k) = \{ \text{at}(i, t, \vec{c}_i^k), \text{speed}(i, t, v_j^k), \text{course}(i, t, c_j^k) \} \)

4.6 Association

For a number of different objects, the semantic registration process of section 4.2 provides an expected observation proposition \( \Phi_i^k(k+1) = \{ \text{at}(q, t_i^k, \vec{c}_i^k), \text{speed}(q, t_i^k, v_j^k), \text{course}(q, t_i^k, c_j^k) \} \) and a probability distribution \( \{ \Phi^k(k+1), p(\Phi^k(k+1)) \} \) and \( \Phi^k(k+1) \) is a set of expected observation propositions \( \hat{\Phi}_i^k(k+1|k) = \{ \text{at}(i, t, \vec{c}_i^k), \text{speed}(i, t, v_j^k), \text{course}(i, t, c_j^k) \} \) for each known state of affairs for object i at time step k. Association theory \( \Lambda(k+1) \) must decide whether the reported observed proposition \( \Phi_i^k(k+1) \) for object q is in fact the predicted observation proposition \( \hat{\Phi}_i^k(k+1|k) \) for object i.

There are a few steps to consider in the association process. As with “level 1” data association, the proposition association benefit from some initial process. As with “level 1” data association, the

\[
\hat{\Phi}_i^k(k+1|k), \Phi_i^k(k+1) \}
\]

and so take \( p(is(\hat{\Phi}_i^k(k+1|k), \Phi_i^k(k+1)) \mid \Phi_i^k(k+1)) \), to be

\[
p((a(t_i, t_i^k, \vec{c}_i^k), \text{speed}(q, t_i^k, v_j^k), \text{course}(q, t_i^k, c_j^k))).
\]

This measure considers the difference between the observed and expected propositions given the degree of uncertainty associated with the observed propositions.

Concerning (b), \( \Gamma = \{ is(\hat{\Phi}_i^k(k+1|k), \Phi_i^k(k+1)), \Phi_i^k(k+1) \} \) is taken to be about observations of an existing object with the approximate location, speed and course of \( \Phi_i^k(k+1) \), while \( I = \{ -is(\hat{\Phi}_i^k(k+1|k), \Phi_i^k(k+1)), \Phi_i^k(k+1) \} \) is taken to be about observations of a new object with the approximate location, speed and course of \( \Phi_i^k(k+1) \). BI

\[
\{ \text{believes}(\text{src}_i, i(k+1) = q(k+1)), \Phi_i^k(k+1), \Phi_i^k(k+1) \}
\]

is taken to be about the source correctly associating an existing object with observations of that existing object with the approximate location, speed and course of \( \Phi_i^k(k+1) \), while BI

\[
\{ \text{believes}(\text{src}_i, i(k+1) = q(k+1)) \}
\]

is taken to be about the source incorrectly associating an existing object with observations of a new object with the approximate location, speed and course of \( \Phi_i^k(k+1) \). Then

\[
p(\text{believes}(\text{src}_i, i(k+1) = q(k+1)) \mid \Gamma) = \frac{p(BI^+)}{p(I^+)}
\]

Object level fusion for some locations, speeds and courses is more difficult than for others, such as near branching sea lanes, and some sources have greater competency at reporting observations with certain locations, speeds and courses than others. The probabilities \( p(\text{believes}(\text{src}_i, i(k+1) = q(k+1)) \mid \Gamma) \) and \( p(\text{believes}(\text{src}_i, i(k+1) = q(k+1)) \mid I) \) capture these aspects, using past joint distribution data. The score

\[
p(is(\hat{\Phi}_i^k(k+1|k), \Phi_i^k(k+1)) \mid \{ \Phi_i^k(k+1), \text{believes}(\text{src}_i, i(k+1) = q(k+1)) \})
\]

is obtained by combining the information in (a) and (b). For \( \sigma = \{ \Phi_i^k(k+1), \Phi_i^k(k+1) \} \) and \( \beta = \{ \text{believes}(\text{src}_i, i(k+1) = q(k+1)) \}

\[
\text{score is} \quad p(\Phi_i^k(k+1), \beta) = \frac{p(\beta | I^+).p(\sigma | \Phi_i^k(k+1)) + p(\beta | I^+).p(\sigma | \Phi_i^k(k+1)) - p(\sigma | \Phi_i^k(k+1))}{p(\beta | I^+).p(\sigma | \Phi_i^k(k+1)) - p(\sigma | \Phi_i^k(k+1))}.
\]

An assignment of reported observations to predicted observations then occurs by forming the matrix of score values and applying an auction assignment process ([12]) or a similar technique, while ensuring that at most one assignment is made from the set \{ \hat{\Phi}_i^k(k+1|k) | j \in N \}. The association process can: (1) associate \( \Phi_i^k(k+1) \) with \( \hat{\Phi}_i^k(k+1|k) \) for some q, i and j, in which case \( \Phi_i^k(k+1) \) becomes \( \Phi_i^k(k+1) \) and the state update process is invoked; (2) associate no observation report with any member of \{ \hat{\Phi}_i^k(k+1|k) | j \in N \} for some i, in which case the state update process is notified of this; or (3) associate \( \Phi_i^k(k+1) \) with nothing for some q, in which case the state initiation process is invoked with \( \Phi_i^k(k+1) \).
4.7 State Initiation

If $\Phi^i_j(k+1)$ is not associated with $\hat{\Phi}^i_j(k+1)$ for any $j$ and existing $i$, then a new state of affairs is to be initiated by theory $l(k+1)$ for a new object $r$ at time step $k+1$, based upon the observation proposition $\Phi^j_l(k+1)$ now termed $\Phi(k+1)$. A Bayesian approach gives

$$p(W^r_j(k+1) \mid \{\Phi^j_l(k+1)\}) = \frac{p(\Phi^j_l(k+1) \mid W^r_j(k+1)) \cdot p(W^r_j(k+1))}{p(\Phi^j_l(k+1))}$$

with

$$p(\Phi^j_l(k+1)) = \sum_{W^r_j(k+1) \in W^r} p(\Phi^j_l(k+1) \mid W^r_j(k+1)) \cdot p(W^r_j(k+1)).$$

Each $p(\Phi^j_l(k+1) \mid W^r_j(k+1))$ term is computed by identifying the intervals comprising $W^r_j(k+1)$ and integrating across the probability density function associated with $\Phi(k+1)$ with those intervals. Each $p(W^r_j(k+1))$ term relates the a priori probability of an object in possible world $W^r_j(k+1)$.

4.8 State Update

If $\Phi^j_l(k+1)$ is associated with $\hat{\Phi}^j_l(k+1)$ to become $\Phi(k+1)$ for some $j$ and existing $i$, then $U(k+1)$ updates the possible worlds distribution for object $i$ at time step $k+1$.

$$p(W^j_i(k+1) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k+1)\}) = \frac{p(W^j_i(k+1) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k+1)\})}{p(\Phi^j_l(1), \ldots, \Phi^j_l(k+1))}$$

But $p(W^j_i(k+1)) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k+1)\}) = p(\Phi^j_l(k+1) \mid \{W^j_i(k+1), \Phi^j_l(1), \ldots, \Phi^j_l(k)\})$.

$$p(W^j_i(k+1) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k+1)\}) = p(\Phi^j_l(k+1) \mid \{W^j_i(k+1), \Phi^j_l(1), \ldots, \Phi^j_l(k)\}) \cdot p(W^j_i(k+1) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k)\})$$

under an assumption of conditional independence, while

$$p(\Phi^j_l(k+1) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k)\}) = \sum_{W^j_i(k+1) \in W^j_i} p(\Phi^j_l(k+1) \mid W^j_i(k+1)) \cdot p(W^j_i(k+1) \mid W^j_i(k+1)).$$

under an assumption of conditional independence. Thus

$$p(W^j_i(k+1) = p(W^j_i(k+1) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k+1)\}) = \frac{p(\Phi^j_l(k+1) \mid W^j_i(k+1)) \cdot p(W^j_i(k+1) \mid \Phi^j_l(1), \ldots, \Phi^j_l(k+1))}{\sum_{W^j_i(k+1) \in W^j_i} p(\Phi^j_l(k+1) \mid W^j_i(k+1)) \cdot p(W^j_i(k+1) \mid \Phi^j_l(1), \ldots, \Phi^j_l(k+1))}.$$

Each $p(W^j_i(k+1) \mid \{\Phi^j_l(1), \ldots, \Phi^j_l(k)\})$ term is obtained from the prediction step outlined in section 4.4. $p(\Phi^j_l(k+1) \mid W^j_i(k+1))$ is computed as suggested in section 4.7.

5 Conclusions

This paper has shown how the STDF model can be applied to form situation assessments for the merchant shipping problem. The formulation is currently being implemented and the results will be presented at the conference. The possible worlds distribution approach provides a means of interfacing “level 1” and “level 2” fusion, so that higher level decisions about vessel alliances, intent, destinations, et cetera can be deduced as consequences of a preferred possible world.

References