On Track Fusion with Communication Constraints

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Abstract—Distributed Kalman filters are often used in multisensor target tracking where the fusion center receives local estimates and fuses them to obtain the global target state estimate. With such a fusion architecture, each local tracker can communicate less frequently with the fusion center than the local filter update rate. The global target state estimate via track fusion is usually less accurate than that of the centralized estimator when local estimation errors are correlated and local trackers communicate to the fusion center with bandwidth constraints lower than the measurement rate. This paper focuses on the tradeoff between bandwidth and tracking accuracy for track fusion with communication constraints. We show that the performance degradation increases for track fusion on demand compared with the centralized estimator as the number of local trackers increases. We relate the steady state analysis of track fusion under bandwidth constraints to noisy Wyner-Ziv source coding problem and compare our results with the theoretical rate distortion curve of the quadratic Gaussian CEO problem. We conclude that track fusion on demand is a side-information unaware strategy while the awareness of the correlated estimation errors at each local tracker can improve the track fusion accuracy significantly.

Keywords: Tracking fusion, steady state analysis, quadratic Gaussian CEO.

I. INTRODUCTION

In multisensor target tracking a centralized fusion architecture has been traditionally adopted as the baseline to achieve the best tracking accuracy. In this architecture all the measurements from different sensors are sent to a single location, called the fusion center (FC), to obtain the global target state estimate [3]. Recent trend in the design of robust and scalable sensor networks has drawn increased interest in distributed fusion architectures where many local data processors exist and they can send the local target state estimates to the fusion center [7], [22]. Due to communication bandwidth constraints, each local tracker only reports its estimate to the fusion center when the corresponding channel is available, i.e., communication on demand, and usually the communication between a local tracker and the fusion center operates at a lower rate than the filter update rate of the local tracker. The fusion center fuses the local track estimates to improve the system performance in terms of tracking accuracy. Compared with the centralized track estimate, the performance of distributed track fusion degrades when the local estimation errors are correlated due to common prior or common process noise being used in the local Kalman filters [3]. Both analytical and numerical studies have shown that track fusion only has less than 5% increase in the mean square error (MSE) of either target position or velocity estimate in comparison with the centralized solution in the two sensor case [3], [4]. However, as the number of sensors increases, the performance difference also increases especially when sensors have similar measurement accuracy and share similar sensor-to-target geometry [5].

This paper focuses on the tradeoff between bandwidth and tracking accuracy for track fusion with communication constraints. We show that the MSE of the global track estimate decreases in a slower rate using track fusion on demand than that of the centralized estimation as the number of sensors increases. We relate the steady state analysis of track fusion under bandwidth constraints to noisy Wyner-Ziv source coding problem [21] and compare our results with the theoretical rate distortion curve of the quadratic Gaussian CEO problem [20]. We conclude that track fusion on demand is a side-information unaware strategy and the awareness of the correlated estimation errors at each local tracker can improve the track fusion accuracy significantly when the number of sensors is large.

II. PROBLEM FORMULATION

A. Distributed Kalman Filter

To simplify the analysis, we assume that the target motion is modeled by the following linear equation

\[ x(t_{k+1}) = F(t_{k+1}, t_k)x(t_k) + v(t_{k+1}, t_k) \]

where \( x(t_k) \) is the target state vector at time \( t_k \); \( F(t_{k+1}, t_k) \) is the state transition matrix from time \( t_k \) to \( t_{k+1} \); and \( v(t_{k+1}, t_k) \) is the process noise from time \( t_k \) to \( t_{k+1} \). We assume that there are \( N \) sensors measuring the same target and report the raw measurements or local estimates to the fusion center. We do not consider possible false alarms and missed detections in the subsequent analysis. Simulation study based on more realistic distributed tracking scenarios including data association issues can be found in [5], [11]. To ease the analysis, we also assume that all sensors are synchronized and have the same measurement rate. In this case, the target state equation can be simplified as

\[ x(k+1) = Fx(k) + v(k) \]  \hspace{1cm} (1)

where \( v(k) \) is a zero-mean white Gaussian process noise sequence with covariance \( Q(k) \). The measurement model for
sensor $i$ is

$$y_i(k) = H_i x(k) + w_i(k)$$  \hfill (2)$$

where $w_i(k)$ is a zero-mean white Gaussian measurement error sequence with covariance $R_i(k)$ and independent of the process noise. The centralized estimator uses the following state prediction

$$\hat{x}_c(k|k-1) = F \hat{x}_c(k-1|k-1)$$  \hfill (3)$$

$$P_c(k|k-1) = FP_c(k-1|k-1)F^T + Q(k-1)$$  \hfill (4)$$

and updates the state estimate using all measurements as follows

$$\hat{x}_c(k|k) = P_c(k|k-1)^{-1}(\hat{x}_c(k|k-1) + \sum_{i=1}^{N} H_i^T R_i(k)^{-1} y_i(k))$$  \hfill (5)$$

$$P_c(k|k) = \left( P_c(k|k-1)^{-1} + \sum_{i=1}^{N} H_i^T R_i(k)^{-1} H_i \right)^{-1}$$  \hfill (6)$$

It yields the optimal global target state estimate in the minimum mean square error (MMSE) sense [3]. In a distributed configuration, the $i$-th sensor (also called local tracker) obtains state prediction $\{\hat{x}_i(k|k-1), P_i(k|k-1)\}$ based on (3)–(4) and updates the state estimate $\{\hat{x}_i(k|k), P_i(k|k)\}$ based on (5)–(6) using measurement $y_i(k)$ only. At the FC, these estimates are fused as follows

$$\hat{x}_d(k) = P_d(k|k-1)^{-1} \hat{x}_d(k|k-1) + \sum_{i=1}^{N} P_i(k|k-1)^{-1} \hat{x}_i(k|k)$$

$$P_d(k) = \left( P_d(k|k-1)^{-1} + \sum_{i=1}^{N} P_i(k|k-1)^{-1} P_i(k|k-1)^{-1} \right)^{-1}$$

The above fusion formula is equivalent to the centralized solution [3]. However, each local tracker needs to communicate to the FC at the same rate as its filter update rate. This is usually not feasible due to limited communication bandwidth since a target state estimate may have a higher dimension than a measurement. Alternatively, a local tracker can communicate to the fusion center less frequently on its local track estimate. The reduced communication rate by a factor of $m$ results in the state prediction at the FC being replaced by the $m$-step prediction $\{\hat{x}_d(k|m), P_d(k|m)\}$. Since the one-step local prediction $\hat{x}_i(k|k-1)$ relies on the local estimate $\hat{x}_i(k-1|k-1)$ which is unavailable at the FC, a simple alternative is to replace it with the $m$-step local prediction $\hat{x}_i(k|m)$ in the above fusion formula. Note that the modified fusion formula is not optimal when $m > 1$. Furthermore, the filter calculated estimation error covariance is usually optimistic [13]. The optimal fusion formula was obtained in [4] for arbitrary $m$. However, the FC needs to know the local target motion model and the time instances at which each local tracker updates its local estimates. More general results on the best linear estimation fusion schemes with various prior knowledge at the FC can be found in [1], [14], [15].

### B. Track Fusion on Demand

Since track fusion is operated on demand depending on the available communication bandwidth, the FC may not know $m$ before receiving local estimates. In the worst case, we assume that $m \to \infty$ and thus the state predictions are useless, i.e., $P_d(k|m)^{-1} \to 0$ and $P_i(k|m)^{-1} \to 0$ ($\forall i$). The fusion formula becomes

$$\hat{x}_d = P_d \sum_{i=1}^{N} P_i^{-1} \hat{x}_i$$  \hfill (7)$$

$$P_d = \left( \sum_{i=1}^{N} P_i^{-1} \right)^{-1}$$  \hfill (8)$$

where the time indices are dropped for brevity. Note that the above fusion formula is optimal in the MMSE sense only when the estimation errors from the local trackers are independent [3]. Due to the common process noise being used by the local Kalman filters, the local estimates do have correlated errors and thus the above fusion formula is overly optimistic about its estimation error covariance. Denote by $P_{ij}$ the crosscovariance of the estimation errors between local trackers $i$ and $j$. The optimal track fusion is [5]

$$\hat{x}_d = P_d \left[ \begin{bmatrix} I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & I & 0 \\ 0 & \cdots & 0 & I \end{bmatrix} P^{-1} \right] \hat{x}$$  \hfill (9)$$

$$P_d = \left( I P^{-1} \right)^{-1}$$  \hfill (10)$$

where

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_N \end{bmatrix}, \quad I = \begin{bmatrix} I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & I & 0 \\ 0 & \cdots & 0 & I \end{bmatrix}, \quad P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1N} \\ P_{21} & P_{22} & \cdots & P_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N1} & P_{N2} & \cdots & P_{NN} \end{bmatrix}$$

The above fusion formula (7)–(8) is not optimal in the MMSE sense when the FC has prior knowledge of the target state, e.g., state prediction based on the previous estimate. However, it has the advantage that the FC does not need to have the knowledge of target motion model.

We are interested in how the performance degrades in terms of the relative error $||P_d - P_c||/||P_c||$ in the steady state when $N$ is large and the communication rate is extremely low, e.g., $m$ is large. Note that by increasing $m$ the effect of quantization error at each local tracker becomes negligible especially for large $N$. It has been shown that in the scalar Gaussian case, the estimator based on the transmission of a single bit per sensor can have the variance as small as $\pi/2$ times that of the clairvoyant estimator without communication constraints for large $N$ [17]. The distributed parameter estimation problem without knowing the noise distribution was studied in [23] where the authors established the $O(1/N)$ scaling law of the MSE for one-bit rate-constrained decentralized estimator. In a distributed Kalman filter setting, [18] showed that the

\[ \text{It is optimal in the MMSE sense given all local estimates, not all measurements and thus still suboptimal compared with the centralized estimator.} \]
MSE scales in $O(1/N)$ when the FC operates in full rate but only receives one bit, namely, the sign of innovation, from each local tracker. Thus we will ignore the quantization error contributed to the MSE of the track fusion scheme when comparing the scaling law in terms of $N$ between track fusion and centralized estimation.

C. Quadratic Gaussian CEO Problem

When $m$ is finite, each tracker can also deploy sophisticated source coding scheme to satisfy the bandwidth constraints. In this case, the FC is treated as the central estimation officer (CEO) who employs $N$ agents observing noise-corrupted versions of a source process $\{x(k)\}_{k=1}^{\infty}$. The quadratic Gaussian CEO problem deals with estimating the Gaussian source $\{x(k)\}$ with observations $\{y_i(k)\}_{i=1}^{N}$ under additive Gaussian noise with communication rate constraints $\{R_i\}_{i=1}^{N}$. To simplify the discussion, we assume both source and observation sequences are scalar. The distortion between the source and its estimate at CEO is measured by

$$d = \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} E[|x(k) - \hat{x}(k)|^2]$$

From the information theoretic viewpoint, an important question is to find the rate region $(R_1, ..., R_N)$ so that a given distortion $d$ is achievable [20]. If we assume that all agents have the same observation noise variance and the same rate constraints, then we only need to characterize the rate-distortion function $(R, d)$ by the sum rate $R = \sum_{i=1}^{N} R_i$ and study how this curve scales with $N$. The rate-distortion region provides the theoretical limit of achievable MSE using distributed track fusion with more sophisticated encoding scheme. It can be related to the steady state analysis of the distributed Kalman filter, to be discussed next.

III. STEADY STATE ANALYSIS

A. Estimation Error Covariance: Track Fusion vs. Centralized Solution

We consider a simple target motion model given by discretized continuous white noise acceleration model [2] with sampling period $T$ so that

$$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad Q(k) = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix} q$$

where $q$ is the power spectrum density of the process noise. Assume that the measurement matrix is $H = [1 \ 0]$ and the measurement error variance is $R(k) = \sigma_w^2$. Then the steady state filter gain is $W = \left[ \alpha \ \frac{\beta}{\sqrt{2}} \right] T$ where

$$\alpha = \beta \sqrt{u}, \quad \beta = \frac{12}{6(u + \sqrt{u}) + 1}, \quad u = \frac{1}{3} + \sqrt{\frac{1}{12} + \frac{4}{\lambda^2}}$$

and $\lambda$ is the target maneuvering index given by

$$\lambda = \sqrt{\frac{T^3 q}{\sigma_w^2}}$$

The steady state estimation error covariance is

$$P = \begin{bmatrix} \alpha & \beta \\ \frac{\beta}{\sqrt{2}} \alpha & \frac{\beta^2}{2} \end{bmatrix} \sigma_w^2$$

If all the $N$ sensors have the same sampling rate and measurement error variance, then the centralized solution has the error covariance

$$P_c = \begin{bmatrix} \alpha_c & \beta_c \\ \frac{\beta_c}{\sqrt{2}} \alpha_c & \frac{\beta_c^2}{2} \end{bmatrix} \sigma_w^2$$

with $\alpha_c$ and $\beta_c$ given by (9) where the local target maneuvering index $\lambda$ is replaced by the global target maneuvering index $\lambda_c = \sqrt{N} \lambda$. The distributed track fusion has the error covariance

$$P_d = \frac{1}{N} P + \frac{N - 1}{N} \Sigma$$

where $\Sigma$ is the crosscovariance of the estimation errors of any two sensors due to the common process noise which can be obtained by solving the following algebraic Lyapunov equation [3]

$$\Sigma = (I - WH)(F \Sigma F^T + Q)(I - WH)^T$$

It has been shown that, as $N$ increases, the relative difference between $P_d$ and $P_c$ also increases [5]. This general trend is also true for other target motion models as well as for asynchronous sensors with independent measurement noises.

B. Performance Limit of Track Fusion

We are interested in the performance limit of track fusion vs. centralized estimation as $N$ increases. We compare the relative difference in the position MSE $(P_{d11} - P_{c11})/P_{c11} \cdot 100\%$ and in the velocity MSE $(P_{d22} - P_{c22})/P_{c22} \cdot 100\%$. In the steady state, the difference in the position MSE increases from $5\%$ ($N = 2$) to $34\%$ ($N = 5$) for small target maneuvering indices ranging from $\lambda < 0.1$ as shown in Fig. 1. Similarly, the difference in the velocity MSE increases from $5\%$ ($N = 2$) to more than $25\%$ ($N = 5$) especially when target maneuvering indices ranging from 0.1 to 1 as shown in Fig. 2. Note that the results are in line with those obtained in [4] for the two-sensor case. Next we will show that the relative performance difference between track fusion and centralized estimation will go unbounded as $N$ increases.

Since we assume that track fusion has very low communication rate ($m \to \infty$), the problem of interest usually should have the target maneuvering index $\lambda$ being small such that both local trackers and the fusion center do not have much uncertainty on the target motion. In this case, the first order approximation $\alpha \approx \sqrt{2\lambda}$ and $\beta \approx \lambda$ are valid. We further assume that $\alpha_c \approx \sqrt{2\lambda_c}$ and $\beta_c \approx \lambda_c$ even for large $N$. Note that without process noise, track fusion has identical performance to that of the centralized estimation. However, small process noise is often assumed even for constant speed target motion model. Here we focus on how $P_c$ and $P_d$ scale with $N$ when $\lambda_c$ is small. In the steady state, the MSE in position using centralized estimation decreases in $O(N^{-3/4})$. 
The MSE between track fusion and centralized estimation will go unbounded as $N$ increases. Note that for finite $m$, each local tracker can use the measurement at time $k$ to update the state prediction based on the local estimate at time $k-m$ and send the local estimate to the fusion center. In this case, the distributed Kalman filter is optimal assuming that measurements from time $k-m+1$ to time $k-1$ are unavailable. In the steady state, the FC assumes the target maneuvering index $\lambda_c = \sqrt{m/N}\lambda$. For small $\lambda$, if $m$ increases with a scaling law lower than $O(N)$, then the MSE in position using distributed Kalman filter at the FC decreases faster than $O(1)$. Intuitively, the MSE obtained at a local tracker using $m$ successive measurements has a scaling law no worse than that using the fused estimate from $m$ sensors, each of which runs Kalman filter update with only a single measurement.

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**Fig. 1.** Percentage difference of position MSEs between track fusion and centralized estimation for 2–5 sensors

**Fig. 2.** Percentage difference of velocity MSEs between track fusion and centralized estimation for 2–5 sensors

However, the MSE in velocity using centralized estimation will decrease in $O(1)$. Intuitively, when one is certain about target velocity from the motion model, having more noisy position measurements can not further improve the estimation accuracy in velocity unless $\lambda_c$ is not too small. It is worth noting that for asynchronous sensors, the best achievable scaling law of the MSE in position using centralized estimation is still $O(N^{-3/4})$ while the MSE in velocity can scale in $O(N^{-3/2})$ due to the fact that the measurement rate seen at the FC can be $N$ times of the synchronous case.

The MSEs in both position and velocity will decrease in $O(1)$ when using track fusion. Intuitively, as $N$ increases, the crosscovariance term in (10) will dominate the fused estimation error covariance no matter what target maneuvering index is chosen. Thus the relative difference in the position MSE between track fusion and centralized estimation will go unbounded as $N$ increases. Note that for finite $m$, each local tracker can use the measurement at time $k$ to update the state prediction based on the local estimate at time $k-m$ and send the local estimate to the fusion center. In this case, the distributed Kalman filter is optimal assuming that measurements from time $k-m+1$ to time $k-1$ are unavailable. In the steady state, the FC assumes the target maneuvering index $\lambda_c = \sqrt{m/N}\lambda$. For small $\lambda$, if $m$ increases with a scaling law lower than $O(N)$, then the MSE in position using distributed Kalman filter at the FC decreases faster than $O(1)$. Intuitively, the MSE obtained at a local tracker using $m$ successive measurements has a scaling law no worse than that using the fused estimate from $m$ sensors, each of which runs Kalman filter update with only a single measurement.

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**Fig. 3.** Crosscorrelation coefficients between local estimation errors for position and velocity

Define the crosscorrelation coefficient between the local estimation errors of position component as $\Sigma_{11}/P_{11}$ and that of velocity component as $\Sigma_{22}/P_{22}$. Fig. 3 shows that the crosscorrelation coefficients of both position and velocity increase as the target maneuvering index $\lambda$ decreases. Note that for small $\lambda$ the position MSE $P_{11}$ is twice that assuming the local estimation errors are independent when $N = 5$. In this case, the velocity MSE $P_{21}$ is four times that assuming independent errors. As $N$ increases or $\lambda$ decreases, the benefit of using more sensors in track fusion diminishes. Similar results have been reported in [12] where the total bit rate is constrained on the distributed estimation using BLUE fusion.

**C. Other Communication Strategies**

Instead of sending local estimates to the FC on demand, the local trackers can also compute some alternative quantities of the same dimension and communicate them with the FC. One possible approach is to transmit tracklets [9]. A tracklet is an estimated target state whose errors are uncorrelated with any other measurements or estimates of the same target.
Typically, local tracks from the same target are decorrelated immediately before the tracklet is transmitted. It is equivalent to the decorrelation at the FC, i.e., finding a linear transform to make $P$ block diagonal. When $m \to \infty$, tracklet fusion has the same accuracy as track fusion. For practical tracking scenarios including target motion and measurement origin uncertainties, tracklet fusion can have a slightly better performance than track fusion [8], [10]. Another approach is to communicate to the FC the compressed measurements having the same dimension as the local estimate. In this case, the FC can achieve the same accuracy as centralization after filter update [6], [24]. However, the FC needs to know $m$ and, in the asynchronous sensor case, every time instance at which a measurement is made at the local tracker after the last communication. Thus this approach can not perform the fusion of compressed measurements on demand with many sensors having very low communication bandwidth.

IV. RATE DISTORTION FUNCTION OF QUADRATIC GAUSSIAN CEO PROBLEM

We consider a tracking example with $N$-sensor track fusion where $m$ is finite. We limit our discussion to the position component of the steady state MSE assuming the target maneuvering index is fairly small. Note that finding the rate-distortion region of the quadratic Gaussian CEO problem with correlated source is still an open problem. Thus we focus on the scalar case and rewrite the sensor measurement model as

$$y_i(k) = x_i(k) + n_i(k), \quad i = 1, \ldots, N$$

where the position component $x_i(k)$ of the target state has a Gaussian prior with variance $\sigma_x^2$ and all measurement errors are independent Gaussian with zero mean and the same variance $\sigma_w^2$. If each sensor has a maximum data rate $R$ to communicate with the FC, then the above quadratic Gaussian CEO problem has the minimum achievable mean square error (MSE) distortion $d$ given by the root of the following equation [16]

$$NR = \frac{1}{2} \log \left\{ \frac{\sigma_x^2}{d} \right\}^{\frac{N}{N+1}} \left\{ \frac{N \sigma_w^2 d}{(N \sigma_x^2 + \sigma_w^2)^2} \right\}^{\frac{1}{N+1}}$$

The above rate distortion function has the following properties. When $R \to \infty$, the minimum distortion is the well known minimum mean square error (MMSE) given by

$$d = \frac{1}{1/\sigma_x^2 + N/\sigma_w^2}$$

It can be achieved by using centralized estimation without communication constraint. When $\sigma_x^2$ and $N$ are large, we have $d \approx \sigma_x^2 e^{-2NR}$ for small $R$. This means that the distortion decays exponentially fast with respect to $R$ and consequently with respect to $m^{-1}$ for large $m$. Note that the track fusion on demand that we considered here corresponds to the low rate regime with $\sigma_x^2 \to \infty$. However, as $R$ becomes large, the distortion decays inversely proportional to $R$ which is the penalty paid by disallowing sensors to communicate with each other. Recall that the position distortion of the centralized estimator scales in $O(N^{-3/4})$ for small $\lambda$ without communication constraints. Taking quantization of the local estimates into account, the distortion still decays exponentially fast in $N$ under the low rate regime. Thus track fusion on demand which scales $O(1)$ for large $N$ is clearly not on the frontier of the rate distortion curve, i.e., it does not fully utilize the communication bandwidth to achieve the minimum distortion.

V. ROBUSTNESS VS. ACCURACY: TOWARDS A BETTER SCALING LAW

From the information theoretic viewpoint, finding the best communication mechanism between local trackers and the fusion center is a distributed source coding problem where each local tracker has to compress the source separately in a lossy fashion. It is often cast into a noisy Wyner-Ziv source coding problem [21]. In general, an achievable rate region can be obtained by extending the Slepian-Wolf coding scheme to a cascade of a suitable vector quantizer with a binning operation for the codeword indices [17]. The achievable distortion scales differently in $N$ under rate constraints depending on whether the decoder at the FC has side information. The side information includes the prior of target state and the crosscovariance of the estimation errors of the local trackers. Intuitively, if we are given two bin indices communicated by the two local sensors, the decoder needs to undo the binning and retrieve the correct quantization cell indices. This requires the two bin indices to be jointly typical with its corresponding source sequences. If one sensor has a communication failure, the decoding at the FC is likely to fail. Thus the efficiency in reducing the communication rate sacrifices the robustness of the decoder at the FC.

Track fusion on demand is robust in the sense that it can operate on an arbitrary number of local trackers. In the steady state, any individual communication failure will not affect the FC to fuse the remaining available information. When $N$ is large, the performance degradation due to communication failure is negligible. The price paid for robustness is that each local tracker updates the target state estimate in a side-information unaware manner. To be more specific, the local target maneuvering index is $\lambda$ while the global target maneuvering index viewed by the centralized estimator is $\lambda_c$. Thus each local tracker is unaware of the existence of other local trackers, which leads to the fused error covariance $P_f$ being dominated by the crosscovariance term.

If each local tracker uses $\lambda_c$ to compute the filter gain and communicate the local target state estimates to the fusion center, then the crosscovariance will decrease as $N$ increases. In this case, the local estimate is not optimal in the MMSE sense. In the steady state, each local track has the estimation error covariance given by the solution to the following algebraic Lyapunov equation [2]

$$P = \left[ I - \tilde{W}H \right] F \hat{P} F^T + Q \left[ I - \tilde{W}H \right]^T + \sigma_w^2 \tilde{W} \tilde{W}^T$$

where $\tilde{W}$ is the filter gain and $\hat{P}$ is larger than $P$ when $\tilde{W} \neq W$. The crosscovariance $\Sigma$ of the estimation errors of any two
sensors is given by the solution to
\[ \Sigma = [I - WH][F \Sigma F^T + Q][I - WH]^T \]

The FC can still use (7)–(8) for track fusion on demand. It can be shown that the position component of \( P_d \) in the new fusion scheme scales at least in \( O(N^{-1/2}) \) when \( \lambda \) is small enough (see Appendix). The scaling law is still suboptimal compared with the centralized solution but it is clearly better than the track fusion scheme using the original local estimates. We suspect that the best achievable scaling law of the MSE in position for the distributed track fusion is \( O(N^{-5/4}) \) when \( \lambda \) is small enough and the FC fully utilizes the side information of the existing local trackers. However, we are unable to establish this. Nevertheless, the resulting quantization scheme can be highly nonlinear where distributed Kalman filter is not directly applicable. A recent work [19] shed light on the rate distortion function in the high correlation asymptotic regime, which seems to be the case when both \( m \) and \( N \) are large and \( \lambda \) is small.

To better understand the scaling law, we compute the normalized MSE in position for centralized tracker (\( P_{c11}/P_{11} \)) and distributed tracker with the new fusion scheme (\( P_{d11}/P_{11} \)) for various target manoeuvring indices. Fig. 4 shows the normalized MSEs in position vs. the number of sensors \( N \) using centralized and distributed fusion schemes. For the centralized scheme, as \( \lambda \) decreases, we expect the scaling law changes from \( O(1/N) \) to \( O(N^{-3/4}) \). For the new distributed fusion scheme, as \( \lambda \) decreases, we expect the scaling law changes from \( O(1/N) \) to \( O(N^{-1/2}) \). From Fig. 4 we can see that the normalized MSE does not saturate as \( N \) increases as opposed to the standard track fusion scheme. Similar comparison is made for the normalized velocity MSE and shown in Fig. 5.

For the centralized scheme, as \( \lambda \) decreases, we expect the scaling law changes from \( O(1/N) \) to \( O(1) \). Clearly, the MSE in velocity does not saturate when \( \lambda = 1 \). Interestingly, we can see that the MSE in velocity saturates for large \( N \) using the new distributed fusion scheme. This indicates the suboptimality of the track fusion scheme compared with the centralized estimation. The above results illustrate a simple tradeoff between robustness and accuracy since the new distributed fusion scheme requires to have \( N \) successful local communicators although the rate can be set arbitrarily low.

VI. CONCLUDING REMARKS

This paper studied the tradeoff between bandwidth and tracking accuracy for track fusion with communication constraints. Using steady state analysis of nearly constant speed target motion, we found that the performance of track fusion on demand degrades significantly compared with the centralized estimation as the number of sensors increases. This general trend stems from the increasing correlation among local track estimates with reduced communication rate. We related the steady state MSE of track fusion under bandwidth constraints to the noisy Wyner-Ziv source coding problem and compared our results with the theoretical rate distortion curve of the quadratic Gaussian CEO problem. We concluded that track fusion on demand is a side-information unaware strategy. It sacrifices the MSE performance in exchange for the robustness against communication failure. We also presented an example where the awareness of the correlated estimation errors at each local tracker can improve the fusion accuracy with a better scaling law in the number of sensors.

APPENDIX

In the appendix, we will show that the track fusion scheme with local trackers using \( \lambda_c \) to compute the filter gain instead of \( \lambda \) achieves a scaling law of the MSE in position diminishing in the number of sensors. Denote by \( p_{11} \) the steady state position MSE from each local tracker and \( \sigma_{11} \) the steady state cross correlation of the position estimation error between two local trackers. Assume that the communication rate \( m^{-1} \).
decays no faster than \( O(N^{-1/3}) \), which implies that the state prediction error variance at the FC will not go unbounded. In addition, we let \( \lambda \to 0 \) and \( \lambda_c \to \infty \) in the sense that \( \lambda \) decays no faster than \( O(N^{-1/2}) \). Note that the fused target position estimate is

\[
p_{d11} = \frac{1}{N}p_{11} + \frac{N - 1}{N} \sigma_{11}^2
\]

It is suffice to show that \( \sigma_{11} \) decays faster than \( O(1) \). Solving the steady state Lyapunov equation, we have

\[
\sigma_{11} = \frac{(1 - \alpha_c)^2 m^3 T^3 q}{3\alpha_c (2 - \alpha_c)} + o(N^{-1})
\]

Using Taylor series expansion, we have

\[
1 - \alpha_c = \left(3 - \sqrt{3}\right) \lambda_c^{-2} + o(\lambda_c^{-2})
\]

In the asymptotic regime, when \( m \) grows in \( O(N^{1/3}) \) and \( \lambda \) decays in \( O(N^{-1/2}) \), using the fact that \( \lambda_c = \sqrt{m^3 N} \), we can see that \( \sigma_{11} \) decays at least in \( O(N^{-1/2}) \). Thus the fused target position estimate has the MSE with a scaling law no worse than \( O(N^{-1/2}) \). Intuitively, as \( N \) increases and the communication rate \( m^{-1} \) decreases, the local innovations become more and more important for the track fusion at the FC. This is in line with the proposal of communicating only the sign of innovation whenever possible as in [18].

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