Evaluation of Wavelet Transform Algorithms for Multi-Resolution Image Fusion

Saim Muhammad, Monica Wachowicz and Luis. M. T. de Carvalho

Center for Geo-Information
Wageningen UR, PO Box 47
6700 AA Wageningen, The Netherlands
Telephone: +31.317.474764 – Fax: +31.317.474567
<http://www.geo-informatie.nl/>

E-mails: muhammad.saim@student.girs.wau.nl
m.wachowicz@alterra.wag-ur.nl,
luis.carvalho@staff.girs.wau.nl

Abstract

Wavelet Transforms can be used for multi-resolution image fusion at pixel level, as they work both in spatial and spectral domains and result in the preservation of spatial of spectral details of input images. Different wavelet transform algorithms have been developed and applied to a variety of applications such as noise suppression, filtering, image restoration, image compression, and astronomical applications. This paper explores the use of current developed wavelet transform algorithms for multi-resolution fusion of satellite images. The aim is to investigate how appropriate these wavelet transform algorithms are for this multi-resolution image fusion. Five different types of wavelet transform algorithms are selected and the results are evaluated by comparing their spatial and spectral quality with the spatial and spectral qualities of their source satellite images (i.e. Ikonos Panchromatic and Multispectral, and Landsat TM). The findings show that different wavelet transform algorithms have a "preservation tradeoff" between the spatial quality and spectral quality. Due to the frequency shift limitation of wavelet transform, it can preserve the spatial and/or spectral details of the input images for a certain number of levels.

Keywords: multi-resolution image fusion, wavelet transform, spatial quality, spectral quality

1 Introduction

A vast literature is available and provides a system-level discussion about multi-sensor fusion technologies as they relate to command, controls, communication, and intelligence [1], [2] and [3]. The technological aspects have been considered within detection (or decision) theory, estimation theory, digital signal processing, and parametric and non-parametric data fusion techniques including fuzzy

logic, neural networks, and voting logic [4] and [5]. However, it is only recently that data fusion techniques have been considered for remote sensing applications to improve the spatial resolution of multi-spectral images [6] and their classification performance [7].

This reality coincides with an exponential increase in digital data generated by Earth Observation Systems. A full set of sweeps from NEXRAD-88D radar can record 13 megabytes of data over a five-minute period over 120 radar sites across the U.S. The Moderate Resolution Imaging Spectroradiometer (MODIS) developed for global remote sensing of clouds, aerosols, water vapor, land, and ocean properties provides 1.29 Gbytes per hour.

With so much data comes the problem of combining (fusing) these data and extracting useful information from them. Most of the models that have been proposed for data fusion consist of three levels. These levels are the signal level (also called pixel, measurement), the feature level (attribute), and the decision level. They have been described in such a way that we begin with the pre-processing of data, followed by feature extraction. Having extracted the features, object recognition (also called identification) is performed by statistical techniques, or geometric models. The results are usually partitioned into groups that represent objects belonging to the same class.

Selection of the level for multi-resolution image fusion depends on the characteristics of the input images and the desired output. In this paper, we focus on multi-resolution image fusion at the pixel level. A multi-resolution fusion model is proposed based on both spectral and spatial characteristics of satellite images (raw data).

Several approaches have been proposed for the implementation of multi-resolution fusion models at the pixel level. Some examples are linear superposition [8], optimization methods [9], image pyramids [10] and wavelet transform [6] and [9]. However, we see a substantial

challenge for research in methods for multi-resolution fusion of satellite images. This challenge is two-fold:

- ✎ extension of the existing methods which were developed for the fusion of stationary input frequencies (spectral characteristics) into a spectral and spatial (frequency and space) domain;
- ✎ development of new methods to determine the quality of the fusion outputs based on the integration of these components (frequency, space, and time).

In this paper we address one aspect of the problem, the evaluation of existing wavelet transform algorithms and how they should be extended to implement a multi-resolution image fusion model, which requires the fusion of satellite images by localizing the frequency both in space and scale. Toward this end, we have selected five wavelet transform algorithms. The following sections will describe our evaluation on the application of different wavelet transform algorithms for multi-resolution image fusion at pixel level.

2 Multi-resolution image fusion at pixel level

“Data fusion is a formal framework, in which are expressed means and tools for the alliance of data originating from different sources. It aims at obtaining information of greater quality: the exact definition of greater quality will depend upon the application” [11].

This definition emphasizes the conceptual fusion model and the related fundamentals in remote sensing underlying data fusion. In this definition, the different observation modalities of one sensor (e.g., multi-spectral channels) are considered as different sources, as well as images taken at different times by the same sensor. Finally, this definition adds, those data fusion shown the importance of obtaining information of greater quality; the exact definition of 'greater quality' will depend upon the application. Here quality does not have a very specific meaning. It is a generic word denoting that the resulting information is more useful for the "customer" than that was available without the fusion process. For example, better quality may be an increase in accuracy of a geophysical parameter or of a classification. It may also be related to the production of more relevant information of increased utility, or to the robustness in operational procedure [11]. Fused data represents an entity in greater detail and with less uncertainty than what is obtainable from any of the individual sources. The fusion process can also extract higher order spatial, temporal and behavioral relationships between those entities. Greater quality may also mean a better coverage of the area of interest, or a better use of financial or human resources allocated to a project. In some cases, quality can be replaced by efficiency.

The objectives for carrying out pixel level image fusion can be enlisted as below [10]:

- ✎ combine the images of different spatial resolution to obtain new image with best characteristics of input imagery,
- ✎ improve geometric correction,
- ✎ provide stereo-viewing capabilities for stereo-photogrammetry,
- ✎ enhance features not visible in either data set alone,
- ✎ detect changes using multi-temporal data,
- ✎ substitute missing information,
- ✎ replace defective data.

The pre-requisites for image fusion at the pixel level are as following:

- ✎ images should have different spatial and spectral resolutions
- ✎ images should represent the same area
- ✎ Images should be accurately registered and
- ✎ no major changes should have occurred in the observed area in the interval between acquisition of the first and last source images

In fact, it is possible for the last requirement not to be satisfied. But in this case, the aim of the data fusion process might be to perform change detection in the observed area [4].

3 Wavelet Transform (WT)

According to [12], “Wavelet transform is a tool that cuts up data or functions or operators into different frequency components, and then studies each component with a resolution matched to its scale”.

This definition leads to a *time-frequency* representation of the data under investigation: in the case of images, it leads to a so-called *scale-space* representation [6]. Hence, the wavelet transform enables the input signal to be decomposed on the basis of a series of elementary functions –known as wavelets”. These are generated by the dilation and translation functions, known as the ‘mother’ and ‘father’ wavelets as shown in Equation 1:

$$\Psi_{a,b} = |a|^{-1/2} \Psi\left(\frac{x-b}{a}\right) \quad (1)$$

where

- a the dilation step/Scale parameter,
- b the translation step/position parameter,
- a, b $\in \mathbb{R}$ and
- a $\neq 0$

The computation of the wavelet transform for each scale and each location of a signal provides a local representation of this signal. The process can be inverted so that the original signal can be reconstructed exactly (*i.e.*, without loss) from the wavelet coefficients as in general by using Equation 2:

$$f(x) = \frac{1}{C_\Psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} WT_f(a,b) \Psi_{a,b}(x) \frac{dad b}{a^2} \quad (2)$$

where C_Ψ is the admissibility condition of the mother wavelet.

Equation 2 can be interpreted in two ways:

- $f(x)$ can be reconstructed exactly if one knows its wavelet transform;
- $f(x)$ is a super-imposition of wavelets.

Each of these interpretations leads to a different application of the wavelet transform: in the first instance, to the processing of the signals; in the second, to their analysis or reconstruction of the image by using the wavelet produced while forward transform. As a result, there are different types of Wavelet transforms. Broadly they can be classified into two main categories :

- ▣ Continuous Wavelet Transform (CWT) and
- ▣ Discrete Wavelet Transform (DWT)

3.1 Continuous Wavelet Transform (CWT)

In the continuous wavelet transform, the input signal/pixel value is correlated with an analyzing continuous wavelet [13]. For the continuous wavelet transform for 1-dimensional signal $f(x) \in L^2(\mathbb{R})$, the space of all integrable function is given in the Equation 3:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \Psi\left(\frac{x-b}{a}\right) dx \quad (3)$$

where $W(a,b)$ is the wavelet coefficient of the function $f(x)$

The continuous wavelet transform has the following characteristics [13]:

- ▣ CWT is a linear transform
- ▣ CWT is covariant under translation
- ▣ CWT is covariant under dilation

The continuous wavelet transform can be computed by changing the scale of the analysis window, shifting the window in time, multiplying by the signal, and integrating over all times [14]. Some well-known examples of CWT algorithms are Morlet's, Mexican hat and Haar wavelet transforms.

3.2 Discrete Wavelet Transform (DWT)

In the discrete case, the wavelet function is sampled at discrete mesh-points. Filters of different cut off frequencies are used to analyze the signal at different scales. The signal is passed through a series of high pass filters (moving difference) to analyze the high frequencies and through a series of low pass filters (moving average) to analyze the low frequencies [14]. Correlations can be performed in the physical or Fourier space, the former has a preference when the support of a wavelet function is small (i.e. it is non-zero on a limited number of grid points) [13]. Within the discrete

wavelet transform [12], discrete wavelet transforms can be distinguished on the basis of:

- ▣ Redundant discrete systems (frames) and
- ▣ Ortho-normal (and other) bases of wavelets

Some examples of well-known algorithms for discrete wavelet transform are Mallat's multi-resolution analysis, Feaveau's non-dyadic resolution factor, a trous and pyramidal algorithms. Discrete wavelet transforms are mostly implemented using digital filters. For example, if the multi-resolution image fusion is 2-dimensional, hence first the filters are applied to each row and afterwards to each column.

3.3 Selection of WT algorithms

Continuous and discrete wavelet transform algorithms can be further classified as redundant or non-redundant [12]. The results of the WT are redundant when the output has more pixels than the original image and not redundant when the output and the original image have the same number of pixels. The structure of implementation can also be used to classify wavelet transforms into dyadic or non-dyadic. Moreover, the structure of the output can be used to distinguish wavelet transforms as cube, pyramid, or an image. In order to find out which algorithm is more appropriate for multi-resolution image fusion at pixel level, we have selected five wavelet transform algorithms from different categories (Table 1). Pyramidal wavelet transform was selected because it decomposes the image by a factor $4/3N^2$ and has previously been used in multi-resolution analysis. The linear and B-Spline types of pyramidal wavelet transform were selected.

Mallat-Daubechies is a dyadic form of WT and it allows decomposition of an image with a resolution factor of 2. This algorithm gives the directional detail images of the resulting resolution. The Feaveau is a non-dyadic WT and allows the image decomposition with a resolution factor of $\sqrt{2}$. Finally, Haar is a continuous wavelet transform with orthonormal basis, and was selected to find out the differences between discrete wavelets and a continuous one.

Table 1 Characteristics of the selected WT algorithms

Multi-resolution WT Algorithm	Type	Class	Redu ndant
Pyramidal linear WT	Discrete	Pyramid	Yes
Pyramidal B-Spline WT	Discrete	Pyramid	Yes
Mallat-Daubechies WT	Discrete	Image	No
Feaveau WT	Discrete	Image	No
Haar WT	Continuous	Image	No

4 Implementation

The main pre-requisites for the implementation of the multi-resolution image fusion model are:

- ▣ the images need to be geo-referenced,

- the sub sets of the geo-referenced images need to be drawn by taking care that the number of rows and columns in all the images must be multiple of 2, and
- HSRI (High Spatial Resolution Image) should have at least double the number of pixels in both rows and columns that of LSRI (Low Spatial Resolution Image).

Figure 1 Implementation of the multi-resolution image fusion model

The study area is located in Hilversum (the Netherlands) and is composed of features from a variety of sizes, ranging from less than 1 meter to more than 30 meters. The high-resolution Ikonos PANchromatic (PAN) image was fused with the Ikonos Multi-Spectral (MS) image, which consisted of four bands, 3 from visible and 1 from NIR part of the spectrum. Both images were captured on April 28th, 2000. In order to evaluate the potentials and constraints of WT for multi-resolution image fusion for more number of scales, the high resolution Ikonos PAN was also fused with the coarse resolution image of Landsat Thematic Mapper (TM). First four bands (3 from visible, 1 from NIR, as these cover the same part of the spectrum as covered by the Ikonos PAN and Ikonos MS) of The Landsat TM image, captured on May 3rd, 1999 were selected for fusion with Ikonos Pan image.

The implementation of the above mentioned prerequisite, and later that of the multi-resolution image fusion model shown in Figure 1, was carried out through following steps.

Geometric Correction of the images

All the three images were geo-referenced by using the Top10 data (National Topographical data of The

Netherlands, Scale 1:10,000) as the reference map and the Root Mean Square (RMS) error was 0.038856 pixels. Images were wrapped by using the first order polynomial equation method. For resampling the nearest neighborhood option was used as it has a less effect to the pixel values as compared to the cubic convolution.

Taking the sub sets of the images

Due to the dyadic nature of the wavelet transform algorithms, it is necessary that the number of pixels in the columns and rows of the input image should be multiple of 2. The original Ikonos PAN image consists of 1997X2360 pixels, Ikonos MS 500X591 pixels and Landsat TM 239X272 pixels. Therefore, sub sets were created having 1024X2048 pixels of Ikonos PAN image, 256X512 pixels of Ikonos MS and 32X64 pixels of Landsat TM. The images had the same co-ordinate system that has allowed us to create the sub sets using upper right and lower left corner co-ordinates limits.

Histogram matching

The histogram of the Ikonos PAN image was matched to each band of Ikonos MS and Landsat TM images. In this way eight Ikonos panchromatic images were generated, four for each of Ikonos MS and Landsat TM. Afterwards, the wavelet transforms were applied on these bands for multi-resolution image fusion.

Image fusion by using the wavelet transform

In the case of the image fusion of Ikonos PAN with Ikonos MS band, each histogram matched PAN band was decomposed using three numbers of scales. This decomposed smooth band was then replaced with the corresponding band of Ikonos MS image to which the histogram of panchromatic image was matched. The wavelet co-efficients that were produced during the decomposition were used to reconstruct the MS band of 1-meter spatial resolution. In this way all the four bands of Ikonos MS were resynthesized. The result is an image having all the spatial details that of Ikonos PAN image and the spectral information that of Ikonos MS image (Figure 1).

In the case of Landsat TM image, the same procedure was adopted for decomposition of Ikonos PAN image, but the number of scale was six in order to gain the spatial resolution of 32m. The Landsat TM image was also resampled using the Nearest Neighborhood method to obtain a resolution of 32m, which could be used for the reconstruction of the Landsat TM image having 1-meter spatial resolution. All the five algorithms were applied and the results were ten MS images of 1meter spatial resolution (five from Ikonos MS and five from Landsat TM).

5 Evaluation of results

Wavelet transform operates in both spectral and spatial domains. Therefore, the quality of the resulting fused

images need to be assessed in both domains in order to evaluate the potential of these different WT algorithms.

5.1 Assessment of spatial quality

The assessment of the spatial quality of the resulting images can be carried out by visual analysis [15] and [16]. Three samples have been selected for describing the visual assessment of spatial quality of our resulting images (See Figure 2, Figure 3 and Figure 4). If we zoom in these samples to the pixel level, the visual comparison shown that the Pyramid Linear and the Mallat-Daubechies WT transform have given the better results for both Ikonos MS and Landsat TM images fusion with Ikonos PAN. However, the Haar wavelet transform gave the worst result since the spatial details were there but the spectral information was vanished. According to [12] “Haar function does not have good time-frequency localization: it’s Fourier transform $\psi(\xi)$ decays like $|\xi|^{-1}$ for $\xi \rightarrow \infty$ ”.

By visually comparing the image fusion results of Ikonos MS with Landsat TM, it was also found that the spectral information for objects having the size more than 10m was in the correct position for both cases (See Figure 2). In this case, the spectral information was mostly about the built up area and surrounding large green areas. But the spectral information for smaller objects especially trees in built up areas couldn't be found in the image result of the Landsat TM as shown in sample 2 (Figure 3). There was also some dispersion of spectral information in the case of Landsat image fusion results as shown in the sample 3 (Figure 4). Although dispersion sometimes happens due to a bad geo-referencing, this was not in our case since the spectral information matches to spatial features in other parts of the image.

As mentioned in Section 4, the date of acquisition of Ikonos PAN and Landsat TM has a gap of one year. However, this was not considered to be the cause of the dispersion because the dispersion has occurred in the built up areas in this case. These built up areas have not changed in the period of one year. They are present in the ground truth data (Top 10 data), which has been updated in 1999, the same year of the Landsat TM’s acquisition.

Therefore, the results have confirmed the limitation of wavelet transforms during the decomposition of satellite images. As inverse wavelet transform works by adding a new column and row after each of the existing ones and then the values for these new cells are interpolated using the wavelet co-efficient, produced during the decomposition. According to [11], one major drawback of the wavelet transform when applied to image fusion is its shift dependency; i.e. a simple shift of the input signal may lead to complete different transform coefficients. When applying wavelet transforms for the decomposition of satellite images, only a certain number of scales (levels) can be carried out. In our research, the decomposition gave good results up to three levels.

By using the Image Quality Measure (IQM) developed by the MITRE organization, we have also evaluated the spatial quality of the fusion outputs. This measure computes the Image Quality (IQ) in Power spectrum using a rating scale from 0 to 1. The IQ is used to compute the visual National Image Interpretation Rating (NIIR) scale, as it was found that the co-efficient of correlation between IQM and NIIR value was 0.93. We have used the IQ values for our spatial quality assessment because they can show us how much the spatial characteristics of the Ikonos PAN image are preserved while decomposing and reconstructing multispectral images. The residual values of the IQ obtained for all WT based image fusion results both for Ikonos MS and Landsat TM, are shown in Figures 5 and 6.

Based on the IQ residual values, the spatial quality of all resulting fused images is better than the original one. The Haar WT gave the highest residual value of IQ for Ikonos MS image (See Figure 5). The main reason for this high spatial quality is that the Haar WT removes the noise during the decomposition of an image.

Figure 5 Comparison of IQ values for the Ikonos MS image

One of the reasons can be due to the number of scales (levels) of decomposition. After a certain level of decomposition the details can not be preserved any more due to the shift in frequency, as previously mentioned. With the Haar WT, we were able to get the best spatial quality than the original one because the number of pixels has remained same.

Figure 6 Comparison of IQ values for the Landsat TM image

Figure 2 Sample 1 for visual analysis of spatial quality

Figure 3 Sample 2 for visual analysis of spatial quality

Figure 4 Sample 3 for visual analysis of spatial quality

Another reason is due to the decay in the Fourier space as the frequency values are going to be distributed equally to the negative and positive part of the Fourier space. The NIIR and contrast were also measured for all the resulting bands for both cases and these were quite similar to the IQ values shown in Figures 5 and 6. Mainly because they are computed on the basis of the IQ values and these are the absolute residuals.

5.2 Assessment of spectral quality

For the spectral quality assessment, the statistical parameters of the fusion outputs were compared with their respective statistical parameters of the original multi-spectral bands. In the case of the fusion of the Ikonos MS image with the Ikonos PAN image, the resulting band's statistical parameters were compared with those of Ikonos MS bands and the same was carried out for the Landsat TM fusion results. The bias (difference in the mean values of the resulting fused image and original image) was computed to measure the difference in central tendency. In both cases, the bias values were similar for all the WT based image fusion results. Only the Haar WT had lower bias values due to the reasons already explained in the Section 5.1.

Standard deviation was also used for the quality assessment. The difference in the standard deviation globally represents the level of error in any pixel. In other words it shows the quantity of spectral information added or lost during the enhancement of the spatial resolution. The differences found in the measurements of the standard deviation for both image fusion results of Ikonos MS and Landsat TM are shown in Figures 7 and 8.

Figure 7 Comparison of standard deviation values for the Ikonos MS image

In both cases, none significant differences were found for the standard deviation values except for the Haar WT. Comparatively, the Pyramidal Linear and Mallat-Daubechies wavelet transforms seem to have less loss of spectral information in the case of Ikonos MS image, as their values are quite near to the original one (Figure 7). In the case of the Landsat TM image, the Haar WT once again gave the best result as the number of levels for decomposition increases. However, it still indicates some loss of spectral information since the bands have lower standard deviation values than the original ones.

Figure 8 Comparison of standard deviation values for the Landsat TM image

Finally, the correlation co-efficients were also computed and the results can explain the similarity of spectral information between the original and the fused images. The values computed for the Ikonos MS showed that Pyramidal Linear wavelet transform gave the better results, as the correlation co-efficient value was higher than 0.8. For Landsat TM almost same trends were found. Figures 9 and 10 illustrate the results.

Figure 9 Comparison of correlation values for the Ikonos MS image

In the case of the fusion with the Landsat TM image, the results were quite different. The gap between the IQ values of Haar with the others is much bigger than in the previous case, which means Haar WT removed more noise as the steps for decomposition were more (Figure 6).

Figure 10 Comparison of correlation co-efficient values for the Landsat TM image

6 Conclusions

On the basis of the results of this research, it can be concluded that there is a "preservation trade off" between the spatial quality and the spectral quality of the fusion output. This is valid for all five wavelet transform algorithms, which have been used for the multi-resolution image fusion at the pixel level. For example, the Haar wavelet transform algorithm provided the highest spatial quality in comparison to others algorithms, but its spectral quality was the lowest.

In general, the results of the multi-resolution image fusion using the Ikonos MS image were satisfactory. However, the results of the multi-resolution image fusion using the Landsat TM were less promising due to the limitation of wavelet transforms caused by frequency shifting. Therefore we can conclude that wavelet transforms do not preserve the spatial and spectral information of satellite images, especially after a large number of scales. This is mainly due to the different parameters of the sensors. As a result our multi-resolution image model at the pixel level needs to incorporate the geometric characteristics of the sensors in order to improve the results. Moreover, the input images may be decomposed into a shift invariant representation to overcome the shift dependency of the wavelet fusion scheme as suggested by [9].

On the basis of the spatial and the spectral quality assessment, we can affirm that there was not a significant difference among the quality of different discrete WT algorithms. A significant difference has occurred only when we compared them to the continuous WT, in our case the Haar wavelet.

In summary, we can conclude that the Pyramidal Linear (discrete, pyramid class) and the Mallat-Daubechies (discrete, image class) wavelet transform provided better results in both spatial and spectral qualities as compared to others. Although the existing wavelet transform algorithms, which were not specially developed for multi-resolution image fusion of satellite images, have given good results but still there is a need to develop new algorithms to overcome their frequency shift limitation.

References

- [1] Gan, Q. and Harris, C.j. (1999). Multisensor data fusion using Kalman filters based on linearised process models. *Proceedings of the International Conference on Data Fusion 99*, UK, pp 105-112.
- [2] Bar-Shalom, Y. and Li, X.R. (1995). *Multitarget-Multisensor Tracking: Principles and Techniques*, YBS Publishing, Storrs, CT.
- [3] Manyika, J. and Durrnat-Whyte. (1994). *Data Fusion and Sensor Mngement: A descentrelized Information Theoretic Approach*, Ellis Horweed, New York, USA.
- [4] Fabre, S., Appriou, A. and Briottet, X. (2001). Presentation and description of two classification methods using data fusion based on sensor management. *Information Fusion*, 2(2001): 49-71.
- [5] Hall, D. (1992), *Mathematical techniques in Multi-sensor Data Fusion*. Artech House, Corwwod, MA, USA.
- [6] Ranchin, T., L. Wald, and M. Mangolini (2001), Improving the spatial resolution of images by means of sensor fusion. A geneal solution: the ARSIS method. In: *Remote Sensing and Urban Analysis*. J. P. Donnay and M. Barnsley (eds), Taylor and Fracis, Greate Britain: pp 19-34.
- [7] Kumar, A.S. and Majumder, K.L. (2001). Information fusion in tree classifiers. *International Journal of Remote Sensing*, 22(5): pp 861-869.
- [8] Lillesand, T.M. and R.W. Keifer (2000), *Remote sensing and image interpretation*, John Wiley and Sons, New York: 513-532.
- [9] Rockinger, O. (1999), *Image Fusion*, url:http://www.rockinger.purespace.de/method7_r.htm
- [10] Smits, C.P. (2001), Geospatial data fusion: Overview and current status, *International Geoscience and Remote Sensing Symposium*.
- [11] Wald, L. (1999), Some terms of reference in data fusion. *IEEE Transactions on Geosciences and Remote Sensing*, 37(3): pp 1190-1193.
- [12] Daubechies, I. (1992), *Ten Lectures on Wavelets*, volume 61 of CMBS-NSF Regional Conference Series in Applied Mathematics (Philadelphia: Society for Industrial and Applied Mathematics (S.I.A.M.)): pp 1-16.
- [13] Starck, J.L., F. Murtagh and A. Bijaoui (1998), *Image Processing and Data Analysis*. University Press, Cambridge: pp 45-67.
- [14] Polikar, R. (2001), *The engineer's ultimate guide to wavelet analysis*, The wavelet tutorial, url:http://engineering.rowan.edu/~polikar/WAVELETS/WT_tutorial.html
- [15] Li, J. (2000), Spatial quality evaluation of fusion of different resolution images, *IAPRS*, vol. XXXIII, Amsterdam.
- [16] Wald, L., T. Ranchin, and M.Mangolini (1997), Fusion of satellite images of different spatial and spectral resolutions: assessing the quality of resulting images. *Photogrammetric Engineering and Remote Sensing*, 63(6): pp 91-699.