On Fusion of Multiple Objectives for UAV Search & Track Path Optimization

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This paper addresses the problem of designing a fused scalar objective function for autonomous surveillance—target search and tracking (S&T)—by unmanned aerial vehicles (UAVs). A typical S&T mission includes multiple, most often inherently conflicting, objectives such as detection, survival, and tracking. A common approach to coping with this issue is to optimize a fused scalar objective—a convex combination (weighted sum) of the individual objectives. In practice, determining the fusion weights of a multiobjective combination is, more or less, a guesswork whose success is highly dependent on the designer's assessment and intuition. An optimal (trade-off) point in the performance space is hard to come up with by varying the weights of the individual objectives. In this paper the problem of designing optimal fusion weights is treated more systematically in a rigorous multiobjective optimization (MOO) framework. The approach is based on finding a set of optimal points (Pareto front) in the performance space and solving the inverse problem—determine the fusion weights corresponding to a chosen optimal performance point. The implementation is done through the known normal boundary intersection (NBI) numerical method for computing the Pareto front. The use of the proposed methodology is illustrated by several case studies of typical S&T scenarios.

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1. INTRODUCTION

Due to the significant advancement of the *unmanned vehicle* (UV) technologies in recent years, a great deal of research effort has been devoted to the problem of path optimization (planning and dynamic replanning) of a single or multiple UVs in uncertain and possibly hostile environments. While various UV mission scenarios have been considered in the literature, this paper is focused on UAV surveillance missions which typically include search (detection and localization) of new targets and possibly tracking of detected targets. The techniques considered, however, can be easily applied to other types of missions as well.

Most of the literature on autonomous UAV surveillance deals with search oriented systems, e.g., [8], [7], [4], [15], [14]. Multiple-UAV tracking has been addressed in [9], [12], and tracking combined with detection has been dealt with in [10], [11]. In all of its variations an S&T mission includes multiple objectives, often conflicting to each other. At a high level these objectives can be grouped into several different types including, but not limited to, target *detection*, target *tracking* (classification, recognition), UAV *survivability*, UAV *cooperation*, UAV *efficiency*, and possibly others [7]. Quantifying various objectives and defining a fused scalar *mission objective function* to be optimized during a mission is a crucial issue in the design of S&T systems. Commonly, search-only systems use mission objective functions made up of, most often probability-based, gain/loss functions—e.g., cumulative detection probability, survival probability, etc. [8], [7], [4], [15], [14]. The tracking oriented systems of [9], [12] use information gain based mission objectives, in terms of the Fisher information matrix (FIM) of the tracking filters, and [10], [11] further include the detection objective measured also in terms of FIM. This makes it possible to use standard estimation fusion techniques [1] to fuse the detection and estimation objectives into a scalar objective. However, expressing all objectives through FIMs is difficult to extend to more complicated practical scenarios, e.g., to include efficiency (UAV flight regime cost) or other objectives.

Achieving the mission goal is inherently a multiobjective optimization (MOO) problem and in this paper the problem of designing a mission objective function is treated as such—within the framework of the MOO methodology. There are two issues associated with the MOO formulation. First, due to the conflict among the individual objectives the solution in general is not unique. There is a set of optimal points (referred to as Pareto front) such that, loosely speaking, each optimal point corresponds to a certain trade-off among the values of the objective functions. A decision has to be made as to which Pareto optimal point provides the "best trade-off" among all the alternatives. The second issue is implementational—solving an MOO problem by the known computational methods is usually associated with solving a great number of single nonlinear optimization problems and thus is not feasible in "real-time" for an S&T mission.

A natural approach that circumvents these issues is to optimize a *weighted sum* (WS)—convex combination—of the individual objectives as a fused scalar mission objective. Any unique solution of a WS optimization problem is Pareto optimal, and for each Pareto optimal point there exists a set of weights such that solving the WS problem yields this point if the MOO problem is convex. These properties as well as its simplicity is what makes the WS objective attractive for online implementation in S&T problems. It should be noted that WS objectives have been used in a number of algorithms for cooperative UAV target search systems [8], [7], [15]. In the sequel we also assume that the mission objective function used for online optimization, referred to as a fused mission objective, is a WS of the individual mission objectives. Our focus is on the problem of determining the fusion weights in an optimal manner when a WS fused mission objective is designed.

Implementing a WS as a fused mission objective for online UAV flight path optimization presumes knowledge of the fusion weights and its effectiveness depends heavily on these weights. In practice, their specification is done *a priori*, based on subjective considerations about, e.g, the importance of the individual objective functions. It is more or less a guesswork whose success is highly dependent on the designer's assessment and intuition, and other uncertain factors. For example, [8], [7], [15] state that priorities to the specific individual objectives can be achieved by "adjusting" the values of the weights. However, to make such an adjustment optimally is a nontrivial task for the designer. The problem is that the choice of the weights based on importance or priorities is not made in the feasible objective space—the real *performance space*. For complicated nonlinear and conflicting objective functions (such as in an S&T mission), a "reasonable choice" of importance weights may lead to a rather unacceptable trade-off (Pareto optimal point) in the performance space. At the same time acceptable trade-offs may be available for other, non-obvious choices of fusion weights. In addition a trade-off point in the performance space is hard to come up with by simply varying the weights of the individual objectives.

We argue that a more systematic and rigorous way for designing the fusion coefficients that achieve the "best trade-off" among the possible alternatives is needed. Our approach is based on finding a set of optimal points (Pareto front) and solving the inverse problem—determine the fusion weights corresponding to a chosen optimal performance point. The implementation is done through the normal boundary intersection (NBI) numerical method of [3] for computing the Pareto front.

The underlying idea for application of our methodology is to design the weights of the fused criterion for path optimization such that an acceptable trade-off is achieved by this criterion when applied online to real-life scenarios. This can be done by a comprehensive trade-off analysis through Monte Carlo simulation of an ensemble of typical S&T mission scenarios with different detection maps, threat models, efficiency functions, etc. Overall, the approach proposed in this paper is intended to facilitate the design of fused mission objective functions through more insightful determination of the fusion coefficients of the individual objectives.

The mathematical formulation of the problem is given in Section 2. Section 3 provides some necessary background information about MOO and describes an algorithmic solution. Results of several case studies, illustrating the use of the proposed methodology, are presented in Section 4. Conclusions are provided in Section 5.

2. PROBLEM FORMULATION

We consider a team of UAVs engaged in searching a given surveillance region for new (undetected) targets and tracking of detected targets in an uncertain, dynamic, and risky environment. A UAV (sensor suite, or just sensor for short) is denoted by s, $s = 1,2,\ldots,N_s$; a (detected) target that is being tracked is denoted by t, $t = 1,2,\ldots,N_t$. The 2D surveillance region is partitioned into N_n cells numbered by $n = 1,2,\ldots,N_n$ and $p_n = (x_n, y_n)$ denotes the center location of the nth cell in Cartesian coordinates. n will also stand for indexing a new (undetected) target at position $p_n = (x_n, y_n)$, i.e., in cell n.

Next we present modeling of several objective functions involved in typical UAV S&T mission scenarios, and then we discuss the multiple objectives of a single UAV.

2.1. Detection

For a sensor s, a detection event is modeled through the *detection probability* $\pi_D^s = \pi_D(p_s, p)$, where $p_s = (x_s, y_s)$ and $p = (x, y)$ denote sensor and target locations, respectively. In general π_D^s is a function of the sensor type and parameters, target type and parameters, sensor-target geometry, environment, etc. Here for simplicity we consider the dependence of π_D^s only on the distance between p_s and p. For example, for a ground moving target indicator (GMTI) sensor a typical *detection function* $\pi_D^s = \pi_D(p_s, p) = \pi_D(\|p_s - p\|)$ with $\|p_s - p\| = \sqrt{(x_s - x)^2 + (y_s - y)^2}$, similar to the one used in [10], is shown in Fig. 1.

The probability of detecting a target t, known to exist, by sensor s is $\pi_D^{s,t} = \pi_D(p_s, p_t) = \pi_D(\|p_s - p_t\|)$. If a new target n exists at a given location $p_n = (x_n, y_n)$ with probability $\pi_E(p_n)$, then the probability of detection by sensor s is $\pi_D^{s,n} = \pi_D(\|p_s - p_n\|)\pi_E(p_n)$. The *target existence*[1] *probabilities* $\{\pi_E(p_n)\}_{n=1}^{N_n}$ are assumed known

[1]More precisely, it should be target *perceivability* [5], which is, however, beyond the scope of this paper.

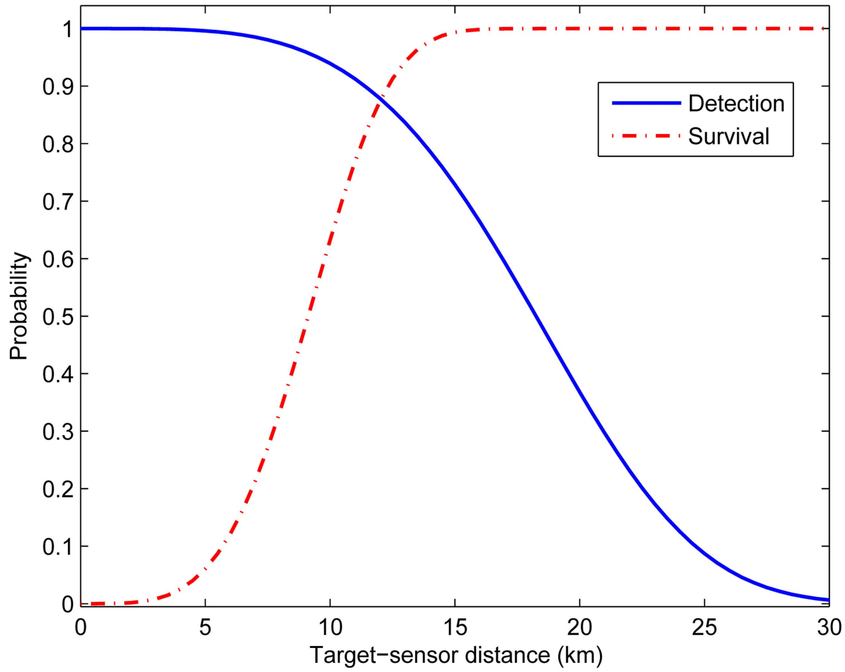

Fig. 1. Detection and survival probabilities.

to the UAVs during the S&T mission via a map of the expected distribution of targets over the surveillance region.

2.2. Survivability

It is assumed that each target (either new or being tracked) poses a threat to a UAV. The event that a sensor s will survive a fire from a given threat θ located at p_θ is modeled through the *survival probability function*[2] $\pi_S(\|p_s - p_\theta\|)$. The probability of surviving the threat from an existing target t is $\pi_S^{s,t} = \pi_S(\|p_s - p_t\|)$. If a threat (new target n) exists at location p_n with probability $\pi_E(p_n)$ then the probability of survival is

$$\pi_S^{s,n} = 1 - (1 - \pi_S(\|p_s - p_n\|))\pi_E(p_n). \tag{1}$$

A survival probability function, assumed in the simulation, is shown in Fig. 1.

2.3. Tracking

For simplicity we assume that a Kalman filter is used for tracking. The tracking objective of a UAV s that tracks a target t can be quantified through the filter *information matrix* (IM) $I = P^{-1}$ where P is the filter covariance matrix.[3] We adopt $\ln|I|$ as a scalar measure[4] of I, and the following approximate relationship for updating the *expected* IM [10]

$$I = \bar{I} + \pi_S \pi_D H' R^{-1} H \tag{2}$$

where $\bar{I}$ is the predicted IM, H is the measurement matrix, and R is the measurement error covariance. Thus the expected *tracking information gain* (TIG) γ_T is measured by

$$\gamma_T = \ln|I| - \ln|\bar{I}| = \ln|\bar{I} + \pi_S \pi_D H' R^{-1} H| - \ln|\bar{I}|. \tag{3}$$

The expected TIG for sensor target pair (s,t) is a function of the distance $\|p_s - p_t\|$, i.e., $\gamma_T^{s,t} = \gamma_T(\|p_s - p_t\|)$ through $\pi_S^{s,t} = \pi_S(\|p_s - p_t\|)$, $\pi_D^{s,t} = \pi_D(\|p_s - p_t\|)$, and $R = R(\|p_s - p_t\|)$ since the observation error depends on the distance. The tracking objective of s is to maximize $\gamma_T(\|p_s - p_t\|)$ with respect to p_s.

2.4. Other Objectives/Constraints

There are a number of other relevant objectives, such as cooperation, engagement, efficiency, whose detailed analysis is not needed for the description of our approach. The reader is referred to, e.g., [7], [15] for a formulation and a more detailed analysis of other objectives. We limit our consideration here to the above three objectives since they are the most significant for an S&T mission but our approach is not limited to these objectives—it allows other objectives to be easily incorporated.

2.5. Fusion of Multiple Objectives

We explain the approach and formulate the problem for a generic single UAV S&T mission scenario since

[2]For simplicity, collision with other UAVs is ignored here but it can be easily modeled in a similar manner.

[3]The superscript indices s, t and the subscript time index k are dropped here to simplify notation.

[4]An alternative scalar measure that can be used is $\text{tr}(I)$.

the mission objectives of a group of UAVs strongly depend on many other factors such as the overall system network architecture (distributed or centralized), cooperation strategy, communication capabilities, whose consideration is beyond the scope of this paper. The approach is, however, directly applicable to such multiple UAV scenarios as well.

Denote by s the sensor under consideration, by $n_1, n_2, \ldots, n_{R_s}$ the cells that can be reached by the sensor in the next time step, and by $t_1, t_2, \ldots, t_{T_s}$ the targets that are being tracked by s. Let

$$\boldsymbol{\pi}_D^s = [\pi_D^{s,n_1}, \ldots, \pi_D^{s,n_{R_s}}]'$$

$$\boldsymbol{\pi}_S^s = [\pi_S^{s,n_1}, \ldots, \pi_S^{s,n_{R_s}}, \pi_S^{s,t_1}, \ldots, \pi_S^{s,t_{T_s}}]'$$

$$\boldsymbol{\gamma}_T^s = [\gamma_T^{s,t_1}, \gamma_T^{s,t_2}, \ldots, \gamma_T^{s,t_{T_s}}]'$$

be the vectors of the detection, survival and tracking objective functions of s.

Given target locations p_{n_i}, $i = 1, \ldots R_s$ and p_{t_j}, $j = 1, \ldots T_s$, the *immediate* (one time-step ahead) goal of s can be rigorously formulated as the following MOO problem

$$\max_{p_s} \begin{bmatrix} \boldsymbol{\pi}_D^s(p_s) \\ \boldsymbol{\pi}_S^s(p_s) \\ \boldsymbol{\gamma}_T^s(p_s) \end{bmatrix} \tag{4}$$

where p_s is the sensor position at the next time.

The dimension of the vector problem (4) can be significantly reduced if the threats are assumed independent. In this case the vector objective $\boldsymbol{\pi}_S^s(p_s)$ can be replaced by the scalar objective

$$\pi_S^s(p_s) = \prod_{i=1}^{R_s} \pi_S^{s,n_i} \prod_{j=1}^{T_s} \pi_S^{s,t_j}.$$

As motivated in Section 1, in order to avoid the problems associated with a complete mathematical solution of (4) for online implementation we assume that the mission objective for online optimization is formulated as the following WS single objective optimization problem

$$\max_{p_s} [\mathbf{w}_D' \boldsymbol{\pi}_D^s(p_s) + \mathbf{w}_S' \boldsymbol{\pi}_S^s(p_s) + \mathbf{w}_T' \boldsymbol{\gamma}_T^s(p_s)] \tag{5}$$

where $\mathbf{w} = [\mathbf{w}_D', \mathbf{w}_S', \mathbf{w}_T']'$ is a fusion weight vector with components $w_i \geq 0$ and $\sum_i w_i = 1$.

We aim at finding numerically (off line) the Pareto front for problem (4) and for each point $(\boldsymbol{\pi}_D^s(p_s^*), \boldsymbol{\pi}_S^s(p_s^*), \boldsymbol{\gamma}_T^s(p_s^*))$ on the front determining the fusion weight vector $\mathbf{w}^*$ such that the solution of (5) is p_s^*, where $\mathbf{w}^*$ is the "best" fused combination of objectives *given* the tradeoff point in the performance space $(\boldsymbol{\pi}_D^s(p_s^*), \boldsymbol{\pi}_S^s(p_s^*), \boldsymbol{\gamma}_T^s(p_s^*))$.

3. SOLUTION METHODOLOGY

3.1. MOO Background Concepts

Here we provide brief information about some basic concepts of the MOO needed later. For details the reader is referred to [6], [13].

A multiobjective optimization problem in mathematical notation is posed as follows

$$\text{minimize } \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_M(\mathbf{x}) \end{bmatrix}, \qquad M \geq 2$$
$$\text{subject to } \mathbf{x} \in C = \{\mathbf{x} : \mathbf{h}(\mathbf{x}) = \mathbf{0}, \mathbf{g}(\mathbf{x}) \leq \mathbf{0}\} \tag{6}$$

where $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, M$, are the objective functions, $\mathbf{x}$ is the *decision variable* vector, C is the *feasible set*, and $\mathbf{h}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are the *constraint* functions. Usually $\mathbf{f}(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$, and $\mathbf{h}(\mathbf{x})$ are assumed twice continuously differentiable. The image of the feasible set $\mathbf{f}(C) \subseteq \mathbb{R}^M$ is referred to as *feasible objective set*, which is a subset of the objective space $\mathbb{R}^M$.

In general, no single $\mathbf{x}$ exists that minimizes every f_i simultaneously. A common concept of optimality for MOO problems is that of Pareto optimality. A decision vector $\mathbf{x}^* \in C$ is *Pareto optimal* (PO) if there does not exist another decision vector $\mathbf{x} \in C$ such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$ for all $i = 1, \ldots, M$ and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one index j. An objective vector $\mathbf{y}^* = \mathbf{f}(\mathbf{x}^*)$ is PO if its corresponding decision vector $\mathbf{x}^*$ is PO. Simply put, an objective vector is PO if any attempt to improve a component (individual objective) will deteriorate at least an other component (individual objective). The set of all PO objective vectors is referred to as the Pareto optimal set, or *Pareto front* (PF). The complete mathematical solution is to find the PF.

There are usually infinitely many PO solutions. In practice solving an MOO means finding a PO solution that satisfies the needs and requirements of a *decision maker.* This is usually a person (e.g., a designer or end-user) that supposedly has an insight into the problem, can express preference relations between different solutions, and can select a final, "best," single solution. Such an approach has been used in engineering applications to facilitate solving complex design problems. The power of using the PF stems from the fact that it reveals the entire spectrum of efficient alternatives for a particular practical problem and allows to select the "best" among them.

Perhaps the most natural approach to the MOO problem is that of the *weighted sum* (WS): minimize a convex combination of the individual objectives

$$\text{minimize } \mathbf{w}'\mathbf{f}(\mathbf{x}) = \sum_{i=1}^{M} w_i f_i(\mathbf{x})$$
$$\text{subject to } \mathbf{x} \in C \tag{7}$$

where $w_i \geq 0$ and $\sum_{i=1}^{M} w_i = 1$. Any unique solution of (7) is PO for the problem (6), and for each PO point of (6) there exists a weighting vector $\mathbf{w}$ such that solving (7) yields this point if the problem (6) is convex [6]. Unfortunately, despite the above properties, finding points on the PF by varying the weighting coefficients **w** has been found to suffer serious drawbacks. It has been observed that small changes in **w** may cause dramatic changes in the objective vectors and large changes in **w** may result in almost unnoticeable changes in the objective vectors. This instability is due to the fact that the WS is not a Lipschitzian function of **w** [6]. Clearly, this makes the relation between weights and performance very complicated and *non-intuitive*. Obtaining a good approximation of the PF directly, by uniform sampling of **w**, may be extremely inefficient since it may lead to very uneven sampling of the PF [2].

An effective method for numerical computation of evenly distributed points on the PF for the MOO problem (6) is the normal boundary intersection (NBI) method of [3]. This method suits very well our "inverse" problem, formulated at the end of Section 3—determine the fusion weights corresponding to a chosen optimal performance point—since it provides a direct link between the NBI computed PF points and their corresponding weights in the problem (7).

3.2. Fusion Weights Determination via NBI

A formal description of the algorithm that we use for determination of the fusion weights through the NBI computed PO points is given below. Its validity follows from Claims 1 and 2, Section 6 of [3].

ALGORITHM

(I) NBI WEIGHTS: *Generate* $\boldsymbol{\beta} = [\beta_1,\ldots,\beta_M]'$ *such that* $\beta_i \geq 0$ *and* $\sum_{i=1}^{M}\beta_i = 1$.

(II) NBI MINIMIZER: *Obtain a point* $\mathbf{x}^* = \mathbf{x}^*_\beta$ *by solving (numerically) the nonlinear optimization problem*

$$\begin{aligned} &\min_{\mathbf{x},t} -t \\ &\text{s.t. } \Phi\boldsymbol{\beta} + t\hat{\mathbf{n}} = \mathbf{f}(\mathbf{x}) - \mathbf{f}^* \\ &\mathbf{x} \in C \end{aligned} \tag{8}$$

determined by computing the following:

1) $\mathbf{x}^*_i = \arg\min_{\mathbf{x}\in C} f_i(\mathbf{x}),\ i = 1,\ldots,M$*—minimizers of the individual objectives of* (6)*;*

2) $\mathbf{f}^* = [f_1(\mathbf{x}^*_1), f_2(\mathbf{x}^*_2),\ldots,f_M(\mathbf{x}^*_M)]'$*—vector of individual minima (utopia point) of* (6)*;*

3) $\Phi = [\mathbf{f}(\mathbf{x}^*_1) - \mathbf{f}^*, \mathbf{f}(\mathbf{x}^*_2) - \mathbf{f}^*,\ldots,\mathbf{f}(\mathbf{x}^*_M) - \mathbf{f}^*]$*—pay-off matrix of* (6)*;*

4) $\hat{\mathbf{n}} = -\Phi[1,1,\ldots,1]'$*—quasi-normal search direction for* (8).

(III) FUSION WEIGHTS: *Obtain* $\mathbf{w}^* = [w^*_1,\ldots,w^*_M]'$ *which corresponds to* $\mathbf{x}^*$ *as*

$$w^*_i = \frac{\lambda_i^{(1)^*}}{\sum_{i=1}^{M}\lambda_i^{(1)^*}}, \qquad i = 1,\ldots,M$$

if all $\lambda_i^{(1)^*}$, $i = 1,\ldots,M$ *have the same signs, where* $\boldsymbol{\lambda}^{(1)^*} = [\lambda_1^{(1)^*},\ldots,\lambda_M^{(1)^*}]'$ *is the vector of the Karush-Kuhn-Tucker (KKT) multipliers for the equality constraint* $\Phi\boldsymbol{\beta} + t\hat{\mathbf{n}} = \mathbf{f}(\mathbf{x}) - \mathbf{f}^*$.

REMARK 1 In our Matlab program implementation of the above algorithm, in Step II, we used the standard function for nonlinear constrained minimization `fmincon` from the Matlab optimization toolbox for minimizing the individual objectives of (6) and solving the problem (8). This function provides the KKT multipliers $\lambda_i^{(1)^*}$, $i = 1,\ldots,M$, needed in Step III, as output parameters.

REMARK 2 As shown in [3], the WS problem (7) with $\mathbf{w} = \mathbf{w}^*$ determined in Step III has the solution $\mathbf{x}^* = \mathbf{x}^*_\beta$ determined in Step II. If $\mathbf{w}^*$ cannot be determined in Step III, i.e., some $\lambda_i^{(1)^*}$ has a sign which is different from the sign of $\sum_{i=1}^{M}\lambda_i^{(1)^*} \neq 0$ then either the NBI computed point $\mathbf{x}^* = \mathbf{x}^*_\beta$ is not PO or $\mathbf{x}^*$ is PO but lies in a nonconvex part of the PF and cannot be obtained by minimizing a WS of the objectives. For convex problems (as most real problems are) such an issue does not exist.

REMARK 3 An even spread of NBI points $\{\mathbf{x}^*_{\beta_\nu}\}_{\nu=1}^{N}$ will be obtained if the set of points $\{\Phi\boldsymbol{\beta}_\nu\}_{\nu=1}^{N}$ forms an uniformly-spaced grid on the simplex $\{\Phi\boldsymbol{\beta}\}_{\boldsymbol{\beta}}$. This is due to the fact that, according to (8), the points obtained by the NBI are restricted to lie on a set of parallel vectors (all parallel to the normal $\hat{\mathbf{n}}$) emanating from the uniformly spread points $\{\Phi\boldsymbol{\beta}_\nu\}_{\nu=1}^{N}$. A simple algorithm to achieve this is to generate the NBI weights $\{\boldsymbol{\beta}_\nu\}_{\nu=1}^{N}$ uniformly, i.e., each component of $\boldsymbol{\beta}_\nu$ has a value in $[0,1/p,2/p,\ldots,1]$ where $p \geq 2$ is an integer and all components sum up to 1. This yields an uniform grid with a total of $N = \binom{M+p-1}{p}$ points.

4. CASE STUDIES

As formulated in Section 2, a WS single objective optimization problem given by (5) is to be solved online during an S&T mission. The fusion weights $\mathbf{w} = [\mathbf{w}'_D, \mathbf{w}'_S, \mathbf{w}'_T]'$ are designed (determined off-line) based on a comprehensive trade-off analysis such that an acceptable trade-off will be achieved by the WS criterion when applied online to real-life scenarios. As illustrated below, such an analysis can be done through Monte Carlo simulation of an ensemble of typical S&T mission scenarios with different detection maps, threat models, efficiency functions, etc. It includes obtaining a representative set of trade-off points $\{(\boldsymbol{\pi}^s_D, \boldsymbol{\pi}^s_S, \boldsymbol{\gamma}^s_T)\}$ for the problem (4), along with their corresponding weights $\{\mathbf{w}\}$ in the problem (5), and can be done efficiently by means of the NBI-based algorithm of Section 3.2. A decision upon the "best" trade-off point $(\boldsymbol{\pi}^{s*}_D, \boldsymbol{\pi}^{s*}_S, \boldsymbol{\gamma}^{s*}_T)$, made by the designer, gives in turn the "best" fusion weights $\mathbf{w}^*$ to be implemented in the online optimization problem (5).

To illustrate the use of the proposed technique in the trade-off analysis for determination of the "best" fusion weights we present next four case studies of UAV S&T scenarios.

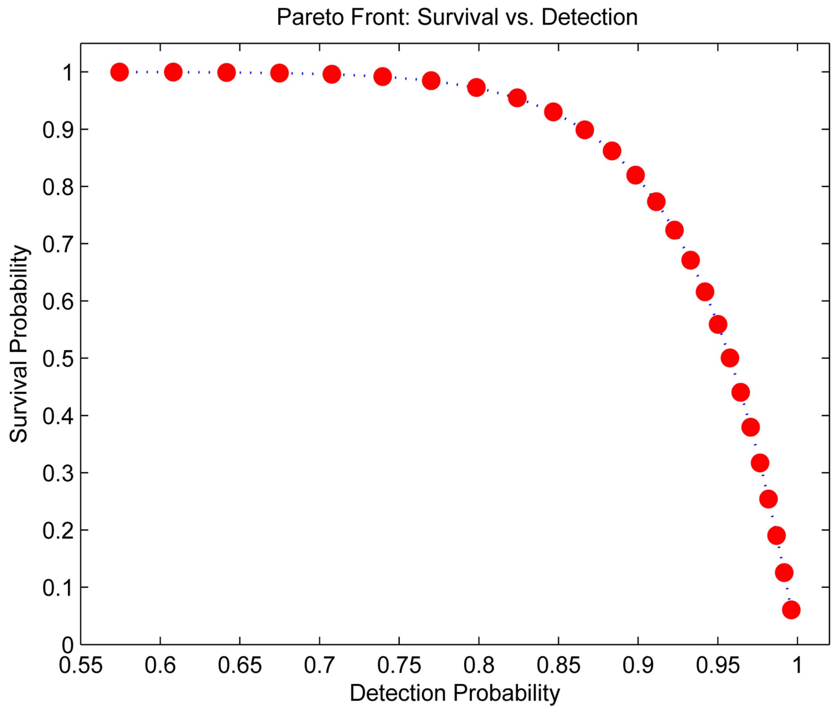

Fig. 2. PF generated by NBI.

4.1. Detection vs. Survivability

1) *Single target search*: For a trade-off analysis between the detection and survivability objectives we consider first a simple scenario of one sensor s and one new target n located at $p_n = (x_n, y_n)$. The UAV s aims at solving the following two-objective optimization problem

$$\max_{p_s} \begin{bmatrix} \pi_D(\|p_s - p_n\|) \\ \pi_S(\|p_s - p_n\|) \end{bmatrix}. \quad (9)$$

By using the algorithm of Section 3.2 we obtain a uniform representation of the Pareto front

$$\pi_D(\|p_s - p_n\|) \text{ vs. } \pi_S(\|p_s - p_n\|)$$

and for each trade-off point $(\pi_D(\|p_s^* - p_n\|), \pi_S(\|p_s^* - p_n\|))$ of the PF we determine the corresponding weights w_D^* and w_S^* such that the solution of the single-objective optimization problem

$$\max_{p_s} [w_D^* \pi_D(\|p_s - p_n\|) + w_S^* \pi_S(\|p_s - p_n\|)] \quad (10)$$

is p_s^*.

The simulated scenario parameters are as follows. The assumed detection and survival functions, shown in Fig. 1, are

$$\pi_D(d) = \exp(-(d/20)^4) \quad (11)$$

$$\pi_S(d) = 1 - \exp(-(d/10)^4) \quad (12)$$

where $d = \|p_s - p_n\|$ is the distance. Without loss of generality it is assumed that $p_n = (0,0)$.

The Pareto front generated by the algorithm of Section 3.2 is shown in Fig. 2. Its computation required solving 24 nonlinear single objective optimization problems of the type (8). For a rough comparison, the direct "bruteforce" WS method for PF determination required a dramatically larger number of problems (5) in order to provide a comparable representation of the PF.

Next, for each optimal point on the PF $(\pi_D(\|p_s^* - p_n\|), \pi_S(\|p_s^* - p_n\|))$ we determined the corresponding fusion weights w_D^* and w_S^* for the equivalent WS single objective optimization problem. The results are given in Table I. It reveals the available trade-offs between the detection and survival probabilities and includes the corresponding weights that yield these trade-offs through maximizing the WS objective. What is left to the user (or designer) is to choose one or more preferable trade-off points and they will be automatically achieved through the corresponding weights. For example, if the selected trade-off performance from Table I is $\pi_D^* = 0.8665$ and $\pi_S^* = 0.899$ (line 12) then the fusion weights $w_D^* = 0.651$ and $w_S^* = 0.349$ are to be used in (10). It should be also noted that Table I allows to design a set of WSs corresponding to different tactical situations and thus give the UAV a capability to operate in different modes depending on the situation by simply switching the weights of the WS objective function.

2) *Multiple target search*: Next we present a trade-off analysis of the detection and survival objectives of a sensor s in a search scenario with two targets n_1 and n_2 known to exist at $p_{n_i} = (x_{n_i}, y_{n_i})$, $i = 1,2$.

For simplicity it is assumed that the threats are independent, and thus the joint survival probability

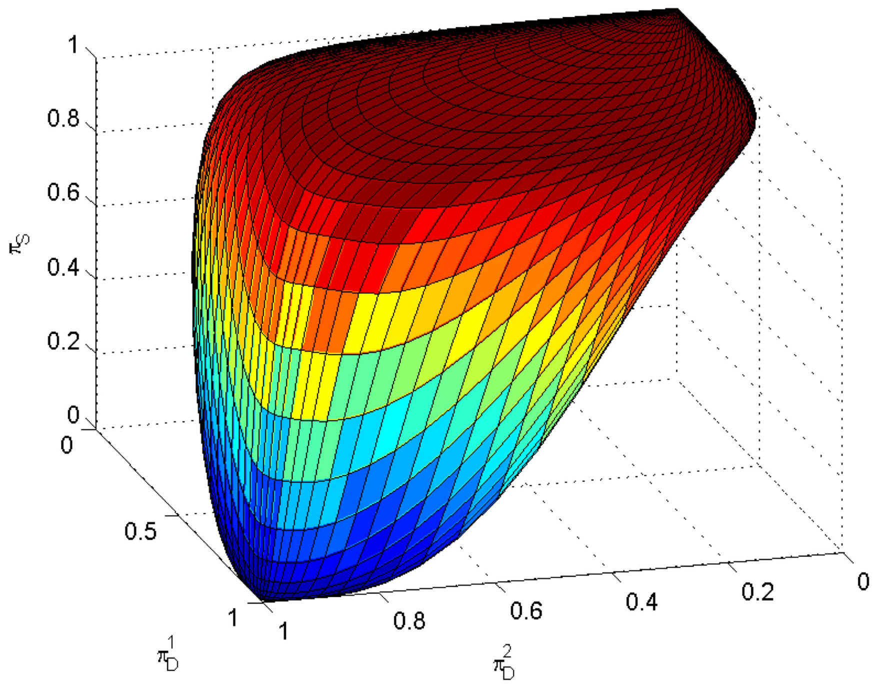

Fig. 3. Feasible objective set.

TABLE I
Trade-Off Points & Fusion Weights

π_D	π_S	w_D	w_S
0.9916	0.1256	0.9338	0.0662
0.9818	0.2541	0.9240	0.0760
0.9764	0.3172	0.9180	0.0820
0.9643	0.4406	0.9027	0.0973
0.9575	0.5006	0.8930	0.1070
0.9419	0.6161	0.8670	0.1330
0.9329	0.6711	0.8494	0.1506
0.9228	0.7237	0.8273	0.1727
0.9114	0.7735	0.7991	0.2009
0.8984	0.8198	0.7624	0.2376
0.8836	0.8619	0.7143	0.2857
0.8665	0.8990	0.6510	0.3490
0.8468	0.9301	0.5690	0.4310
0.8241	0.9548	0.4677	0.5323
0.7984	0.9727	0.3534	0.6466
0.7701	0.9847	0.2412	0.7588
0.7396	0.9920	0.1478	0.8522
0.7077	0.9960	0.0822	0.9178
0.6749	0.9981	0.0421	0.9579
0.6417	0.9992	0.0202	0.9798
0.6081	0.9996	0.0091	0.9909

is

$$\pi_S^s = \pi_S^{s,n_1}\pi_S^{s,n_2} = \pi_S(\|p_s - p_{n_1}\|)\pi_S(\|p_s - p_{n_2}\|)$$

where the function $\pi_S(d)$ is given by (12).

The UAV s aims at solving the following three-objective optimization problem

$$\max_{p_s} \begin{bmatrix} \pi_D(\|p_s - p_{n_1}\|) \\ \pi_D(\|p_s - p_{n_2}\|) \\ \pi_S^s(\|p_s - p_{n_1}\|, \|p_s - p_{n_2}\|) \end{bmatrix} \qquad (13)$$

where $\pi_D(d)$ is given by (11).

TABLE II
Trade-Off Points & Fusion Weights

$\pi_D^{n_1}$	$\pi_D^{n_2}$	π_S^s	$w_D^{n_1}$	$w_D^{n_2}$	w_S^s
0.98	0.98	0.04	0.4240	0.4240	0.1520
0.95	0.95	0.27	0.4466	0.4466	0.1068
0.92	0.92	0.50	0.4383	0.4383	0.1234
0.88	0.88	0.72	0.4112	0.4112	0.1776
0.83	0.83	0.89	0.3298	0.3298	0.3404
0.73	0.73	0.98	0.1130	0.1130	0.7740
0.59	0.59	0.99	0.0059	0.0059	0.9882
0.44	0.44	1.00	0.0001	0.0001	0.9998

Note that without the above threat independence assumption π_S^{s,n_1} and π_S^{s,n_2} should be considered as individual objectives (as in the general MOO problem formulation (4)), which would lead to a four dimensional problem.

The parameters of the simulation are the same as in the previous scenario. In addition, the second target n_2 is located at $p_n = (10,0)$.

Fig. 3 shows the *feasible objective region* for detection and survival, and Fig. 4 shows the obtained Pareto front.

Table II gives the fusion weights $w_D^{n_1*}$, $w_D^{n_2*}$ and w_S^{s*} of the WS objective function

$$w_D^{n_1*}\pi_D(\|p_s - p_{n_1}\|) + w_D^{n_2*}\pi_D(\|p_s - p_{n_2}\|) + w_S^*\pi_S^s(\|p_s - p_{n_1}\|, \|p_s - p_{n_2}\|)$$

corresponding to the obtained Pareto optimal points $(\pi_D(\|p_s^* - p_{n_1}\|), \pi_D(\|p_s^* - p_{n_2}\|), \pi_S^s(\|p_s^* - p_{n_1}\|, \|p_s^* - p_{n_2}\|))$.

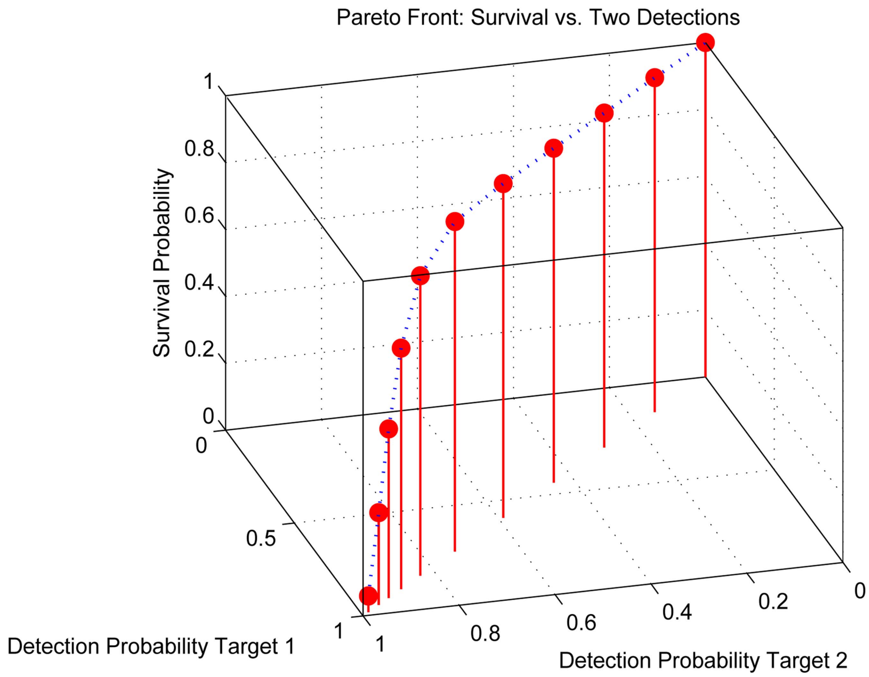

Fig. 4. PF generated by NBI.

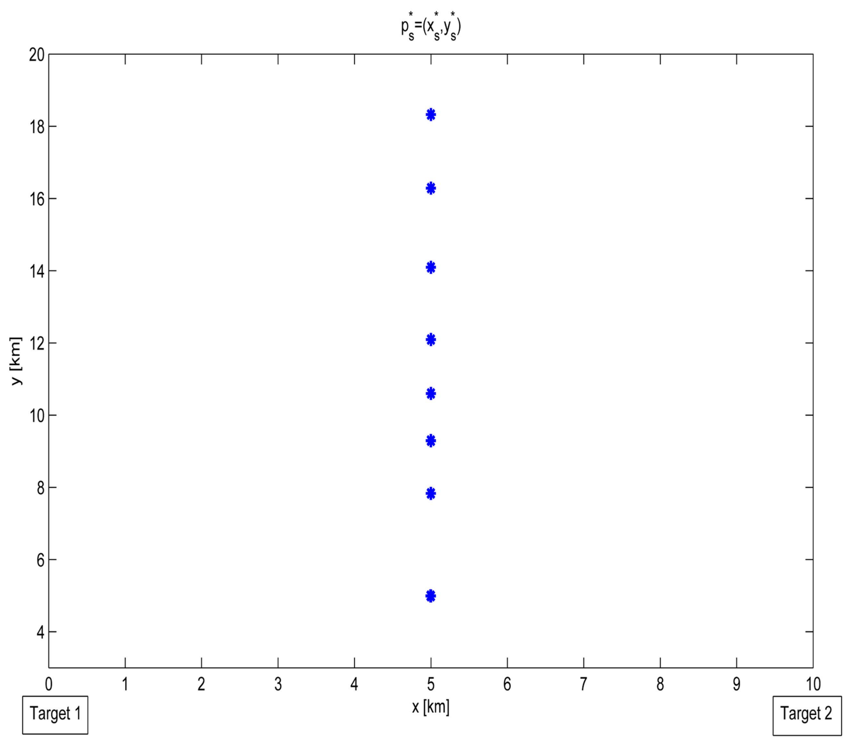

Fig. 5. Pareto optimal locations of the UAV.

In addition, Fig. 5 provides information about the locations of the UAV $p_s^* = (x_s^*, y_s^*)$ that achieve Pareto optimal performance.

4.2. Detection vs. Survivability vs. Tracking

1) *Single target tracking*: This scenario includes tracking a single target t by a UAV s. According to (4) the UAV aims at solving the following two-objective optimization problem

$$\max_{p_s} \begin{bmatrix} \pi_S(\|p_s - p_t\|) \\ \gamma_T(\|p_s - p_t\|) \end{bmatrix}. \tag{14}$$

The parameters of the simulation are as follows. The target is located at $p_t = (10, 0)$. The assumed detection

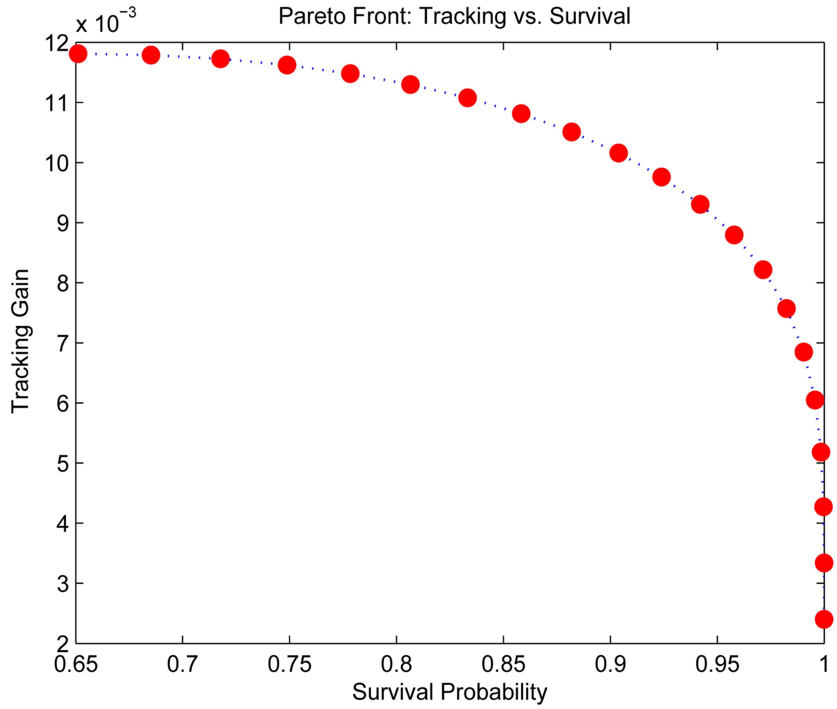

Fig. 6. PF generated by NBI.

and survival functions $\pi_D(d)$ and $\pi_S(d)$ are the same as in Case A.1 (see Fig. 1). It is assumed that the measurement error covariance is $R(d) = (\sigma^2_{\min} + d^2)\mathrm{I}_2$ with $\sigma^2_{\min} = 0.1$, and $\bar{I} = \mathrm{I}_4$ where $d = \|p_s - p_t\|$ and I denotes the identity matrix. $H = [\mathrm{I}_2 \ \mathrm{O}_2]$ where O denotes the null matrix. Under these assumptions it can be calculated from (3) that

$$\gamma_T(d) = 2\ln\left(1 + \frac{\pi_S(d)\pi_D(d)}{\sigma^2_{\min} + d^2}\right)$$

where $\pi_S(d)$ and $\pi_D(d)$ are given by (12) and (11), respectively (see Fig. 1).

Fig. 6 shows the obtained Pareto front. Table III gives the fusion weights w^*_S and w^*_T of the WS objective

$$w^*_S\pi_S(\|p_s - p_t\|) + w^*_T\gamma_T(\|p_s - p_t\|)$$

corresponding to the obtained Pareto optimal points $(\pi_S(\|p^*_s - p_t\|), \gamma_T(\|p^*_s - p_t\|))$.

2) *Joint search & tracking*: This scenario includes tracking a single target t and detecting a new target n by a UAV s. The UAV aims at solving the following three-objective optimization problem

$$\max_{p_s} \begin{bmatrix} \pi_D(\|p_s - p_n\|) \\ \pi^s_S(\|p_s - p_n\|, \|p_s - p_t\|) \\ \gamma_T(\|p_s - p_t\|) \end{bmatrix} \quad (15)$$

where under the independent threats assumption

$$\pi^s_S = \pi^{s,n}_S \pi^{s,t}_S = \pi_S(\|p_s - p_n\|)\pi_S(\|p_s - p_t\|)$$

The parameters of the simulation are as follows. The targets' locations are $p_n = (0,0)$ and $p_t = (10,0)$. For the target under track it is assumed that $R = (\sigma^2_{\min} + \|p_s - p_t\|^2)\mathrm{I}_2$ with $\sigma^2_{\min} = 0.1$, and $\bar{I} = \mathrm{I}_4$. $H = [\mathrm{I}_2 \ \mathrm{O}_2]$. Under these assumptions it can be calculated from (3) that

$$\gamma_T = 2\ln\left(1 + \frac{\pi_S(\|p_s - p_n\|)\pi_S(\|p_s - p_t\|)\pi_D(\|p_s - p_t\|)}{\sigma^2_{\min} + \|p_s - p_t\|^2}\right)$$

Fig. 7 shows the feasible objective region for detection, survival and tracking, and Fig. 8 shows the obtained Pareto front.

TABLE III
Trade-Off Points & Fusion Weights

π_S	γ_T	w_S	w_T
1.0000	0.0033	0.9013	0.0987
0.9997	0.0043	0.6154	0.3846
0.9985	0.0052	0.3278	0.6722
0.9957	0.0060	0.1742	0.8258
0.9904	0.0068	0.1028	0.8972
0.9824	0.0076	0.0672	0.9328
0.9715	0.0082	0.0475	0.9525
0.9579	0.0088	0.0354	0.9646
0.9420	0.0093	0.0274	0.9726
0.9239	0.0098	0.0218	0.9782
0.9038	0.0102	0.0175	0.9825
0.8819	0.0105	0.0142	0.9858
0.8584	0.0108	0.0115	0.9885
0.8332	0.0111	0.0092	0.9908
0.8065	0.0113	0.0073	0.9927
0.7784	0.0115	0.0056	0.9944
0.7488	0.0116	0.0040	0.9960
0.7177	0.0117	0.0026	0.9974
0.6852	0.0118	0.0013	0.9987

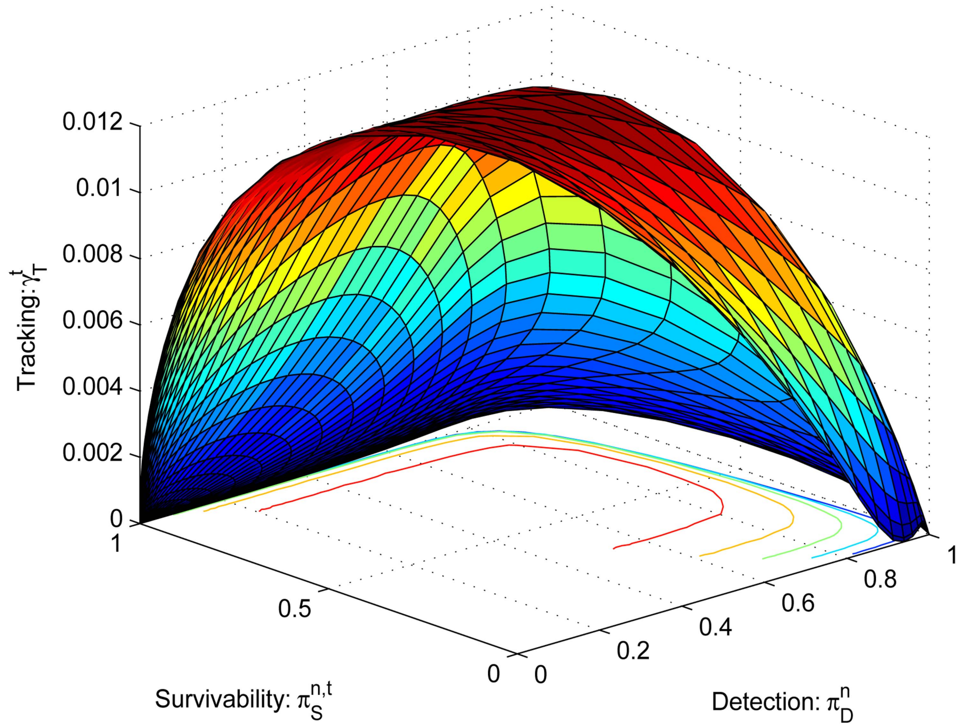

Fig. 7. Feasible objective set.

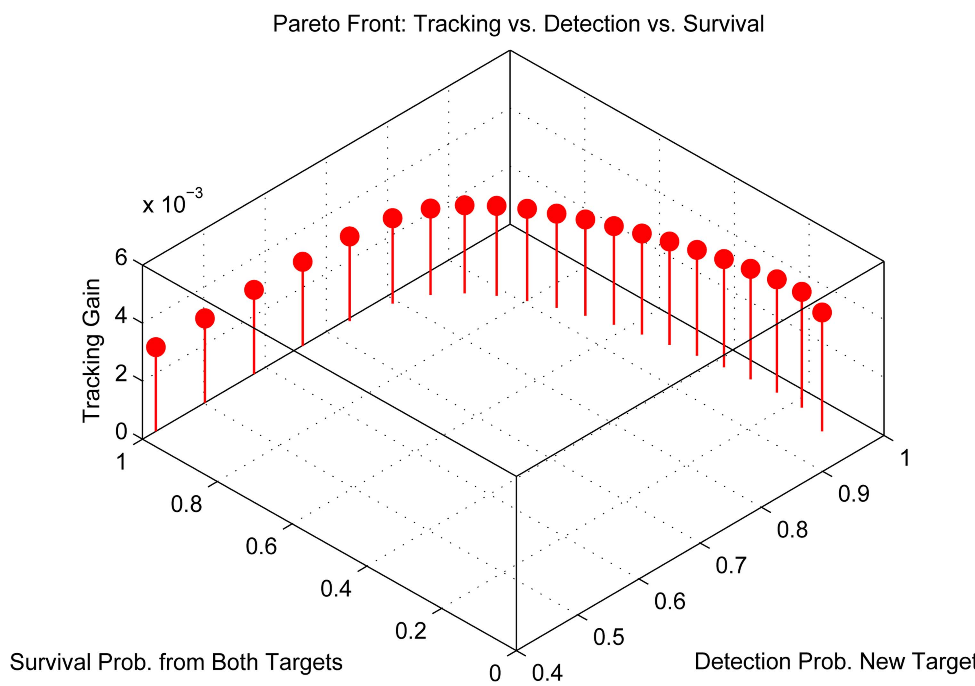

Fig. 8. PF generated by NBI.

Table IV gives the weights w_D^*, w_S^* and w_T^* of the WS objective

$$w_D^* \pi_D(\|p_s - p_n\|) + w_S^{s*} \pi_S^s(\|p_s - p_n\|, \|p_s - p_t\|)$$
$$+ w_T^* \gamma_T(\|p_s - p_t\|)$$

corresponding to the obtained Pareto optimal points $(\pi_D(\|p_s^* - p_n\|), \pi_S^s(\|p_s^* - p_n\|, \|p_s^* - p_t\|), \gamma_T(\|p_s^* - p_t\|))$.

In addition, Fig. 9 provides information about the locations of the UAV $p_s^* = (x_s^*, y_s^*)$ that achieve the Pareto optimal performance.

5. CONCLUSIONS

A systematic and rigorous multiobjective optimization based approach for designing a fused scalar ob-

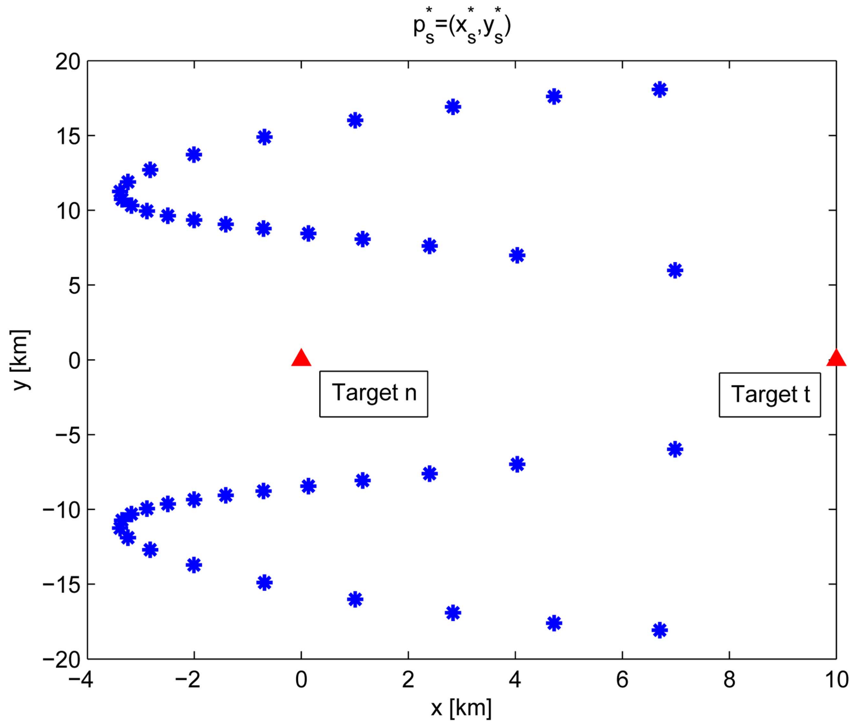

Fig. 9. Pareto optimal locations of the UAV.

TABLE IV
Trade-Off Points & Fusion Weights

π_D	π_S^s	γ_T	w_D	w_S^s	w_T
0.9750	0.2455	0.0039	0.1352	0.0031	0.8617
0.9728	0.3117	0.0038	0.1886	0.0107	0.8007
0.9686	0.3763	0.0037	0.2579	0.0205	0.7216
0.9632	0.4397	0.0036	0.3466	0.0339	0.6195
0.9567	0.5023	0.0036	0.4495	0.0516	0.4989
0.9491	0.5639	0.0035	0.5526	0.0736	0.3738
0.9404	0.6246	0.0034	0.6387	0.0990	0.2623
0.9304	0.6841	0.0033	0.6967	0.1276	0.1757
0.9187	0.7422	0.0032	0.7239	0.1607	0.1154
0.9047	0.7984	0.0032	0.7217	0.2018	0.0765
0.8875	0.8517	0.0031	0.6899	0.2573	0.0528
0.8655	0.9009	0.0030	0.6222	0.3385	0.0393
0.8356	0.9434	0.0030	0.5037	0.4636	0.0327
0.7933	0.9754	0.0029	0.3220	0.6466	0.0314
0.7339	0.9929	0.0029	0.1297	0.8372	0.0331
0.6606	0.9987	0.0029	0.0299	0.9356	0.0345
0.5819	0.9998	0.0029	0.0045	0.9605	0.0350
0.5019	1.0000	0.0029	0.0005	0.9645	0.0350
0.4218	1.0000	0.0029	0.0001	0.9649	0.0350

jective function for search and track missions of unmanned aerial vehicles through weighted combinations of objectives has been proposed. It allows to obtain a representative set of possible trade-off optimal alternatives and determine the weights for the combination of objectives that meets a selected "best" trade-off. The proposed methodology can greatly facilitate the design of mission objective functions through performing insightful trade-off analysis. Its usefulness has been illustrated by results from several case studies of typical search and track mission scenarios.

It should be kept in mind that the method used as well as all numerical methods for general multiobjective optimization can at best provide only *local* Pareto optimality and thus it can be hard sometimes to find initial solutions leading to the trade-off region of practical interest.

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