Tracking Targets with Multiple Measurements per Scan Using the Generalized PHD Filter

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The task of tracking targets, that generate more than one measurement per scan appears in several applications such as extended object and group tracking. In this case, the target (or group) extent implies that multiple measurements, drawn according to a spatial probability distribution, are measured per sensor-scan. However, applications exist where targets generate several measurements per sensor-scan, which are not geometrically correlated according to a distribution in the measurement space. An example for such an application is Blind Mobile Localization, which is the passive non-cooperative localization and tracking of mobile terminals in urban scenarios. In this paper a Probability Hypothesis Density filter for general models of target-generated measurements is applied to track targets with multiple measurements per scan, where the measurements do not necessarily have to be spatially related in the measurement space. Furthermore, the problem of numerical feasibility is identified and two ways of approximating the update equation of the generalized Probability Hypothesis Density filter are proposed. Finally, two numerical evaluations are carried out to assess sequential Monte Carlo-implementations of the generalized PHD-filter.

Manuscript received June 18, 2014; released for publication June 29, 2015.

Refereeing of this contribution was handled by Paolo Braca.

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I. INTRODUCTION

The purpose of this paper is to investigate the applicability of the generalized PHD-filter [7] to scenarios with arbitrary target-measurement models. To this end, approximation conditions and a generalization of the probability of detection are developed, applied to two different scenarios and compared to existing standard approaches.

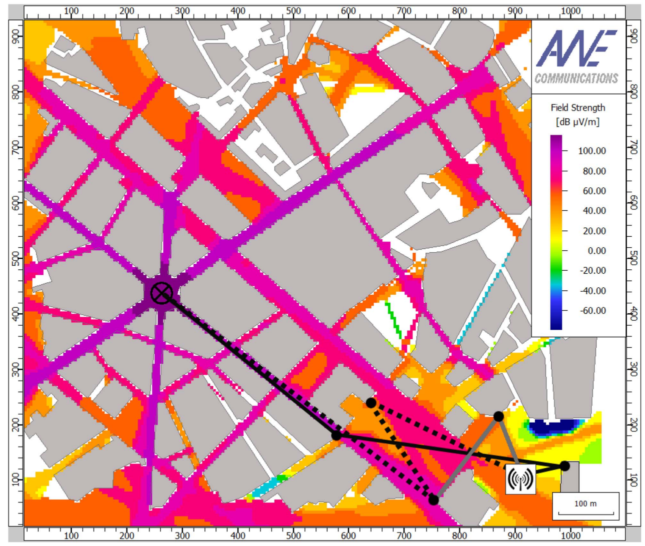


Fig. 1. Visualization of the field-strength prediction given by the ray-tracing simulation: For a given observer (black cross) mobile station (antenna) constellation the color at the emitter location indicates the received field-strength at the observer. Three multipaths are visualized (black solid, block dotted, gray) and the interaction points are plotted as black dots.

Due to the assumption that targets generate conditionally independent observations with at most one observation per target, the standard Probability Hypothesis Density (PHD)-filter [26] is not suited for applications where a target may generate multiple measurements in one sensor-scan. However, for the problem of extended object tracking several modifications of the standard PHD-filter are available (an excellent overview about existing methods is given in [28]). In [26] the target extent is modeled by a set of point scatterers, where each scatterer generates an individual measurement. In [27] an approximation is presented, based on the approximate Poisson model of Gilholm, Godsill, Maskell and Salmon [15], where the target extent is modeled by a spatial probability distribution. Furthermore, the set of measurements is preprocessed into associated groups which represent the individual targets. In [22] the target extent is modeled by random matrices and in [16], [17], [18], [20], [19] the approach is combined using PHD and cardinalized PHD-filters. Furthermore, techniques for reducing the number of measurement set partitions, which are essentially based on clustering measurements, are presented in these references. All methods mentioned have in common that they make explicit use of the target extent. However, scenarios exist where targets generate multiple measurements which are not spatially related in the measurement space.

An example is given by Blind Mobile Localization (BML) [9], [2], [1], where an observer station (OS) tracks the state of an electromagnetic emitter in an urban environment. The boundary conditions of the problem imply that the OS has to determine the location of the mobile terminal by only inspecting the transmitted electromagnetic waves. In urban scenarios the effect of multipath propagation often is inevitable. This is due to physical effects on the electromagnetic wave of the signal such as reflection, diffraction, and scattering. As a consequence, the observer station receives multiple signals that have traveled along different paths. Each of these paths is distinct in either the time of arrival (ToA), azimuth (angle) of arrival (AoA) or the elevation (angle) of arrival (EoA). Hence, the mobile station (MS), which is a point target in the sense of [26], generates several observations, the so-called multipaths. A multipath is defined by a relative time of arrival (RToA), an AoA and an EoA. The distribution of measurements in the measurement space strongly depends on the environment and a measurement function would be discontinuous and difficult to model and calculate. This makes it impossible to find a general distribution of the multipath-measurements in the measurement space, which can be used for the association of targets and multipaths as it is done in [26] and [27], when preprocessing the measurement set.

In [9] a sequential Monte Carlo (SMC)-implementation of the standard Intensity filter (iFilter) [33], [32], which is closely related to the standard PHD-filter [26] and uses the same assumptions, is applied to the problem of BML. For the formulation of the likelihood-function context information of the urban environment is used in terms of a ray-tracing simulation, which predicts a set of multipaths for a given OS-MS constellation (see Figure 1, for details on the formulation of the likelihood-function see [9]). Due to the standard scatterer measurement model of the iFilter, it is assumed that each multipath represents an individual and independent measurement. Therefore, the estimated number of targets is not equal to the true number of targets, but to the number of multipaths which belong to a target. A sophisticated method for state extraction, considering the association possibilities between different multipath-sets and targets, is also presented in [9]. This state extraction scheme needs to be considered since the target generated measurement models of the standard PHD-filter and BML differ. Even though this solution yields satisfying results, it is an approximation, which assumes that each multipath represents an individual target, i.e., the creation of at most one observation per target, which is obviously not given for the application of BML. Taking into account all of the previous considerations a PHD-filter for targets which generate multiple measurements per sensor-scan without a common distribution in the measurement space is needed.

The PHD-filter derivation using probability generating functionals (PGFLs) can be found in [26], [23], [24], [25] and [35]. A very detailed derivation of the PHD-filter using the PGFL-framework is presented in [21]. There, the PHD-filter is derived by modeling a PGFL, using Bayes theorem and afterwards the update (and prediction) equation of the PHD-filter is obtained by functional differentiation. To derive a PHD-filter for scenarios, which do not fulfill the standard assumptions, PGFLs represent an appropriate approach. In [36] and [26] examples of PGFLs for nonstandard targets are given and the calculation of the respective Gâteaux derivatives using compositions of so-called secular functions with functionals is proposed. In [7] the authors present a general chain rule (GCR) for functional-derivatives. This result is extended to locally convex topological spaces in [6] and closely related to the ideas presented in [36]. It can be used to determine the Gâteaux derivatives of complex PGFLs, e.g., for PGFLs which model target interaction [21, chapter 3]. Furthermore, in [7] a generalized PHD-filter is developed for arbitrary models of target-generated measurements and general clutter processes. The generalized PHD-filter possesses the ability to track targets, which are themselves point scatterers and create multiple measurements per scan, that are not drawn according to a spatial probability distribution in the measurement space.

In [10] the generalized PHD-filter is investigated and approximation conditions are developed. A small numerical evaluation is carried out to demonstrate that the proposed methods are applicable. This work extends the considerations made there by introducing a generalization of the probability of detection for targets that generate multiple measurements per sensor-scan. Furthermore, extensive numerical evaluations are carried out. Especially, the proposed methods are applied to the challenge of BML for the first time and compared to an existing approach.

In this paper, the generalized PHD-filter, using a Poisson-model for the clutter process, is investigated for the purpose of tracking targets with multiple measurements per sensor-scan. Thereby, a spatial distribution of the measurements is not assumed. Two approximations of the update equation of the generalized PHD-filter with Poisson-clutter are presented to reduce the number of partitions and thus the computational effort. To assess the proposed approach two numerical evaluations are carried out. First, a multi-target scenario, where two targets generate multiple correlated measurements, is investigated and different parameterizations of an SMC-implementation of the generalized PHD-filter are applied. The results are compared in terms of the estimated number of targets, the mean runtime per update, the optimal sub pattern assignment (OSPA) metric and the root mean squared error (RMSE). Second, the generalized PHD-filter is applied to a single-target BML-scenario and it is compared to an adaption of the standard iFilter, presented in [9] in terms of the RMSE and the processing time. A detailed investigation of the different likelihood-functions in the BML-scenario of the two compared approaches closes the numerical evaluation.

This paper is organized as follows. Section II gives an overview of the relation between PGFLs and the PHD-filter. Section III considers the formulation of the problem. In III-A the generalized PHD-filter with Poisson clutter model is presented. Section III-B investigates the computational complexity for scenarios where target measurements are not spatially related. Section IV-A proposes two ways of approximating the update equation of the generalized PHD-filter with Poisson-clutter and without using a distance-criterion in the measurement space. Two numerical evaluations are carried out in Section V. In Section V-A different parameterizations of SMC-implementations of the generalized PHD-filter are applied to a multi-target scenario.The generalized PHD-filter is applied to a BML-scenario and compared to the standard iFilter adaption presented in [10] in Section V-B. Conclusions are drawn and future work is presented in Section VI.

II. PROBABILITY GENERATING FUNCTIONALS AND THE PHD-FILTER

This section follows the considerations and notation of [21], which is based on [26], [23], [24], [25], [34], [8] and [30]. It has to be pointed out that all the work presented in this section has been presented first in [26], [23], [24], [25], [34], [8] and [30]. However, the notation of this paper is based on [21] and [35] since the authors consider it to be more intuitive.

This section provides background information about the connection between PGFLs and the PHD-filter which are necessary to understand following sections. However, for details the authors refer to [26], [21], [34] or [35], which explains the connection between the PHD- and the Intensity-filter, using PGFLs.

To begin with, let $\mathcal{X}$ be a separable metric space. A typical choice for $\mathcal{X}$ is $\mathbb{R}^d$, $d > 0$, which is sufficient for the most applications appearing in target tracking. Then the space of sets of points in $\mathcal{X}$ is defined by

$$E_{\mathcal{X}} := \emptyset \cup \bigcup_{n\geq 1} \mathcal{X}^{(n)}, \tag{1}$$

where $\mathcal{X}^{(n)}$ is the space of sets of size $n \in \mathbb{N}$, that is

$$\mathcal{X}^{(n)} := \{\{x_1,\ldots,x_n\} \mid x_i \in \mathcal{X}, i = 1,\ldots,n\}. \tag{2}$$

When interpreting $\mathcal{X}$ as the target state space in a classical multi-target tracking scenario, where the number of present targets are not known, $E_{\mathcal{X}}$ can be interpreted as the collection of all possible combinations of target states at a given time-step. It is assumed, that each element $\varphi \in E_{\mathcal{X}} \setminus \emptyset$ is locally finite, that is each bounded subset of $\mathcal{X}$ must only contain a finite number of points of φ and simple, i.e.,

$$\forall x_i, x_j \in \varphi, \quad x_i = x_j \Rightarrow i = j. \tag{3}$$

In terms of target tracking this translates to the assumptions that only finitely many targets can be present in a scenario and that no two targets share the same state. A stochastic process in the sense of [34] is defined as a measurable mapping

$$\Phi : (\Omega, \mathcal{F}, \mathbb{P}) \to (E_{\mathcal{X}}, \mathcal{B}(E_{\mathcal{X}})), \tag{4}$$

where $(\Omega, \mathcal{F}, \mathbb{P})$ is an arbitrary probability space and $\mathcal{B}(E_{\mathcal{X}})$ denotes the Borel σ-algebra of $E_{\mathcal{X}}$. Note that due to this definition the stochastic model of the point process is defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, since Φ is defined to be a measurable mapping. The associated counting function for an arbitrary $B \in \mathcal{B}(\mathcal{X})$ is defined by

$$\begin{aligned} N_{\cdot}(B) : (E_{\mathcal{X}}, \mathcal{B}(E_{\mathcal{X}})) &\to (\mathbb{N}, \mathcal{B}(\mathbb{N})) \\ \varphi &\mapsto N_{\varphi}(B) := |B|, \end{aligned} \tag{5}$$

which counts the number of elements in B and is measurable. Then the composition,

$$N_{\cdot}(B) \circ \Phi = N_{\Phi(\cdot)}(B) : (\Omega, \mathcal{F}, \mathbb{P}) \to (\mathbb{N}, \mathcal{B}(\mathbb{N})) \tag{6}$$

is measurable, since the composition of measurable functions is measurable again. It can be interpreted as a counting-function of the outcomes of the stochastic process. In target-tracking for example $N_{\cdot}(B) \circ \Phi$ yields the number of targets in the area $B \in \mathcal{B}(\mathcal{X})$ for a specific element in the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The intensity measure (first order moment, PHD, etc.) is defined for an arbitrary $B \in \mathcal{B}(\mathcal{X})$ by the expectation value

$$\mathbb{E}[N_{\Phi(\cdot)}(B)] = \int_{\Omega} N_{\Phi(\omega)}(B)\mathbb{P}(d\omega) = \int_{E_{\mathcal{X}}} N_{\varphi}(B) P_{\Phi}(d\varphi) \tag{7}$$

$$=: \mu_{\Phi}(B), \tag{8}$$

and thus yields the expected number of points in B. In target tracking $\mu_{\Phi}(B)$ for $B \in \mathcal{B}(\mathcal{X})$ denotes the expected number of targets in some area $B \subseteq \mathcal{X}$. Therefore, the intensity measure is not a probability density function. Instead, it can be described as a function of subsets of $\mathcal{X}$ that determines the expected number of elements therein. P_{Φ} denotes the pushforward (image) measure of $\mathbb{P}$, using the point process Φ. For any bounded and Lebesgue-integrable function

$$h : (\mathcal{X}, \mathcal{B}(\mathcal{X})) \to (\mathbb{R}, \mathcal{B}(\mathbb{R})) \tag{9}$$

the PGFL of the point process Φ is defined by

$$G_\Phi[h] := \sum_{n\geq 0} \int_{\mathcal{X}^{(n)}} \prod_{i=1}^{n} h(x_i) P_\Phi(d\{x_1,\ldots,x_n\}) \tag{10}$$

$$= \sum_{n\geq 0} \frac{1}{n!} \int_{\mathcal{X}^n} \prod_{i=1}^{n} h(x_i) f_\Phi(x_1,\ldots,x_n) dx_1 \ldots dx_n, \tag{11}$$

where $f_\Phi : \mathcal{X}^n \to \mathbb{R}$ is the multi-object density of the corresponding Jannossy measure and defined such that

$$\int_B n! P_\Phi(d\{x_1,\ldots,x_n\}) = \int_B f_\Phi(x_1,\ldots,x_n) dx_1 \ldots dx_n \tag{12}$$

holds for all $B \in \mathcal{B}(E_X)$. Equation (11) holds due to the assumed absolute continuity of P_Φ and the application of the Radon-Nikodym Theorem (for details see [26] or [21, p. 16]).

To illustrate the connection of PGFLs and the PHD-filter, the update equation (the prediction equation is derived analogously) is exemplary derived using Bayes theorem, the definition of PGFLs and functional differentiation. For details see [26], [21, p. 17–p. 24] or [35].

Let $X \subseteq \mathbb{R}^d$, $d > 0$ be the target and $Z \subseteq \mathbb{R}^l$, $l > 0$ be the measurement space. Then

$$E_X = \emptyset \cup \bigcup_{n\geq 1} X^{(n)} \tag{13}$$

and

$$E_Z = \emptyset \cup \bigcup_{n\geq 1} Z^{(n)} \tag{14}$$

are defined analogously to (1). According to the definition of conditional probability, the multi-joint object density is defined on the product space $E_Z \times E_X$ by

$$f_{Z,X} : E_Z \times E_X \to \mathbb{R}$$

$$(Z,X) \mapsto f_{Z,X}(Z,X) = L_{Z|X}(Z \mid X) f_X(X), \tag{15}$$

where $L_{Z|X} : E_Z \times E_X \to \mathbb{R}$ denotes the multi-object likelihood-function on $E_Z \times E_X$, $f_X : E_X \to \mathbb{R}$ is the distribution of X on E_X. In the same way Bayes theorem for point processes is given by

$$f_{X|Z} : E_X \times E_Z \to \mathbb{R}$$

$$(X,Z) \mapsto f_{X|Z}(X \mid Z) = \frac{L_{Z|X}(Z \mid X) f_X(X)}{\int_{E_X} \frac{1}{|X'|!} L_{Z|X}(Z \mid X') f_X(X') dX'}, \tag{16}$$

where $f_{X|Z} : E_X \times E_Z \to \mathbb{R}$ denotes the conditional multi-object density on $E_X \times E_Z$. By multiplying (16) with $(1/|X|!) \prod_{x\in X} h(x)$ and integrating over E_X the PGFL-form of Bayes theorem is obtained by

$$G_{X|Z}[h \mid Z] = \frac{\int_{E_X} \frac{1}{|X|!} \prod_{x\in X} h(x) L_{Z|X}(Z \mid X) f_X(X) dX}{\int_{E_X} \frac{1}{|X'|!} L_{Z|X}(Z \mid X') f_X(X') dX'}. \tag{17}$$

In the following we are interested in an alternative representation of (17). To this end, we first determine the PGFLs of the multi-object likelihood-function and derive afterwards the PGFL of the joint state under the following assumptions (see [26], [35, chapter 5.2]).

1) The target process is a Poisson Point Process (PPP) on X with intensity function $\mu s(\cdot)$.
2) Conditioned on the event $X = \{x_1,\ldots,x_n\}$, the measurement process is the superposition of n mutually independent, identical, target-oriented measurement-processes and a given PPP clutter process on Z with intensity function $\lambda c(\cdot)$.
3) A target generates at most one measurement in Z.

First, the PGFL of the multi-object likelihood-function $L_{Z|X}(Z \mid X)$ is considered for different cases. If $X = \emptyset$, the PGFL is given by

$$G_{Z|X}[g \mid \emptyset] := G_{\text{clutter}}[g]$$

$$:= \exp\left(\lambda\left(\int_Z c(z)g(z)dz - 1\right)\right) \tag{18}$$

due to assumption (2), where $\lambda \in \mathbb{R}$ denotes the average number of clutter and each false alarm is distributed according to $c : Z \to [0,1]$. Second, let $X = \{x\}$. Then it holds, that

$$G_{Z|X}[g \mid \{x\}] = \int_{E_Z} \frac{1}{|Z|!} \prod_{z\in Z} g(z) L_{Z|X}(Z \mid \{x\}) dZ$$

$$= L_{Z|X}(\emptyset \mid \{x\}) + \int_Z g(z) L_{Z|X}(\{z\} \mid \{x\}) dz$$

$$=: G_{\text{obs}}[g \mid x], \tag{19}$$

where (19) is given due to assumption (3) and

$$g : (Z, \mathcal{B}(Z)) \to (\mathbb{R}, \mathcal{B}(\mathbb{R})) \tag{20}$$

denotes a bounded and Lebesque integrable test-function. Let $\hat{L}_{Z|X}$ and p_D be defined on $Z \times X$ and X, respectively such that

$$L_{Z|X}(\{z\} \mid \{x\}) = p_D(x) \hat{L}_{Z|X}(z \mid x) \tag{21}$$

and

$$L_{Z|X}(\emptyset \mid \{x\}) = 1 - p_D(x), \tag{22}$$

where $x \in X$, $z \in Z$. This implies

$$G_{\text{obs}}^{\text{PHD}}[g \mid x] = 1 - p_D(x) + p_D(x) \int_Z g(z) \hat{L}_{Z|X}(z \mid x) dz, \tag{23}$$

where $x \in X$. From now on due to simplicity $\hat{L}_{Z|X}$ is denoted by $L_{Z|X}$. Third, let $X = \{x_1,\ldots,x_n\}$. Then due to assumption (2) the PGFL is given by

$$G_{Z|X}[g \mid \{x_1,\ldots,x_n\}] = G_{\text{clutter}} \prod_{i=1}^{n} G_{\text{obs}}^{\text{PHD}}[g \mid x_i]. \tag{24}$$

The next step is to find an expression for the PGFL of the joint state. It holds that

$$G_{Z,X}[g,h] = \int_{E_X} \frac{1}{|X|!} \prod_{x\in X} h(x) \int_{E_Z} \frac{1}{|Z|!} \prod_{z\in Z} g(z) \tag{25}$$

$$L_{Z|X}(Z \mid X) f_X(X) dZ\, dX$$

$$= \int_{E_X} \frac{1}{|X|!} \prod_{x\in X} h(x) G_{\text{clutter}} \prod_{i=1}^{n} G_{\text{obs}}[g \mid x] f_X(X) dX \tag{26}$$

$$= G_{\text{clutter}} G_X[h G_{\text{obs}}^{\text{PHD}}[g \mid \cdot]], \tag{27}$$

where due to assumption (1) G_X is given by

$$G_X[h] := \exp(\mu(\int_X s(x)h(x)dx - 1)). \tag{28}$$

Equation (25) is given due to the definition of conditional probability and (26) holds since

$$G_{Z|X}[g \mid x] = \int_{E_Z} \frac{1}{z!} \prod_{z\in Z} g(z) L_{Z|X} dZ \tag{29}$$

and (24). The second factor of (27) denotes a composition of functionals, called branching form of the respective PGFL and holds due to the definition of PGFLs. Note, that (27) is obtained by considering

$$G_{\text{clutter}} \prod_{i=1}^{n} G_{\text{obs}}[g \mid \cdot] : X \to \mathbb{R} \tag{30}$$

as a test-function with respect to $x \in X$. In (28) μ denotes the average number of targets and $s : \mathbb{R} \to [0,1]$ is the distribution of the targets.

In the following the derivative of a PGFL is needed. Therefore, let G be a PGFL defined as in (11). Then the Gâteaux derivative of $G[h]$ with respect to the variation ω is defined by

$$\delta G[h;\omega] := \lim_{\epsilon \searrow 0} \frac{G[h+\epsilon\omega] - G[h]}{\epsilon}, \tag{31}$$

where ω is a real-valued, bounded and Lebesgue-integrable function on X (or Z). The differentiation with respect to multiple real-valued, bounded and integrable variations $\omega_1, \ldots, \omega_m$ is defined iteratively, that is

$$\delta^m G[h;\omega_1,\ldots,\omega_m] = \delta(\delta^{m-1} G[h;\omega_1,\ldots,\omega_{m-1}];\omega_m). \tag{32}$$

In [26] and [35] it is shown that

$$L_{Z|X}(Z \mid X) = \delta^m G_{Z|X}[g \mid X; \delta_{z_1},\ldots,\delta_{z_m}]|_{g=0} \tag{33}$$

and thus

$$G_{X|Z}[h \mid Z] = \frac{\delta^m G_{Z,X}[g,h;\delta_{z_1},\ldots,\delta_{z_m}]|_{g=0}}{\delta^m G_{Z,X}[g,1;\delta_{z_1},\ldots,\delta_{z_m}]|_{g=0}} \tag{34}$$

holds. Here, δ_a denotes Dirac delta at the point a.

The Gâteaux derivative with respect to the Dirac delta from (33) and (34) has to be investigated carefully, since Dirac delta is not a proper function [40], [12], [14], [38] and thus the ordinary Gâteaux derivative [13, p. 406] is not defined. However, it can be proven that (31) is well-defined for a large class of PGFLs [11], in the sense that Dirac delta is approximated by a series of test-functions, called approximate identities or Dirac sequences [3, p. 114]. In [11] it is shown that the Gâteaux derivative with respect to the Dirac delta from (33) and (34) can be defined using approximate identities for the class of PGFLs

$$\mathcal{P}_2 \equiv \left\{ \Psi : \mathcal{H} \to \mathbb{R} \mid \Psi(h) = \sum_{n\geq 0} \frac{a_n}{n!} \cdot \int_{\mathcal{X}^n} \prod_{i=1}^{n} h(x_i) f_\Phi(x_1,\ldots,x_n) dx_1 \cdots dx_n \right\}, \tag{35}$$

$$\mathcal{H} \equiv \{h : \mathcal{X} \to \mathbb{R} \mid h \text{ is bounded and Lebesgue-integrable}\}, \tag{36}$$

$a_n \in [0,1]$, where the multi-object density $f_\Phi(x_1,\ldots,x_n)$, $x_1,\ldots,x_n \in \mathcal{X} \subseteq \mathbb{R}$ is either continuous, bounded and in $L^1(\mu;\mathcal{X}^n)$ or in $C_0^0(\mathcal{X}^n)$, that is a continuous function with compact support, $n \in \mathbb{N}$. Furthermore, in [11] and [37] it is shown that many well known tracking filters can be represented by PGFLs from (35).

Finally, the update equation for the first order moment (or PHD) $\mu_{X|Z}$ is given by an additional functional derivative

$$\mu_{X|Z}(x \mid z_1,\ldots,z_m) = \delta G_{X|Z}[h \mid Z; \delta_x]|_{h=1}. \tag{37}$$

A computation of (34) and (37) yields the update equation of the PHD-filter

$$\mu_{X|Z}(x \mid z_1,\ldots,z_m) = \mu s(x) \Bigg((1 - p_D(x)) + \sum_{z\in Z} \frac{p_D(x) L_{Z|X}(z \mid x)}{\lambda c(z) + \mu \int_X p_D(x) s(x) \hat{L}_{Z|X}(z \mid x)} dx \Bigg). \tag{38}$$

III. FORMULATION OF THE PROBLEM

A. The Generalized PHD-Filter

The previous section shows how to obtain the update equation of the PHD-filter by inspecting PGFLs. When deriving the update equation using PGFLs, essentially three steps can be identified.

1) Definition of the PGFL of the multi-object likelihood-function $G_{Z|X}[g \mid \{x\}]$ (see (19)–(24))
2) Definition of the PGFL of the joint state $G_{Z,X}$ (see (25)–(28)).
3) Functional differentiation to determine the intensity (see (34) and (37)).

In [7] the authors present the GCR, which is a generalization of the fourth chain rule for functional derivatives from [26]. This can be used to differentiate complex PGFLs. Furthermore, a PHD-filter for general target-generated measurement models and a general clutter process is developed using the GCR. If the clutter is assumed to be Poisson, the update equation of the general PHD-filter for arbitrary number of measurements per target is given by

$$\mu_{X|Z}(x \mid z_1,\dots,z_m) = \mu s(x)\left(L_{Z|X}(\emptyset \mid x) + \frac{\sum_{\pi\in\Pi_{(1:m)}}\left(\sum_{j=1}^{|\pi|} L_{Z|X}(i(\pi_j)\mid x)\prod_{k=1,k\neq j}^{|\pi|}\eta_{\pi,k}\right)}{\sum_{\pi\in\Pi_{(1:m)}}\prod_{j=1}^{|\pi|}\eta_{\pi,j}}\right), \tag{39}$$

where

$$\eta_{\pi,j} := 1_{\{a:|a|=1\}}(\pi_j)\lambda c(i(\pi_{j,1})) + \mu\int s(x)p_D(x) \cdot L_{Z|X}(i(\pi_{j,1}),\dots,i(\pi_{j,|\pi_j|})\mid x)dx, \tag{40}$$

and $\Pi_{(1:m)}$ denotes the set of all partitions of $\{\delta_{z_1},\dots,\delta_{z_m}\}$, e.g., $\Pi_{(1:2)} = \{\{\{\delta_{z_1}\},\{\delta_{z_2}\}\},\{\{\delta_{z_1},\delta_{z_2}\}\}\}$. The value of the likelihood-function $L_{Z|X}(\emptyset \mid x)$ represents the probability of a non-detection of target x. The function $i : \Pi_{(1:m)} \to \mathcal{P}(Z)$ is defined as $i(\{\delta_{z_1},\dots,\delta_{z_m}\}) = (z_1,\dots,z_m)$, for all $j \in \{1,\dots m\}$ and

$$1_{\{a:|a|=1\}}(\pi) = \begin{cases} 1, & \text{if } |\pi| = 1 \\ 0, & \text{otherwise} \end{cases} \tag{41}$$

defines the indicator function. As mentioned in [7] the probability of detection p_D in (39) is defined more generally as in the standard PHD-filter. A discussion on this can be found in Section IV-C.

The derivation of (39) can be done analogously to the standard PHD-filter as described in Section II for

$$G_{\text{obs}}[g \mid x] := L_{Z|X}(\emptyset \mid x) + \sum_{n\geq 1}\frac{1}{n!}\int_{Z^{(n)}}\prod_{j=1}^{n} g(z_j)L_{Z|X}(z_1,\dots,z_n \mid x)dz_1\dots dz_n \tag{42}$$

instead of (23), using the GCR. The difference between (42) and (23) is that in the second summand of (42) a sum occurs. This is due to the fact that the generalized PHD filter does not assume that a target generates at most one measurement. Instead a target can generate an arbitrary number of measurements and thus all sets of size n, that is $Z^{(n)}$ (see definition (1)) need to be considered. Another possible approach for deriving the update equation of the generalized PHD-filter is to use so-called secular functions and the technique of automatic differentiation as it is done in [36].

Note that (39) can handle correlated measurements originating from a specific target, since only the assumption that the measurement process is the superposition of n mutually independent (conditioned on $X = \{x_1,\dots,x_n\}$) target-oriented measurement-processes is needed for the derivation of the update equation. In particular, the measurements are not assumed to be independent conditioned on a specific target state. Measurements originating from different targets cannot be correlated since in the derivation of the generalized PHD-filter in [7] the corresponding measurement processes need to be mutually independent.

B. Computational Complexity of the General PHD-Filter

The update equation of the PHD-filter for targets with a general target-generated measurement model (39) and Poisson clutter is highly complex due to the combinatorial sum numerically. The number of partitions is growing exponentially with the number n of measurements and is given by the Bell number B_n. The exponential-growth of the Bell number is visualized in Figure 2 and one can see that for an application of (39) approximations are inevitable. In [16], [17], [18], [20], [19] and [39] clustering approaches, which are essentially based on the spatial relation of measurements, are used to reduce the number of partitions. These approximations are possible, if the measurements of a target are spatially related in the measurement space. However, scenarios exist where a target generates multiple measurements per scan which are not spatially related in the measurement space, e.g., BML. For such scenarios, the partitions in equation (39) need to be reduced without using any information about the distribution of measurements in the measurement space.

IV. APPROXIMATION OF THE UPDATE EQUATION

As mentioned in the previous section an evaluation of all feasible measurement set partitions is not possible due to the exponential-growth of the number of partitions with increasing set size. Moreover, a reduction of the number of partitions by the application of clustering methods is not applicable if measurements that belong to a specific target are not spatially related in the measurement space. To this end, two novel approaches are presented in the following section, which approximate the update equation of the generalized PHD filter, by reducing the number of investigated partitions without assuming an underlying spatial distribution of the measurements belonging to a specific target in the measurement space. Furthermore, a generalized definition of the probability of detection is presented for the generalized PHD filter.

A. Incorporation of a Priori Information

The first proposed approximation of equation (39) considers available a priori information about the number of generated measurements per target- and sensor-scan. The idea is to restrict the possible number of generated measurements, that is to assume that a target generates at least $N_{\min} \in \mathbb{N}$ and at most $N_{\max} \in \mathbb{N}$ measurements per sensor-scan. Even though it might seem obvious how a restriction of the number of measurements per target will influence the update equation of the generalized PHD-filter, a detailed derivation is carried out in the following to demonstrate how a priori information and specific assumptions can be incorporated via a mathematically correct approach into an existing generalized PHD-filter.

To derive the respective PHD-update equation the general higher order chain rule, presented in [6] is used. As in the previous section let

$$g : (Z, \mathcal{B}(Z)) \to (\mathbb{R}, \mathcal{B}(\mathbb{R})) \tag{43}$$

and

$$h : (X, \mathcal{B}(X)) \to (\mathbb{R}, \mathcal{B}(\mathbb{R})) \tag{44}$$

be bounded and Lebesgue-integrable test-functions. First, the PGFL of the joint state is given analogously to equation (27) by

$$G_{Z,X}[g,h] = G_{\text{clutter}} G_X[h G_{\text{obs}}[g \mid \cdot]] = (\exp \circ f)[g,h], \tag{45}$$

where

$$f[g,h] := \lambda \left(\int c(z) g(z) dz - 1 \right) + \mu \left(\int s(x) h(x) G_{\text{obs}}[g \mid x] dx - 1 \right) \tag{46}$$

and the approximated PGFL of the likelihood-function $G_{\text{obs}}[g \mid \cdot]$, which incorporates the a priori knowledge on the number of measurements per target is defined by

$$G_{\text{obs}}[g \mid x] := L_{Z|X}(\emptyset \mid x) + \sum_{n=N_{\min}}^{N_{\max}} \frac{1}{n!} \int_{Z^n} \prod_{j=1}^{n} g(z_j) L_{Z|X,n}(z_1, \ldots, z_n \mid x) dz_1 \ldots dz_n. \tag{47}$$

Note, that the key to obtain a PGFL-derivable filters with specific target-generated measurement models, only $G_{\text{obs}}[g \mid \cdot]$ needs to be adapted. Therefore, incorporating assumptions/information that is scenario specific, can be done by modifying $G_{\text{obs}}[g \mid \cdot]$ accordingly. Applying the general higher order chain rule to determine the functional derivative of (45) with respect to impulses yields

$$\begin{aligned} \delta^m G_{Z,X}[g,h;\delta_{z_1},\ldots,\delta_{z_m}] &= \delta^m (\exp \circ f)[g,h;\delta_{z_1},\ldots,\delta_{z_m}] \\ &= \sum_{\pi \in \Pi_{(1:m)}} \delta^{|\pi|} \exp(f[g,h]; \xi_{\pi_1}[g,h], \ldots, \xi_{\pi_{|\pi|}}[g,h]) \\ &= \sum_{\pi \in \Pi_{(1:m)}} \exp(f[g,h]) \prod_{j=1}^{|\pi|} \xi_{\pi_j}[g,h], \end{aligned} \tag{48}$$

where

$$\begin{aligned} \xi_\omega[g,h] &= \delta^{|\omega|} f[g,h;\omega_1,\ldots,\omega_{|\omega|}] \\ &= \mu \int s(x) h(x) \delta^{|\omega|} G_{\text{obs}}[g;\omega_1,\ldots,\omega_{|\omega|}] dx. \end{aligned} \tag{49}$$

For the evaluation of (49) the functional derivative of definition (47) has to be considered. Therefore, let ω be an arbitrary element of a partition from $\Pi_{(1:m)}$. Then, the Gâteaux derivative of the functional is given by

$$\delta^{|\omega|} G_{\text{obs}}[g;\omega_1,\ldots,\omega_{|\omega|}] = \sum_{n=N_{\min}}^{N_{\max}} \frac{1}{n!} \cdot n \cdot (n-1) \cdot \ldots \cdot (n - |\omega| + 1) \cdot \int_{Z^{n-|\omega|}} \prod_{j=1}^{n-|\omega|} g(z'_j) L_{Z|X}(i(\omega), z'_1, \ldots, z'_{n-|\omega|} \mid x) dz'_1 \ldots dz'_{n-|\omega|} \tag{50}$$

if $|\omega| < N_{\min}$. If $|\omega| \in \{N_{\min}, \ldots, N_{\max} - 1\}$ it is given by

$$\begin{aligned} \delta^{|\omega|} G_{\text{obs}}[g;\omega_1,\ldots,\omega_{|\omega|}] = \Bigg(& L_{Z|X}(i(\omega) \mid x) \\ & + \sum_{n=N_{\min}}^{N_{\max}} \frac{1}{n!} \cdot n \cdot (n-1) \cdot \ldots \cdot (n - |\omega| + 1) \\ & \cdot \int_{n=N_{\min}}^{N_{\max}} \prod_{j=1}^{n-|\omega|} g(z'_j) L_{Z|X}(i(\omega), z'_1, \ldots, z'_{n-|\omega|} \mid x) dz'_1 \ldots dz'_{n-|\omega|} \Bigg) \end{aligned} \tag{51}$$

and if $|\omega| = N_{\max}$ it is equal to

$$\delta^{|\omega|} G_{\text{obs}}[g;\omega_1,\ldots,\omega_{|\omega|}] = L_{Z|X}(i(\omega) \mid x). \tag{52}$$

If $|\omega| > N_{\max}$ the derivative is

$$\delta^{|\omega|} G_{\text{obs}}[g;\omega_1,\ldots,\omega_{|\omega|}] = 0. \tag{53}$$

Thus,

$$\begin{aligned} \delta^{|\omega|} G_{\text{obs}}[0;\omega_1,\ldots,\omega_{|\omega|}] &= \begin{cases} L_{Z|X}(i(\omega) \mid x), & \text{if } |\omega| \in \{N_{\min}, \ldots, N_{\max}\} \\ 0, & \text{otherwise} \end{cases} && (54) \\ &= 1_A(\omega) L_{Z|X}(i(\omega) \mid x), && (55) \end{aligned}$$

where $A := \{a : |a| \in \{N_{\min}, \ldots, N_{\max}\}\}$. In the following, the short-hand notation from (55) is used. Given the functional derivative of the PGFL of the joint state with respect to impulses the update equation of the corresponding PHD-filter can be determined. It is given by

$$\mu_{X|Z}(x \mid z_1, \ldots, z_m) = \frac{\delta^{m+1} G_{Z,X}[0,1;\delta_{z_1},\ldots,\delta_{z_m},\delta_x]}{\delta^m G_{Z,X}[0,1;\delta_{z_1},\ldots,\delta_{z_m}]} \tag{56}$$

$$= \left(\sum_{\pi \in \Pi_{(1:m)}} \prod_{j=1}^{|\pi|} \xi_{\pi_j}[0,1] \right)^{-1} \left(\sum_{\pi \in \Pi_{(1:m)}} \delta B_\pi[0,1;\delta_x] \right), \tag{57}$$

where

$$B_\pi[g,h] := f[g,h] \cdot \prod_{j=1}^{|\pi|} \xi_{\pi_j}[g,h] \tag{58}$$

and

$$\delta B_\pi[g,h;\delta_x] = \mu s(x) G_{\mathrm{obs}}[g \mid x] \prod_{j=1}^{|\pi|} \xi_{\pi_j}[g,h]$$

$$+ \sum_{j=1}^{|\pi|} \mu s(x) \delta^{|\pi_j|} G_{\mathrm{obs}}[g; \pi_{j,1}, \ldots, \pi_{j,|\pi_j|} \mid x]$$

$$\cdot \prod_{k=1,k\neq j}^{\pi} \xi_{\pi_k}[g,h] \tag{59}$$

The evaluation of (57) yields the update equation of the approximated generalized PHD-filter with Poisson-clutter. It is given by

$$\mu_{X|Z}(x \mid z_1,\ldots,z_m) = \mu s(x) \Bigg(L_{Z|X}(\emptyset \mid x)$$

$$+ \frac{\sum_{\pi\in\Pi_{(1:m)}} \sum_{j=1}^{|\pi|} 1_A(\pi_j) L_{Z|X}(i(\pi_j) \mid x) \prod_{k=1,k\neq j}^{|\pi|} \eta_{\pi,k}}{\sum_{\pi\in\Pi_{(1:m)}} \prod_{j=1}^{|\pi|} \eta_{\pi,j}} \Bigg), \tag{60}$$

where

$$\eta_{\pi,j} := \xi_{\pi_j}[0,1] = 1_{\{a:|a|=1\}}(\pi_j) \lambda c(i(\pi_{j,1})) +$$

$$\mu \int s(x) 1_A(\pi_j) p_D(x) L_{Z|X}(i(\pi_j) \mid x) dx. \tag{61}$$

Due to the fact that some summands of equation (60) are zero, computational effort can easily be saved. A summand of the sum over all partitions in (60) is zero if for the respective partition $\pi \in \Pi_{(1:m)}$ holds

$$\exists j \in \{1,\ldots,|\pi|\} : |\pi_j| \notin \{1, N_{\min}, \ldots, N_{\max}\}, \tag{62}$$

since then either $1_{\{a:|a|=1\}}(\pi_j) = 0$ or $1_A(\pi_j) = 0$. Therefore, the computational effort can be reduced by rejecting the partitions which fulfill condition (62). After rejecting the partitions, equation (39) can be evaluated, since except for the appearance of $1_A(\cdot) = 0$ it is identical to equation (60).

Note, that partitions are not rejected, if they have a subset, which is of cardinality one. This is independent of $N_{\min}$ and $N_{\max}$ and holds since a Poisson-clutter model is chosen. Therefore, clutter is modeled as single measurements in the measurement space. However, more enhanced clutter models could be included. For example, in a BML-scenario the context information, which is available due to a ray-tracer, does not consider cars and other road users. Therefore, typical clutter sources in a BML-scenario can be road users, which reflect the signal emitted by the mobile station and act as new point sources of the reflected electromagnetic wave(s). Thus, multipaths which are received due to the same clutter source are not independent and hence clutter models which enable multiple measurements per clutter source could enhance data fusion algorithms. Obviously, condition (62) then needs to be adapted.

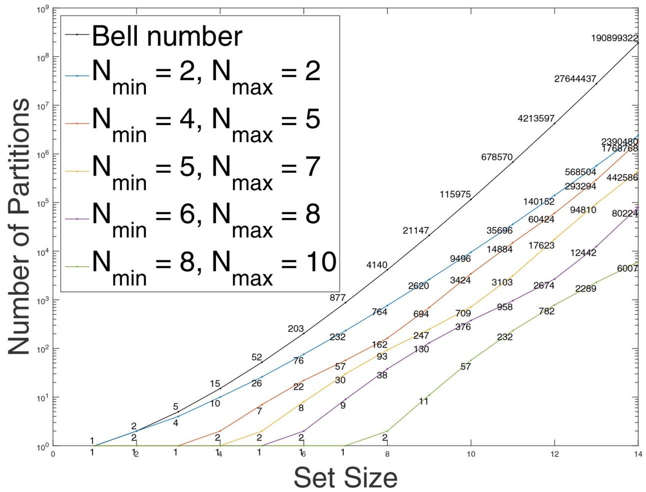


Fig. 2. Comparison of the Bell number and the number of partitions due to approximation (60).

B. Evaluation of Significant Summands

In practical applications, the likelihood-function is close to zero or might even be represented by zero for unlikely events due to the numerical resolution of the computer. Therefore, another practical approach of reducing the number of partitions which have to be considered in equation (60) is to evaluate only the terms for which the likelihood-function value is above a specific significance-threshold. To this end, a criterion based on the cardinality of the partition elements is developed to determine those partitions. Let $\pi \in \Pi_{(1:m)}$ be an arbitrary partition which does not fulfill criterion (62) and $x \in X$ be an arbitrary target position. Then, if

$$\exists j \in \{1,\ldots,|\pi|\} : |\pi_j| > 1 \quad \text{and} \quad L_{Z|X}(i(\pi_j) \mid x) \le \tau \tag{63}$$

is fulfilled

$$\sum_{j=1}^{|\pi|} 1_A(\pi_j) L_{Z|X}(i(\pi_j) \mid x) \prod_{k=1,k\neq j}^{|\pi|} \tilde{\eta}_{\pi,k} \approx 0 \tag{64}$$

approximately holds, where $\tau > 0$ is a chosen threshold, which is suitable small, for the significance of a partition. Note that $|\pi_j| > 1$ in (63) has to be fulfilled due to the first summand in $\tilde{\eta}_{\pi,j}$, since otherwise it might happen that only the jth summand of (64) is approximately zero, while the other summands are significantly larger than zero. Hence condition (63) can be used to reduce the number of the considered partitions. If $\tau = 0$ in (63), "$\approx$" can be replaced by "=" in (64).

Note that for the application of this condition the likelihood-function has to be evaluated for all possible subsets and all particle positions. The number of all possible subsets is given by the binomial series, e.g., for a set of m measurements,

$$N_{\text{Subsets}} = \sum_{k=0}^{m} \binom{m}{k} = 2^m \tag{65}$$

subsets have to be evaluated. However, depending on $N_{\min}$ and $N_{\max}$ the application of condition (62) already reduces the number of subsets which have to be considered significantly, that is for $m > 0$ and $1 < N_{\min} \leq N_{\max} \leq m$ an application of condition (62) reduces the number of subsets of the measurement set, which have to be considered to

$$N_{\text{Subsets}} = 2^{(N_{\max}+1)-N_{\min}+1}. \tag{66}$$

C. Generalization of the Probability of Detection

In [7] the authors emphasize that the probability of detection in [7, (27)] is defined more generally than in the standard PHD-filter, where the detection is modeled by a single Bernoulli-process. If targets generate multiple measurements per scan, the detection process can be modeled by a discrete probability distribution over the number of measurements. For (39) and (60) the single-target likelihood function can be formulated by

$$L_{Z|X}(i(\pi_j) \mid x) = p(|\pi_j|, x) \cdot \hat{L}_{Z|X}(i(\pi_j) \mid x), \tag{67}$$

where the sensor likelihood function $\hat{L}_{Z|X}(i(\pi_j) \mid x)$ is given by

$$\hat{L}_{Z|X}(i(\pi_j) \mid x) = \tilde{L}(i(\pi_j) \mid x) \tag{68}$$

and

$$\hat{L}_{Z|X}(\emptyset \mid x) = 1. \tag{69}$$

Here, $x \in X$, where $X \subseteq \mathbb{R}^d$, $d > 0$ denotes the target space and $\pi_j \in \pi$, where $\pi \in \Pi_{(1:m)} \cup \emptyset$ denotes the set of partitions for a set of measurements of cardinality m, defined analogously to Section III-A.

For the definition of the generalized probability of detection $p(\cdot, x)$, $x \in X$ the detection process of the measurements, which are generated by the same target, needs to be investigated. If the detections (each considered as a random variable) of the single measurements are conditionally (conditioned on a specific target) independent and have the same distribution (same detection probability), the detection process can be modeled by a series of Bernoulli-trials. If this assumption is fulfilled, a possible choice for $p(\cdot, x)$ is the Poisson-distribution, that is

$$p(n, x) = \frac{\lambda^n}{n!} e^{-\lambda}, \tag{70}$$

for all $n \in \mathbb{N}$ and $x \in X$, where the parameter $\lambda \in \mathbb{R}_{>0}$ is the expected number of measurements per target. If the number of measurements, which are generated by a single target can be restricted to $N_{\max} \in \mathbb{N}$ the Binomial-distribution can be used to model the detection process. It is given by

$$p(n, x) = \binom{N_{\max}}{n} q^n (1-q)^{N_{\max}-n}, \tag{71}$$

for all $n \in \mathbb{N}$ and $x \in X$, where $q \in [0,1]$ denotes the detection probability of an individual measurement.

Note, that the Binomial-distribution can be considered as a special case of the Poisson-distribution. To this end, let

$$q := \frac{\lambda}{N_{\max}}. \tag{72}$$

Then

$$\lim_{N_{\max}\to\infty,\ q\to 0,\ N_{\max}q\to\lambda} \binom{N_{\max}}{n} q^n (1-q)^{N_{\max}-n} = \frac{\lambda^n}{n!} e^{-\lambda}, \tag{73}$$

for all $n \in \mathbb{N}$ [29, p. 79].

Thus the Poisson-distribution can be used to model the detection process for small detection probabilities of the individual measurements and a large number of trials, that is in scenarios where the target may generate a large number of measurements. The Binomial-distribution can be used if the maximal number of measurements per target $N_{\max}$ is known. Note, that the two proposed definitions for $p(\cdot, x)$, $x \in X$ are only valid if the detections of the single measurements belonging to the same target are conditionally independent and identically distributed. If this assumption is not valid, other distributions need to be considered.

V. NUMERICAL EVALUATION

To verify the applicability of the considerations of Section IV two numerical evaluations are carried out in the following.

A. Multi-Target Scenario with Correlated Measurements

In this section, a two-target scenario is considered (see Figure 3 (a)), where the targets are moving linearly with a constant speed of 5 m/s (target 1) and 3 m/s (target 2). The trajectory of the first target starts at $(10,5)^T$ and is directed to $(650,400)^T$. The second target starts at $(10,500)^T$ and is directed to $(700,10)^T$. In each iteration (the scan time is 1 s) two correlated measurements are drawn per target according to a Gaussian-distribution with the following parameters. The mean is given by the position of the target and the covariance matrix is given by $\Sigma = \begin{pmatrix} R & C \\ C^T & R \end{pmatrix}$, $R = \text{diag}(10,10)$ and $C = \text{diag}(5,5)$. Since the probability of detection per measurement is $p_D = 0.8$, the covariance matrix Σ is restricted according to the size of available measurements per target. Furthermore, two clutter measurements are drawn uniformly per iteration in the field of view, which is given by

$$\text{FOV} := [0,700] \times [0,700]. \tag{74}$$

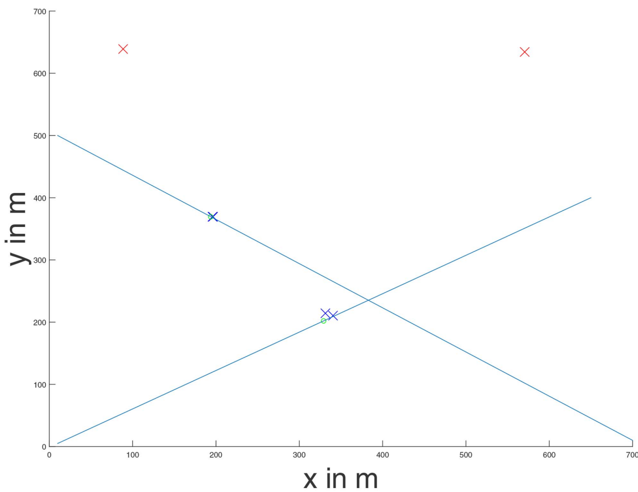


Fig. 3. Visualization of the two-target scenario. Two targets (green circle) are linearly moving with a constant velocity on their trajectory (blue line). In one iteration a target generates two correlated measurements (blue crosses), each with probability of detection $p_D = 0.8$. The measurements are drawn around the targets true position according to a Gauss-distribution with covariance matrix Σ. Furthermore, two clutter measurements (red crosses) are generated uniformly over the field of view in each iteration.

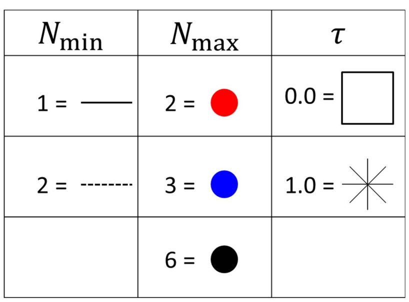


Fig. 4. Legend for Figures 5–8.

For the evaluation of (60), an SMC-implementation is used. The prediction is the same as for the standard PHD-filter, where the probability of survival is set to $p_S = 1.0$ for all particles and the single-object transition density is defined by the continuous white-noise acceleration model from [4] with $\tilde{q} = 1.5$. To reduce the computational complexity, in each iteration 50 newborn particles are generated around the measurements of the previous iteration. For the initialization 100 particles are uniformly drawn in the field of view. A standard resampling-algorithm (see [33]) is carried out and the maximal number of particles is restricted to 150. For the update of the filter, (60) is implemented and conditions (62) and (63) are used to restrict the number of partitions. The likelihood-function is defined as follows. Let $Z \subseteq \mathbb{R}^2$ be the measurement space. Then,

$$E_Z := \emptyset \cup \bigcup_{n \geq 1} Z^{(n)} \tag{75}$$

is defined analogously to Section II by the space of sets of points in Z. The likelihood-function is given by

$$L_{Z|X}(i(\cdot) \mid \cdot) : E_z \setminus \emptyset \times \mathbb{R}^4 \to \mathbb{R}. \tag{76}$$

Let in iteration $k \in \{1, \ldots, 100\}$ be $m_k \in \{2, \ldots, 6\}$ (number of clutter fixed to two, maximum number of measurements per target is two) the number of all received measurements. Let $X \subseteq \mathbb{R}^4$ be the target state space (position + velocity in 2 dimensions, respectively). Then, for a subset $z = \{z_1, \ldots, z_n\} \in E_Z \setminus \emptyset$, $n \in \{1, \ldots, m_k\}$ and $x \in X$ arbitrary the likelihood-function is defined by

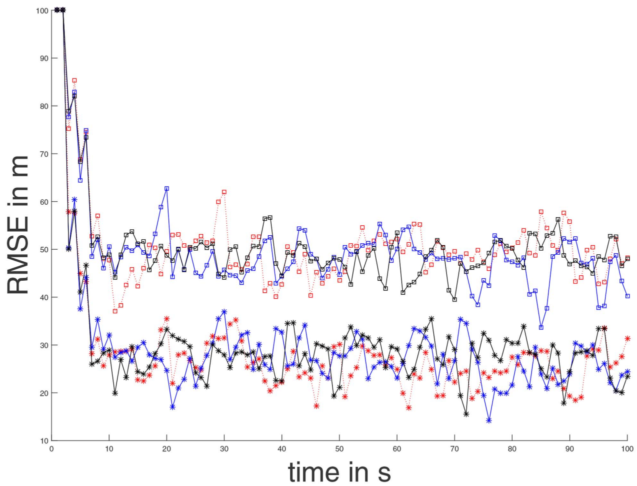
$$L_{Z|X}(i(z) \mid x) = p(|z|) \cdot \mathcal{N}\left(\begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}, \begin{pmatrix} Hx \\ \vdots \\ Hx \end{pmatrix}, \begin{pmatrix} R & C & \ldots & C \\ C^T & R & \ldots & C \\ \vdots & \vdots & \ddots & \vdots \\ C^T & C^T & \ldots & R \end{pmatrix} \right), \tag{77}$$

where $H := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ and $p : \mathbb{N} \to [0,1]$ defines the probability of observing a set of measurements with the respective number of elements. It is defined by

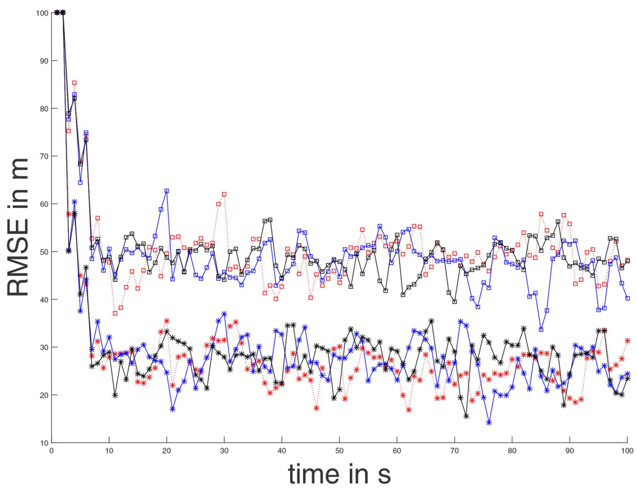
$$p(n) = \begin{cases} 1 - P_D, & \text{if } n = 0 \\ P_D \cdot \binom{2}{n} q^n (1-q)^{2-n}, & \text{if } n = 0, \end{cases} \tag{78}$$

where $P_D = q = 0.8$. Note that given a specific target state $x \in X$ the measurements are not conditionally independent. Since only the mutual independence of the target-oriented measurement processes is needed in the derivation of the generalized PHD-filter, the definition (77) makes sense. The mean number of clutter λ from (40) is set to two and the distribution of clutter $c : \text{FOV} \to [0,1]$ is uniform in the field of view. To extract a state estimate in each iteration, the k-means algorithm is applied to the set of particles, which were present in the previous iteration. The number of clusters k is given by the rounded number of estimated target states from the PHD-filter in each iteration. Note that the enhanced state extraction scheme, presented in [31] needs to be modified for applying it to a scenario where several targets generate multiple measurements.

To assess the proposed PHD-filter 100 Monte Carlo runs are performed for different parameterizations of the approximation conditions (63) and (62). Figure 7 visualizes the results of the different parameterizations in terms of the estimated number of targets. It can be seen that the estimated number of targets does not depend on the chosen partition-sizes, that is it does not depend on approximation criterion (63). This is due to the fact that the number of significant partitions with significance threshold $\tau = 0.0$ (and $\tau = 1.0$) is more or less equal for all three investigated parameterizations $N_{\text{min}} = 2/N_{\text{max}} = 2$, $N_{\text{min}} = 1/N_{\text{max}} = 3$ and $N_{\text{min}} = 1/N_{\text{max}} = 6$. Figure 9 visualizes exemplary for one parametrization the mean number of partitions, where in each Monte Carlo run the mean number of significant partitions is computed



(a) RMSE Target 1



(b) RMSE Target 1

Fig. 5. RMSE with respect to target 1 (a) and target 2 (b). Due to the fact that the generalized PHD-filter over estimates the number of present targets if the significance threshold τ is set to 1.0 (see Figure 7) and the fact that the target state extraction is based on the rounded number of estimated targets, the parametrization using $\tau = 1.0$ perform better in terms of the RMSE than the parametrizations using $\tau = 0.0$.

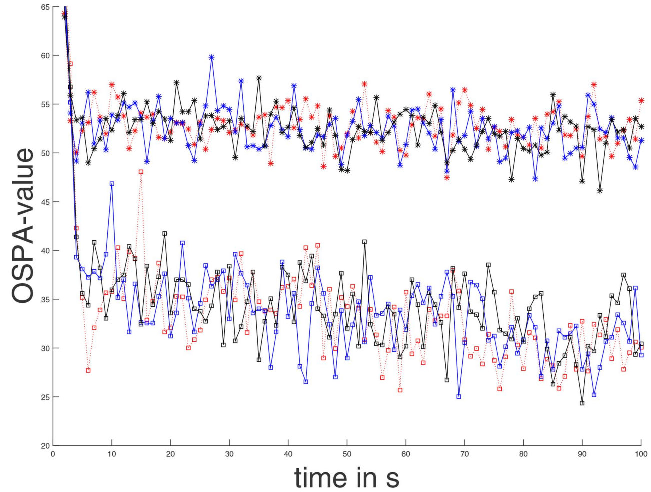


Fig. 6. Mean of the OSPA-values with order $p = 2$ and cut-off value $c = 100$. In terms of the OSPA-metric the parametrizations using $\tau = 0.0$ perform better compared to those that use $\tau = 1.0$, since the over-estimation of the number of targets (see Figure 7) is penalized by the OSPA-metric. The change of the number of investigated partitions does not yield a significant alteration of the results.

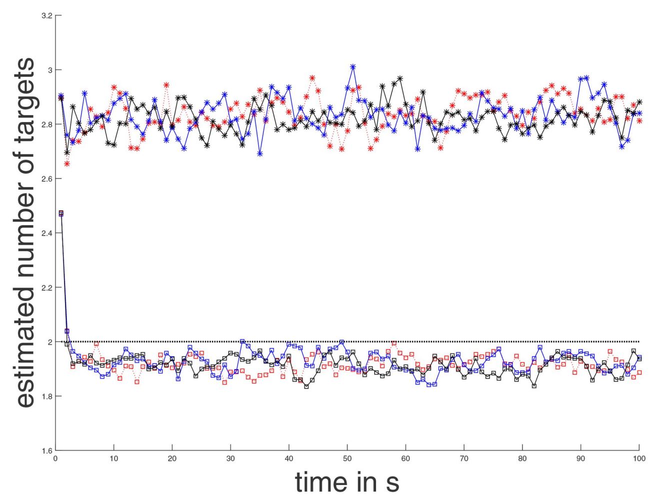


Fig. 7. Estimated number of targets, where the dashed black line shows the true number of present target.

for each time-step over all particles. Furthermore, it can be seen from Figure 7 that the parameterizations with significance threshold $\tau = 1.0$ have a larger deviation in terms of the estimated number of targets than the parameterizations using the significance threshold $\tau = 0.0$ and over-estimate the true number of present targets. This yields to a better performance of the parameterizations using the significance threshold $\tau = 1.0$ in terms of the root mean squared error (RMSE) with respect to the two true target states, which can be seen in Figures 5. This is due to the fact that the target state extraction is done using a k-means clustering algorithm, where k is given by the rounded estimated number of targets. Therefore, the over-estimation of the number of targets by the parameterizations with significance threshold $\tau = 1.0$ yields to a clustering that always estimates at least two clusters. In contrast to that, the parameterizations with significance threshold $\tau = 0.0$ under-estimate the true target number for some iterations and thus the k-means clustering algorithm estimates only one cluster for these iterations. In iterations, where no estimate for a specific target is produced by the generalized PHD-filter, the squared error is set to 100^2 m = 10000 m. However, the over-estimation yields to a worse performance of the parameterizations using significance threshold $\tau = 1.0$ compared to the parameterizations, which use a significance threshold $\tau = 0.0$, since each over-estimation is penalized by the OSPA-metric. The result in terms of the mean time consumption per iteration is shown in Figure 8. It can be seen that the parameterizations using the significance threshold $\tau = 1.0$ are faster compared to the parameterizations using a threshold of $\tau = 0.0$. Furthermore, the parameterizations with $N_{\min} = 2/N_{\max} = 2$ perform bet-

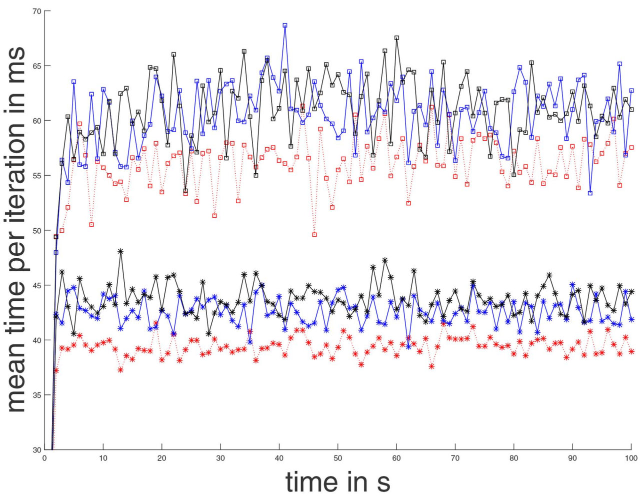


Fig. 8. Comparison of the mean time for updating the generalized PHD-filter of different parametrizations. It can be seen that the more partitions the filter processes and the smaller the significance threshold τ is chosen the longer the update takes.

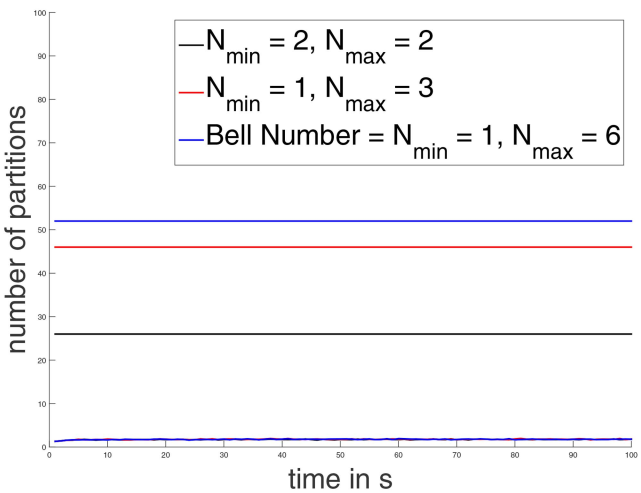


Fig. 9. Mean number of partitions resulting from condition (62) and mean number of partitions due to condition (63) for two parameterizations ($N_{\text{min}} = 2/N_{\text{max}} = 2$ (black), $N_{\text{min}} = 1/N_{\text{max}} = 3$ (red)). The number of all partitions is given by the Bell number (blue). For the computation of the mean number of significant partitions in each Monte Carlo run the mean number of significant partitions is computed for each time-step over all particles. Afterwards, the mean of the number of significant partitions is computed over all Monte Carlo runs. The number of significant partitions is almost the same for the two parameterizations.

ter in terms of time consumption than $N_{\text{min}} = 1/N_{\text{max}} = 3$ and $N_{\text{min}} = 1/N_{\text{max}} = 6$. In summary: the less partitions and the larger the significance-threshold is, the faster and worser the algorithm performs.

Also a parameterization without using the two approximation conditions has been investigated in terms of processed time per iteration. Since for this non-approximated SMC generalized PHD-filter one iteration took up to $5.30 \cdot 10^3$ s, only one MC-run has been performed. Thereby, the mean computation time was $2.28 \cdot 10^3$ s, which shows, that even if $N_{\text{min}} = 1$ and $N_{\text{max}} = 6$ (no approximation in terms of (62) has been made) and

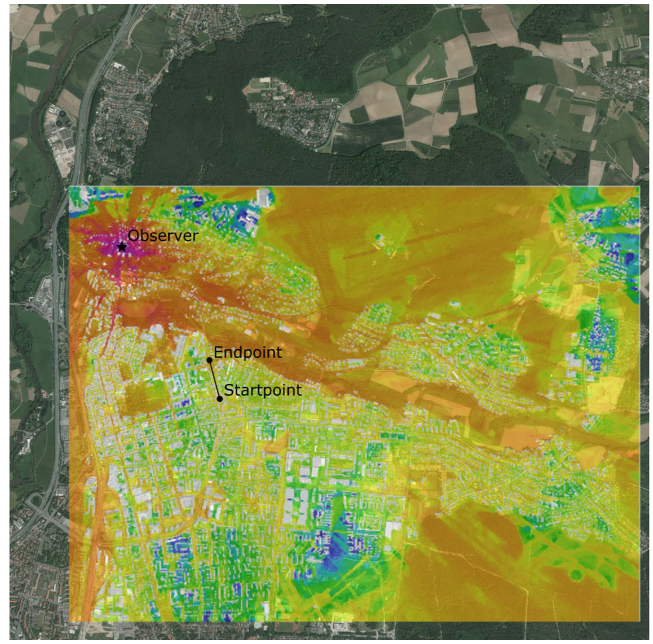


Fig. 10. A single target moves on a linear trajectory with constant speed in an urban environment. At each instance of time multipaths are created using a ray-tracer on a grid and the measurement process is simulated using a Gaussian distributed noise for each multipath—parameter. Furthermore, the detection process is simulated. The colors indicate the received field—strength at the observer (star). Map Data: ©GeoBasis-DE/BKG 2015. Ray-Tracer Visualization: AWE Communications.

$\tau = 0.0$ is chosen, the respective generalized PHD-filter parameterization ($N_{\text{min}} = 1/N_{\text{max}} = 3, \tau = 0.0$) performs about 45 times faster than the standard version, which does not use any approximation condition at all.

All in all, it is numerically shown that the proposed methods of approximation for the generalized PHD-filter can be applied to scenarios where targets generate multiple measurements. It should be noted that the definition of the likelihood-function does depend on the considered scenario and is not part of the numerical evaluation of this work. Furthermore, the following should be kept in mind. The integral of the clutter-intensity $\lambda c(\cdot)$ yields the number of false measurements (not false targets). Thus λ denotes the mean number of false measurements per iteration. Hence, clutter is defined in terms of elements of the measurement space, not as clutter targets in the target space. Thus, in scenarios where clutter scatterer generate multiple clutter-measurements per scan, enhanced clutter models need to be investigated.

B. Single Target Blind Mobile Localization Scenario

To demonstrate the connection of the presented approach to the challenge of BML a single-target scenario in a simulated urban environment is presented. For generating multipath-measurements, a database for a fixed OS and a grid of MS locations is generated by using a ray-tracing simulation. The distance between two grid points is set to 10 m. The number of received multipaths

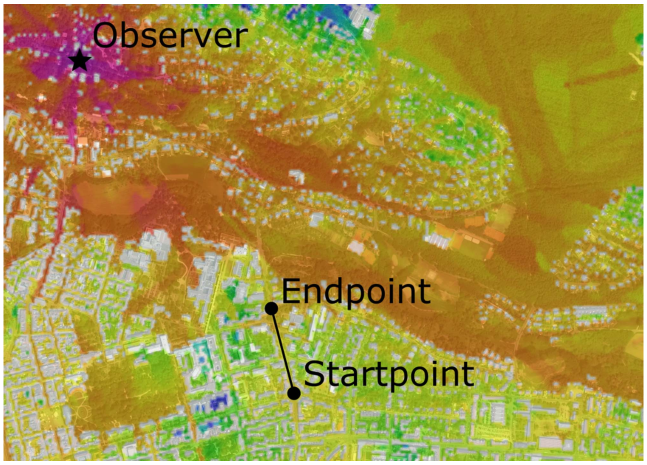


Fig. 11. Zoom of the investigated scenario. Map Data: ©GeoBasis-DE/BKG 2015. Ray-Tracer Visualization: AWE Communications.

is restricted to six and each multipath is characterized by its azimut (angle) of arrival (AoA), its elevation (angle) of arrival (EoA) and its relative time of arrival (RToA) with respect to the first received multipath. Thus,

$$Z := [0,2\pi] \times [-\pi,\pi] \times \mathbb{R}_{>0} \tag{79}$$

and

$$E_Z := \emptyset \cup \bigcup_{n\geq 1} \mathcal{Z}^{(n)} \tag{80}$$

analogously to Section II. Afterwards, a linear ground-truth for the target (that is an electromagnetic emitter), which is moving with a constant velocity of 2.4 m/s, is simulated (see Figure 11). Then, in each time-step the lower left grid point of the box, in which the target is located in, is determined. The multipaths which correspond to the chosen grid point are taken to generate the multipath-measurements, referred to as the true multipaths in the following. First, in each iteration Gaussian-distributed noise is added to the true multipaths, where the standard deviations are set to $\sigma_{\text{AoA}} = \sigma_{\text{EoA}} = 0.001$ rad for the azimuth and elevation of arrival and $\sigma_{\text{RToA}} = 1.0/c$ s, where $c := 299792458$ m/s defines the speed of light. Furthermore, the detection is simulated by a binomial detection process with probability of detection of $p_D = 0.95$. No clutter is added to the measurements.

The generalized PHD-filter is implemented including the approximations proposed in (62) and (63), where $N_{\min} := 3$, $N_{\max} := 6$ and the threshold for significance of a partition $\tau := 1.0 \cdot 10^{10}$. The field of view (FOV) of the considered scenario is given by

$$\text{FOV} := [645259.0, 645999.0] \times [5495257.0, 5496747]. \tag{81}$$

Furthermore, the probability of detection is independent of the target's state space, that is $p(n,x) = p(n)$ for all $x \in X = \text{FOV} \times \mathbb{R}^2$, $n \in \{1,\ldots,6\}$. It is modeled by (71), where $q := p_D := 0.95$. Then, it is incorporated into the likelihood-function, which is defined for a hypothetical emitter position $\xi \in X$ and a set of multipath-measurements $\mathcal{Z}^K := \{z^k\}_{k=1}^K$, where $z^k \in Z$ (according to the ideas presented in [1]) by

$$p(\{z^k\}_{k=1}^K \mid \xi) := p(n) \cdot \lambda_\Phi^{K-n} \cdot \prod_{j\in I} \mathcal{N}(h_\xi^j; z^{i_j}, C^{i_j}), \tag{82}$$

where

$$h_\xi := \{h_\xi^k\}_{k=1}^M \tag{83}$$

denotes the set of predicted multipaths with respect to ξ and the fixed OS coming from the ray-tracer. The occurrence of clutter in a set of multipaths is modeled by

$$\lambda_\Phi := \frac{0.1}{\text{FOV}}, \tag{84}$$

which is equal to the clutter density of the generalized PHD-filter. In [chapter 4.4][1] and [2] the probabilistic likelihood-function is defined by the sum over all possible data interpretation, that is all possible associations between measured and predicted multipaths. Therefore, a possible data interpretation is denoted by $E^K_{i_1,\ldots,i_M}$, where

$$i_j := \begin{cases} 0, & \text{no association, measured multipath is not detected} \\ k \in \{1,\ldots,K\}, & j\text{th predicted multipath is} \cdot \text{associated with measured multipath } k \end{cases} \tag{85}$$

However, due to the computational effort, we only use the best data association, which is determined by applying the Munkres-algorithm [5] to the set of measured and predicted multipaths, using the Mahalanobis-distance with the covariance matrix

$$C^{i_j} = C = \text{diag}[\sigma^2_{\text{AoA}}\ \sigma^2_{\text{EoA}}\ \sigma^2_{\text{RToA}}] \tag{86}$$

for the construction of the cost matrix. Thus, the index i_j in (82) denotes the best (global) association for the specific predicted path. The generalized PHD-filter is realized by an SMC-implementation, since the likelihood-function can be computed only point-wisely. The maximal number of particles used by the generalized PHD-filter is given by 700 and particles are only drawn and predicted to grid points, where at least one multipath can be received due to the available database. In each iteration 200 newborn targets are uniformly drawn over the FOV. The single-object transition density is defined by the continuous white-noise acceleration model from [4], with $\tilde{q} = 1.5$ and the probability of survival $p_S = 1.0$. To extract the target states the weighted mean of all particles is computed.

To compare the result of the proposed generalized PHD-filter, the approach proposed in [9] is considered. There, standard PHD and intensity filters (iFilter) are applied to BML. Since the standard PHD and iFilter make the assumption that a target generates at most one measurement per sensor-scan, the integral of the intensity, that is the estimated number of targets, that are present in the FOV, is equal to the estimated number of measurements. Due to the fact, that one target in BML can emit several multipaths a post-processing is needed for the state-extraction. Thus, in [9] a generalization of the so-called particle grouping from [31] is presented. In the following the generalized mean computation from [9, Section 4.B] together with an SMC-implementation of the iFilter [33] is applied. The likelihood-function of a hypothetical emitter position $x_i \in X$ and one multipath $z^k \in \mathcal{Z}^K$, $k \in \{1,\dots,K\}$ is defined by

$$p(z^k \mid \xi) := \begin{cases} \mathcal{N}(h_\xi^j; z^{i_j}, C^{i_j}), & \text{if } \exists j \in \{1,\dots,M\} \\ & \text{such that } i_j = k \\ 0, & \text{otherwise} \end{cases} \tag{87}$$

The assignment is done via Munkres Algorithm between the set of measured multipaths Z and the set of predicted multipaths h_ξ of ξ. Therefore, the index i_j denotes the assigned measured multipath $\mathcal{Z}^K$ of the jth predicted multipath from h_ξ. The probability of detection is set to $p^D(x) = 0.9$ for all $x \in X$ and the detection probability in the space of hypothesis S_ϕ is defined by $p^D(\phi) = 0.4$. The transition probability from $\mathcal{S}_\phi$ to $\mathcal{S}$ is set to $\Psi(x \mid \phi) = 0.2$, the transition probability in $\mathcal{S}_\phi$ is defined as $\Psi(\phi \mid \phi) = 0.01$ and the transition probability from $\mathcal{S}$ to $\mathcal{S}_\phi$ is given by $\Psi(\phi \mid x) = 0.1$. The number of particles is restricted to 1500. The thresholds for target existence of the standard iFilter and the generalized PHD-filter are set to 0 to make the filter comparable in their RMSE-performance.

To assess both filters with respect to accuracy 100 Monte-Carlo runs of the presented scenario are performed. The results in terms of the RMSE is shown in Figure 12. It can be seen that both filters perform more or less equivalent after iteration 30 (the generalized PHD-filter is slightly better in terms of its RMSE-performance). However, it also can be seen that until iteration 20 the generalized PHD-filter performs worse than the standard iFilter. First, it can be seen that the initialization of the generalized PHD-filter is not as good as the initialization of the standard iFilter. This is essentially due to the fact that the likelihood-function of the generalized PHD-filter is much more restrictive than the likelihood-function of the standard iFilter. This is visualized in Figure 14 (a) and (c) which shows the sum of the likelihood-functions given in (82) and (87), that is

$$\sum_{\pi \in \Pi_{(1,3:6)}^{\mathcal{Z}^K}} p(\pi \mid \xi), \tag{88}$$

where $\Pi_{(1,3:6)}^{\mathcal{Z}^K}$ denotes the set of all partitions of $\mathcal{Z}^K$, where the subsets of one partition possess cardinality $c \in \{1,3,\dots,6\}$, that is

$$\Pi_{(1,3:6)}^{\mathcal{Z}^K} := \{\{\pi_{Z_1},\dots \pi_{z_m}\} : \pi_{z_i} \subseteq \mathcal{Z}^K, |\pi_{z_i}| \in \{1,3,\dots,6\}\} \tag{89}$$

$$\sum_{k=1}^{K} p(z^k \mid \xi) \tag{90}$$

for the standard iFilter and all $\xi \in X$ respectively. The number of investigated partitions is restricted due to the approximation condition (62), where $N_{\min} = 3$ and $N_{\max} = 6$. For better visualization only values of the likelihood-function, which are larger than $1 \cdot 10^{10}$ are plotted. It is obvious that the shape of (88) is sharper and therefore more restrictive than (90). This is due to the fact that the likelihood-function in (82) is given by a product of Gaussians. Therefore, partitions with at least one unlikely subset of multipaths (with respect to a hypothetical emitter position) possess a small likelihood-function value. This contrasts the likelihood-function of the standard iFilter, which only assesses single multipaths (see 87). Thus, the time of convergence (until iteration 10–11) of the generalized PHD-filter is longer than the time of convergence of the standard iFilter. Furthermore, it can be seen from Figure 12 that the localization of both filters around iteration 15 gets worse, while the generalized PHD-filter performs worse than the standard iFilter. This is due to the fact, that the likelihood-functions produce ambiguities in terms of the most likely hypothetical emitter position, which is shown in Figures 14 (b) and (d) for iteration 15. Since in some MC-runs a correct initialization of the generalized PHD-filter was not performed until the occurrence of these ambiguities, the generalized PHD-filter performs worse than the standard iFilter. However, the generalized PHD-filter performs better in iteration 20–25, since the restriction cancels out the ambiguities earlier than the likelihood-function of the standard iFilter.

The comparison of both filters in terms of estimated number of targets, that is the integral of the intensity function over FOV is presented in Figure 13. Due to the assumption that one target generates at most one measurement per iteration the standard iFilter estimates the number of multipaths which belong to a target. In contrast to this the generalized PHD-filter estimates after a few iterations the correct number of present targets.

In terms of time consumption the standard iFilter clearly outperforms the generalized PHD-filter: For one MC-run the standard iFilter needs 82614 ms, where the generalized PHD-filter takes 20250085 ms, which shows that it is of factor 245 slower than the standard iFilter.

VI. CONCLUSION AND FUTURE WORK

In this paper two different ways of approximation for the generalized PHD-filter update from [7] are proposed. In contrast to approximations for extended object and group tracking, the spatial relation of the measurements in the measurement space is not used. The approximations are based on incorporating the a pri-

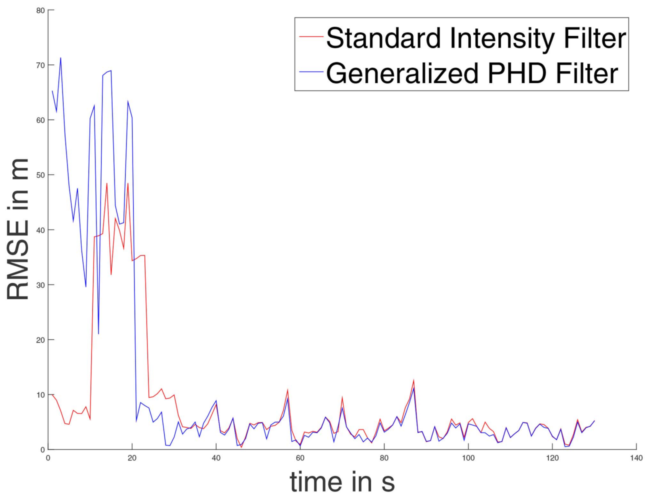


Fig. 12. RMSE of the standard iFilter, which uses an enhanced post-processing scheme for target state extraction and the generalized PHD-filter applying the proposed approximation conditions with $N_{\min} = 3$, $N_{\max} = 6$ and $\tau = 1.0 \cdot 10^{10}$.

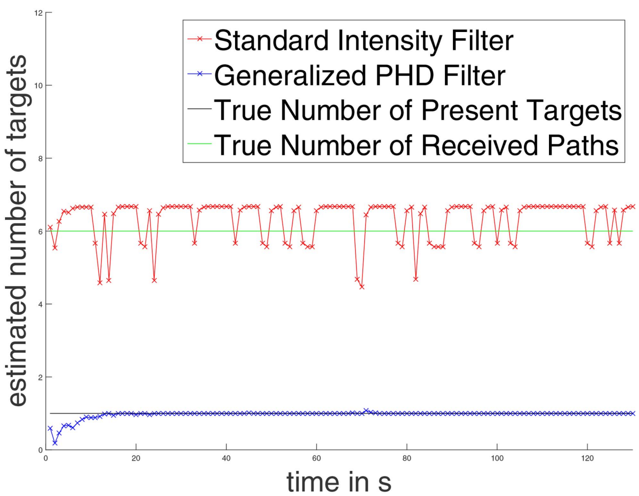


Fig. 13. Estimated number of targets, that is the sum of all particle weights before resampling of the standard iFilter and the generalized PHD-filter. Due to the assumption that one target generates at most one measurement per iteration, which is violated in the considered scenario, the standard iFilter estimates the number of multipaths, which belong to target. The generalized PHD-filter is able to estimate the correct number of present targets.

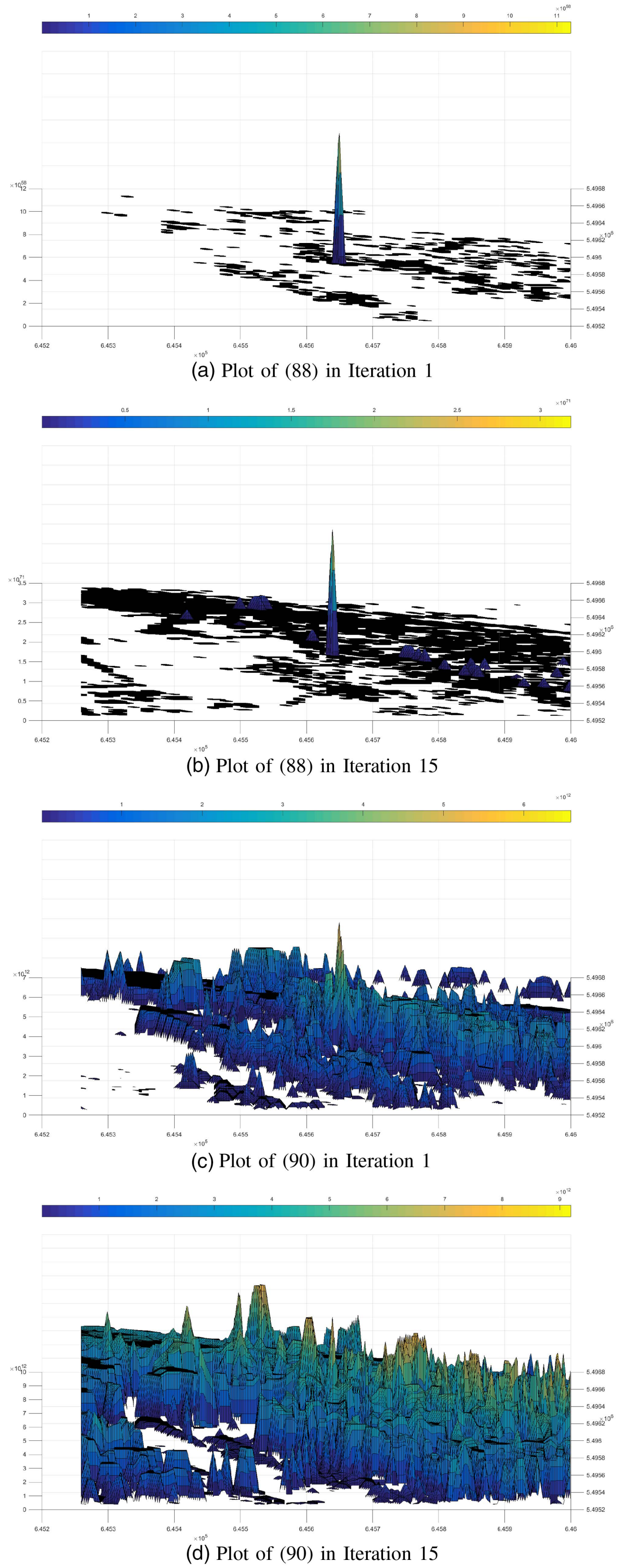


Fig. 14. Visualization of the likelihood-functions (88) and (90) at two instances of time.

ori knowledge on the number of measurements per target and the significance of a partition in terms of the likelihood-function. Therefore, the proposed approximations can be applied to scenarios, where a spatial distribution of the measurements is not available. Furthermore, the detection process is modeled as a function of target state and number of measurements and the usage of the Binomial- and Poisson-distribution for conditionally independent and identical distributed detection processes of the single measurements is motivated. An example for such a kind of scenario is BML where mobile terminals have to be tracked passively and non-cooperatively in an urban environment (see [9], [1], [2]). Two numerical examples for assessing the proposed methods are presented. First, a two-target scenario, where each target generates multiple correlated measurements is used to show the applicability of the proposed methods and to discuss the number of partitions that have been reduced. Several parameterizations are investigated and compared to each other. Second, a single-target BML-scenario is investigated and the generalized PHD-filter, using the proposed approximations and the generalization of the probability of detection is compared against an adaption of the standard intensity filter in terms of runtime, the estimated number of targets and the RMSE performance.

Future work will investigate improved schemes for state extraction and enhanced clutter modulation.

ACKNOWLEDGMENT

The authors would like to thank AWE Communications from Böblingen/Germany for providing a latest version of their ray-tracing simulation "WinProp" to evaluate the developed algorithms in Section V-B.

The authors would also like to thank Stephan Häfner, Martin Käske and Prof. Dr. Reiner Thomä from the Technical University of Ilmenau for providing the simulation of the estimation process used in the numerical evaluation of Section V-B.

This work was supported by the Federal Ministry of Education and Research of Germany (BMBF), within the Project EiLT: "Emitter Lokalisierung unter Mehrwegeausbreitungsbedingungen," http://eilt.medav.de/.

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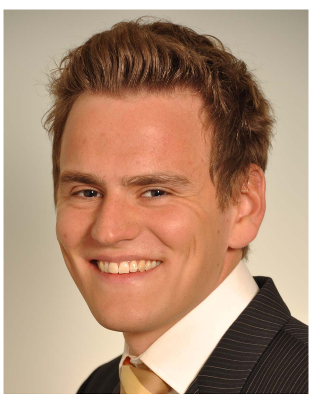

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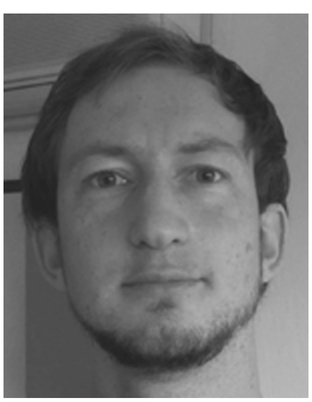

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