

Statistical Efficiency of Simultaneous Target State and Sensor Bias Estimation

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In this paper we provide a new methodology using an exoatmospheric target of opportunity seen in a satellites borne sensor's field of view to estimate the sensor's biases simultaneously with the state of the target. Each satellite is equipped with an Infra Red (IR) sensor that provides the Line Of Sight (LOS) measurements azimuth and elevation to the target. The measurements provided by these sensors are assumed to be noisy but perfectly associated, i.e., it is known perfectly that they belong to the same target. The evaluation of the Cramér-Rao Lower Bound (CRLB) on the covariance of the bias estimates, and the statistical tests on the results of simulations show that both the target trajectory and the biases are observable and this method is statistically efficient.

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I. INTRODUCTION

A space-based tracking system provides many advantages for missile defense as well as space situational awareness as a part of a system of systems that contribute to an overall picture. It can cover gaps in terrestrial radar coverage and expand the capabilities of a Ballistic Missile Defense System (BMDS), allow interceptors to engage enemy missiles earlier in their trajectories, discriminate between warheads and decoys, and provide warhead hit assessment. However, systemic errors in sensing systems hinder accurate threat identification and target state estimation, and, in this way, the space-based tracking systems present some unique challenges [7].

Multisensor systems use fusion of data from multiple sensors to form accurate estimates of a target track. To fuse multiple sensor data the individual sensor data must be expressed in a common reference frame. A problem encountered in multisensor systems is the presence of errors due to sensor bias. Bias error in a spacecraft and sensors can result from a number of different sources [8], including:

- Errors in spacecraft position (spacecraft navigation bias).
- Errors in spacecraft attitude (wheel assembly controller error, coordinate system translation round-off error).
- Errors in sensor calibration (residual pointing error, degradation of sensor alignment).
- Errors in timing caused by bias in the clocks of the sensors.

In [9] time varying bias estimation based on a non-linear least squares formulation and the singular value decomposition using truth data was presented. However, this work did not discuss the CRLB for bias estimation. An approach using maximum a posteriori (MAP) data association for concurrent bias estimation and data association based on sensor-level track state estimates was proposed in [10] and extended in [11].

For angle-only sensors, imperfect registration leads to LOS angle measurement biases in azimuth and elevation. If not corrected, the registration errors can seriously degrade the global surveillance system performance by increasing the tracking errors and even introducing ghost targets. In [6] the effect of sensor and timing bias error on the tracking quality of a space-based IR tracking system that utilizes a Linearized Kalman Filter (LKF) for the highly non-linear problem of tracking a ballistic missile was presented. This was extended in [7] by proposing a method of using stars observed in the sensor background to reduce the sensor bias error. In [4] simultaneous sensors bias and targets position estimation using fixed passive sensors was proposed. A solution to the related observability issues discussed in [4]

TABLE I

Symbols associated with coordinate systems and measurements:

Symbol	Definition
r	The range from the sensor to the target
α, ϵ	Azimuth and Elevation angles
\mathbf{b}	Bias vector
ϕ, ψ, ρ	Roll, pitch and yaw
ω	Orientation of a sensor
T	A rotation matrix
x, y, z	Target Positions in Cartesian coordinates
$\dot{x}, \dot{y}, \dot{z}$	Target velocities in Cartesian coordinates
θ	Parameters vector
\mathbf{z}	Measurements vector
ξ, η, ζ	Sensor Locations
R	Covariance matrix
H	Jacobian matrix
F	Transition matrix

is proposed in [5] using space based sensors. In [3] a simultaneous target state and passive sensors bias estimation was proposed. However, this work did not discuss the statistical efficiency of the estimates. The new bias estimation algorithm developed in this paper is validated using a hypothetical scenario created using System Tool Kit (STK) [1]. The tracking system consists of two optical sensors (space based) tracking a ballistic target. We assume the sensors are synchronized, their locations are known, and the data association is correct and we estimate their orientation biases (assumed constant during the entire tracking time) while simultaneously estimating the state of the target (position and velocity). We evaluate the Cramér-Rao lower bound (CRLB) on the covariance of the bias estimates, which is the quantification of the available information on the sensor biases, and show via statistical tests that the estimation is statistically efficient—it meets the CRLB.

Section II presents the problem formulation and solution in detail. Section III describes the simulations performed and gives the results. Finally, Section IV discusses the conclusions and future work.

II. PROBLEM FORMULATION AND ANNOTATIONS

A. List of Symbols and Acronyms

Table I is a list of the symbols used throughout the paper. In many sections, symbols are given additional subscripts or superscripts to make them more specific.

B. Problem Formulation

In order to fuse measurements from multiple sensors, all the sensors measurements must be expressed with respect to a common frame of reference. The fundamental frame of reference used in this paper is the Earth Centered Inertial (ECI) Coordinate System.

The sensor reference frame associated with sensor platform s (measurement frame of the sensor) is defined by the orthogonal set of unit vectors ($e_{\xi_s}, e_{\eta_s}, e_{\zeta_s}$). The origin of the measurement frame of the sensor is a

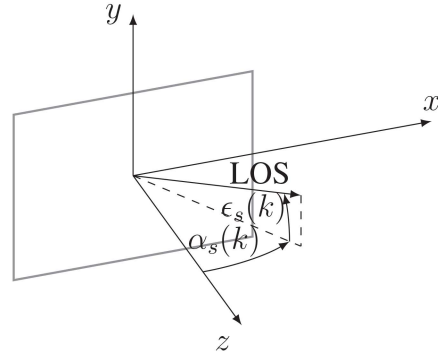


Fig. 1. Optical sensor coordinate system with the origin in the center of the focal plane.

translation of the ECI origin, and its axes are rotated with respect to the ECI axes. The rotation between these frames can be described by a set of Euler angles. We will refer to these angles $\phi_s + \phi_s^n$, $\rho_s + \rho_s^n$, $\psi_s + \psi_s^n$ of sensor s , as roll, pitch and yaw respectively, where ϕ_s^n is the nominal roll angle, ϕ_s is the roll bias, etc.

Each angle defines a rotation about a prescribed axis, in order to align the sensor frame axes with the ECI axes. The xyz rotation sequence is chosen, which is accomplished by first rotating about the x axis by ϕ_s^n , then rotating about the y axis by ρ_s^n , and finally rotating about the z axis by ψ_s^n . The rotations sequence can be expressed by the matrices

$$T_s(\psi_s^n, \rho_s^n, \phi_s^n) = T_z(\psi_s^n)T_y(\rho_s^n)T_x(\phi_s^n) \quad (1)$$

The explicit expressions of the elements of (1) can be found in [3]. Assume there are N_S synchronized passive sensors, with known positions in ECI coordinates,

$$\xi_s(k) = [\xi_s(k), \eta_s(k), \zeta_s(k)]', \quad s = 1, 2, \dots, N_S, \quad k = 0, 1, 2, \dots, K \quad (2)$$

where K is the final tracking time. The sensors get biased noisy measurements for tracking a single target at unknown positions

$$\mathbf{x}_p(k) = [x(k), y(k), z(k)]' \quad (3)$$

also in ECI coordinates. With the previous convention, the operations needed to transform the position of the target location expressed in ECI coordinates into the sensor s coordinate system (based on its nominal orientation) is

$$\mathbf{x}_s^n(k) = T(\omega_s(k))(\mathbf{x}_p(k) - \xi_s(k)) \quad s = 1, 2, \dots, N_S, \quad k = 0, 1, 2, \dots, K \quad (4)$$

where $\omega_s(k) = [\phi_s^n(k), \rho_s^n(k), \psi_s^n(k)]'$ is the nominal orientation of sensor s , $T(\omega_s(k))$ is the appropriate rotation matrix, and the translation ($\mathbf{x}_p(k) - \xi_s(k)$) is the difference between the vector position of the target and the vector position of the sensor s , both expressed in ECI coordinates. The superscript “n” in (8) indicates that the rotation matrix is based on the nominal sensor orientation.

Each passive sensor provides LOS measurements of the target position. As shown in Figure 1, the azimuth angle $\alpha_s(k)$ is the angle in the sensor xz plane between the sensor z axis and the line of sight to the target, while the elevation angle $\epsilon_s(k)$ is the angle between the line of sight to the target and its projection onto the xz plane, i.e.,

$$\begin{bmatrix} \alpha_s(k) \\ \epsilon_s(k) \end{bmatrix} = \begin{bmatrix} \tan^{-1} \left(\frac{x_s(k)}{z_s(k)} \right) \\ \tan^{-1} \left(\frac{y_s(k)}{\sqrt{x_s^2(k) + z_s^2(k)}} \right) \end{bmatrix} \quad (5)$$

The model for the biased noise-free LOS measurements is then

$$\begin{bmatrix} \alpha_s^b(k) \\ \epsilon_s^b(k) \end{bmatrix} = \begin{bmatrix} h_1(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \omega_s(k), \mathbf{b}_s) \\ h_2(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \omega_s(k), \mathbf{b}_s) \end{bmatrix} \triangleq \mathbf{h}(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \omega_s(k), \mathbf{b}_s) \quad (6)$$

where h_1 and h_2 denote the sensor Cartesian coordinates-to-azimuth/elevation angle mapping that can be found by inserting (8) and (5) into (6)

$$\begin{bmatrix} h_1(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \omega_s(k), \mathbf{b}_s) \\ h_2(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \omega_s(k), \mathbf{b}_s) \end{bmatrix} = \begin{bmatrix} \tan^{-1} \left(\frac{x_s^b(k)}{z_s^b(k)} \right) \\ \tan^{-1} \left(\frac{y_s^b(k)}{\sqrt{(x_s^b(k))^2 + (z_s^b(k))^2}} \right) \end{bmatrix} \quad (7)$$

and

$$\mathbf{x}_s^b(k) = T(\omega_s^b(k))(\mathbf{x}_p(k) - \boldsymbol{\xi}_s(k)) \quad s = 1, 2, \dots, N_s, \quad k = 0, 1, 2, \dots, K, \quad (8)$$

where

$$\omega_s^b(k) = [\phi_s^n(k) + \phi_s, \rho_s^n(k) + \rho_s, \psi_s^n(k) + \psi_s]' \quad (9)$$

is the biased orientation of sensor s , and the bias vector of sensor s is

$$\mathbf{b}_s = [\phi_s, \rho_s, \psi_s]' \quad (10)$$

At time k , each sensor provides the noisy LOS measurements

$$\mathbf{z}_s(k) = \mathbf{h}(\mathbf{x}_p(k), \boldsymbol{\xi}_s(k), \omega_s(k), \mathbf{b}_s) + \mathbf{w}_s(k) \quad (11)$$

Let \mathbf{z} be an augmented vector consisting of the batch stacked measurements from all the sensors up to time K

$$\mathbf{z} = [z_1(1), z_2(1), \dots, z_{N_s}(1), \dots, z_1(K), z_2(K), \dots, z_{N_s}(K)] \quad (12)$$

and

$$\mathbf{w}_s(k) = [w_s^\alpha(k), w_s^\epsilon(k)]' \quad (13)$$

The measurement noises $\mathbf{w}_s(k)$ are zero-mean, white Gaussian with

$$R_s = \begin{bmatrix} (\sigma_s^\alpha)^2 & 0 \\ 0 & (\sigma_s^\epsilon)^2 \end{bmatrix} \quad s = 1, 2, \dots, N_s \quad (14)$$

and are assumed mutually independent. The problem is to estimate the bias vectors for all sensors and the state vector (position and velocity) of the target of opportunity, i.e.,

$$\boldsymbol{\theta} = [x(K), y(K), z(K), \dot{x}(K), \dot{y}(K), \dot{z}(K), \mathbf{b}'_1, \dots, \mathbf{b}'_{N_s}]' \quad (15)$$

from

$$\mathbf{z} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{w} \quad (16)$$

where

$$\mathbf{h}(\boldsymbol{\theta}) = [h_{11}(\boldsymbol{\theta})', h_{21}(\boldsymbol{\theta})', \dots, h_{N_s 1}(\boldsymbol{\theta})', \dots, h_{1K}(\boldsymbol{\theta})', h_{2K}(\boldsymbol{\theta})', \dots, h_{N_s K}(\boldsymbol{\theta})']' \quad (17)$$

$$\mathbf{w} = [\mathbf{w}_1(1)', \mathbf{w}_2(1)', \dots, \mathbf{w}_{N_s}(1)', \dots, \mathbf{w}_1(K)', \mathbf{w}_2(K)', \dots, \mathbf{w}_{N_s}(K)]' \quad (18)$$

and the covariance of the stacked process noise (18) is the $(N_s K \times N_s K)$ block-diagonal matrix

$$R = \begin{bmatrix} R_1 & 0 & \dots & 0 \\ 0 & R_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & R_{N_s} \end{bmatrix} \quad (19)$$

C. Space target dynamics

The state space model for a noiseless discrete-time system¹ is of the general form

$$\mathbf{x}(k+1) = f[\mathbf{x}(k), \mathbf{u}(k)] \quad k = 0, 1, 2, \dots, K-1 \quad (20)$$

With small time steps ($\leq 10s$) we can approximate the motion model with the discrete-time dynamic equation

$$\mathbf{x}(k+1) = F\mathbf{x}(k) + G\mathbf{u}(k) \quad (21)$$

where

$$\mathbf{x}(k) = [x(k), y(k), z(k), \dot{x}(k), \dot{y}(k), \dot{z}(k)]', \quad k = 0, 1, 2, \dots, K \quad (22)$$

is the 6 dimensional state vector at time k , F is the state transition matrix, and \mathbf{u} is a known input representing the gravitational effects acting on the target (given in (25)). The state transition matrix for a target with acceleration due to gravity is

$$F = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (23)$$

¹Since we are dealing with exoatmospheric motion it is reasonable to assume that it is noiseless.

and the known input gain matrix (multiplying the appropriate components of the gravity vector) is

$$G = \begin{bmatrix} \Delta t^2/2 & 0 & 0 \\ 0 & \Delta t^2/2 & 0 \\ 0 & 0 & \Delta t^2/2 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix} \quad (24)$$

where Δt is the sampling interval. The gravity term is given by

$$\mathbf{u}(k) = g \frac{\mathbf{x}_p(k)}{a(\mathbf{x}_p(k))} \quad (25)$$

where \mathbf{x}_p is the position part of the state \mathbf{x} in (22), $g = 9.8 \text{ m/s}^2$, and

$$a = \sqrt{x(k)^2 + y(k)^2 + z(k)^2} \quad (26)$$

is the distance from the target to the origin of the coordinates system. For simplicity we assume g to be constant. The ratio \mathbf{x}_p/a yields the time-varying components of the gravity acting on the target and provides the scaling factor for the gravity term. Note that in view of (25), the state model (21) is not linear.

We shall obtain the maximum likelihood (ML) estimate of the augmented parameter vector (15) consisting of the (unknown) target position, velocity and sensor biases, by maximizing the likelihood function (LF) of $\boldsymbol{\theta}$ based on \mathbf{z}

$$\Lambda(\boldsymbol{\theta}; \mathbf{z}) = p(\mathbf{z} | \boldsymbol{\theta}) \quad (27)$$

where

$$p(\mathbf{z} | \boldsymbol{\theta}) = |2\pi R|^{-1/2} \exp\left(-\frac{1}{2}[\mathbf{z} - \mathbf{h}(\boldsymbol{\theta})]' R^{-1} [\mathbf{z} - \mathbf{h}(\boldsymbol{\theta})]\right) \quad (28)$$

and \mathbf{h} is defined in (17)

The ML estimate (MLE) is then

$$\hat{\boldsymbol{\theta}}(\mathbf{z})^{\text{ML}} = \arg \max_{\boldsymbol{\theta}} \Lambda(\boldsymbol{\theta}; \mathbf{z}) \quad (29)$$

In order to find the MLE, one has to solve a nonlinear least squares problem. This will be done using a numerical search via the Batch Iterated Least Squares (ILS) technique.

D. Bias Estimability

Intuitively, the observability of a system guarantees that the sensor measurements provide sufficient information for estimating the unknown parameters. As discussed in [3] the two requirements for bias estimability are:

First requirement for bias estimability. Each sensor provides a two-dimensional measurement (the two LOS angles to the target) at time K . We assume that each sensor sees the target at all the times $0, 1, 2, \dots, K$. Stacking together all the measurements results in an overall measurement vector of dimension $2KN_S$. Given that

the position, velocity of the target and bias vectors of each sensor are three-dimensional, and knowing that the number of equations (size of the stacked measurement vector) has to be at least equal to the number of parameters to be estimated (target state and biases), we must have

$$2KN_S \geq 3N_S + 6 \quad (30)$$

This is a necessary condition but not sufficient because (29) has to have a unique solution, i.e., the parameter vector has to be estimable. This is guaranteed by the second requirement.

Second requirement of bias estimability. This is the invertibility of the Fisher Information Matrix (FIM). In order to have parameter observability, the FIM must be invertible. If the FIM is not invertible (i.e., it is singular), then the CRLB (the inverse of the FIM) will not exist—the FIM will have one or more infinite eigenvalues, which means total uncertainty in a subspace of the parameter space, i.e., ambiguity [2].

For the example of bias estimability discussed in the sequel, to estimate the biases of 2 sensors (6 bias components) and 6 target components (3 position and 3 velocity components), i.e., the search is in an 12-dimensional space in order to meet the necessary requirement (30). As stated previously, the FIM must be invertible, so the rank of the FIM has to be equal to the number of parameters to be estimated ($6 + 6 = 12$, in the previous example). The full rank of the FIM is a necessary and sufficient condition for estimability. There exists, however, a subtle unobservability for this example that will necessitate the use of more measurements than the strict minimum number of measurements given by (30).

E. Iterated Least Squares for Maximization of the LF of $\boldsymbol{\theta}$

Given the estimate $\hat{\boldsymbol{\theta}}^j$ after j iterations, the batch ILS estimate after the $(j + 1)$ th iteration will be

$$\hat{\boldsymbol{\theta}}^{j+1} = \hat{\boldsymbol{\theta}}^j + [(H^j)' R^{-1} H^j]^{-1} (H^j)' R^{-1} [\mathbf{z} - \mathbf{h}(\hat{\boldsymbol{\theta}}^j)] \quad (31)$$

where

$$\mathbf{h}(\hat{\boldsymbol{\theta}}^j) = [h_{11}(\hat{\boldsymbol{\theta}}^j)', h_{21}(\hat{\boldsymbol{\theta}}^j)', \dots, h_{N_S1}(\hat{\boldsymbol{\theta}}^j)', \dots, h_{1K}(\hat{\boldsymbol{\theta}}^j)', h_{2K}(\hat{\boldsymbol{\theta}}^j)', \dots, h_{N_SK}(\hat{\boldsymbol{\theta}}^j)'] \quad (32)$$

where

$$H^j = \left. \frac{\partial \mathbf{h}(\boldsymbol{\theta}^j)}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}^j} \quad (33)$$

is the Jacobian matrix of the vector consisting of the stacked measurement functions (32) w.r.t. (15) evaluated at the ILS estimate from the previous iteration j . In this case, the Jacobian matrix is, with the iteration index omitted for conciseness,

$$H = [H_{11} \ H_{21} \ H_{N_S1} \ \dots \ H_{1K} \ H_{2K} \ H_{N_SK}]' \quad (34)$$

where

$$H_{sk} = \begin{bmatrix} \frac{h_{1_s}(k)}{\partial x(k)} & \frac{h_{1_s}(k)}{\partial y(k)} & \frac{h_{1_s}(k)}{\partial z(k)} & \frac{h_{1_s}(k)}{\partial \dot{x}(k)} & \frac{h_{1_s}(k)}{\partial \dot{y}(k)} & \frac{h_{1_s}(k)}{\partial \dot{z}(k)} & \frac{h_{1_s}(k)}{\partial b_{\alpha_1}} & \frac{h_{1_s}(k)}{\partial b_{\epsilon_1}} & \frac{h_{1_s}(k)}{\partial b_{\rho_1}} & \dots & \frac{h_{1_s}(k)}{\partial b_{\alpha_{N_S}}} & \frac{h_{1_s}(k)}{\partial b_{\epsilon_{N_S}}} & \frac{h_{1_s}(k)}{\partial b_{\rho_{N_S}}} \\ \frac{h_{2_s}(k)}{\partial x(k)} & \frac{h_{2_s}(k)}{\partial y(k)} & \frac{h_{2_s}(k)}{\partial z(k)} & \frac{h_{2_s}(k)}{\partial \dot{x}(k)} & \frac{h_{2_s}(k)}{\partial \dot{y}(k)} & \frac{h_{2_s}(k)}{\partial \dot{z}(k)} & \frac{h_{2_s}(k)}{\partial b_{\epsilon_1}} & \frac{h_{2_s}(k)}{\partial b_{\epsilon_1}} & \frac{h_{2_s}(k)}{\partial b_{\rho_1}} & \dots & \frac{h_{2_s}(k)}{\partial b_{\epsilon_{N_S}}} & \frac{h_{2_s}(k)}{\partial b_{\epsilon_{N_S}}} & \frac{h_{2_s}(k)}{\partial b_{\rho_{N_S}}} \end{bmatrix} \quad (35)$$

The appropriate partial derivatives with respect to the target positions and the bias terms can be found in [3], and the partial derivatives with respect to the target velocity components are:

$$\frac{\partial h_{1_s}(k)}{\partial \dot{x}_s(k)} = \Delta t \frac{\partial h_{1_s}(k)}{\partial x_s(k)} \quad (36)$$

$$\frac{\partial h_{1_s}(k)}{\partial \dot{y}_s(k)} = 0 \quad (37)$$

$$\frac{\partial h_{1_s}(k)}{\partial \dot{z}_s(k)} = \Delta t \frac{\partial h_{1_s}(k)}{\partial z_s(k)} \quad (38)$$

$$\frac{\partial h_{2_s}(k)}{\partial \dot{x}_s(k)} = \Delta t \frac{\partial h_{2_s}(k)}{\partial x_s(k)} \quad (39)$$

$$\frac{\partial h_{2_s}(k)}{\partial \dot{y}_s(k)} = \Delta t \frac{\partial h_{2_s}(k)}{\partial y_s(k)} \quad (40)$$

$$\frac{\partial h_{2_s}(k)}{\partial \dot{z}_s(k)} = \Delta t \frac{\partial h_{2_s}(k)}{\partial z_s(k)}. \quad (41)$$

F. Initialization

Assuming that the biases are null, the LOS measurements from the first and the second sensor $\alpha_1(k)$, $\alpha_2(k)$ and $\epsilon_1(k)$ can be used to solve for each initial Cartesian target position, in ECI coordinates, using (42)–(44). The two Cartesian positions formed from (42)–(44) can then be differenced to provide an approximate velocity. This procedure is analogous to two-point differencing [2] and will provide a full six-dimensional state to initialize the ILS algorithm.

$$x(k)^0 = \frac{\xi_2(k) - \xi_1(k) + \zeta_1(k) \tan \alpha_1(k) - \zeta_2(k) \tan \alpha_2(k)}{\tan \alpha_1(k) - \tan \alpha_2(k)} \quad (42)$$

$$y(k)^0 = \frac{\tan \alpha_1(k)(\xi_2(k) + \tan \alpha_2(k)(\zeta_1(k) - \zeta_2(k))) - \xi_1(k) \tan \alpha_2(k)}{\tan \alpha_1(k) - \tan \alpha_2(k)} \quad (43)$$

$$z(k)^0 = \eta_1(k) + \tan \epsilon_1(k) \left| \frac{(\xi_1(k) - \xi_2(k)) \cos \alpha_2(k) + (\zeta_2(k) - \zeta_1(k)) \sin \alpha_2(k)}{\sin(\alpha_1(k) - \alpha_2(k))} \right| \quad (44)$$

G. Cramér-Rao Lower Bound

In order to evaluate the efficiency of the estimator, the CRLB must be calculated. The CRLB provides a lower bound on the covariance matrix of an unbiased estimator as [2]

$$E\{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})'\} \geq J(\boldsymbol{\theta})^{-1} \quad (45)$$

where J is the FIM, $\boldsymbol{\theta}$ is the true parameter vector to be estimated, and $\hat{\boldsymbol{\theta}}$ is the estimate. The FIM is

$$J(\boldsymbol{\theta}) = E\{[\nabla_{\boldsymbol{\theta}} \ln \Lambda(\boldsymbol{\theta})][\nabla_{\boldsymbol{\theta}} \ln \Lambda(\boldsymbol{\theta})]'\} |_{\boldsymbol{\theta}=\boldsymbol{\theta}_{\text{true}}} \quad (46)$$

where the log-likelihood function is

$$\lambda(\boldsymbol{\theta}) \triangleq \ln \Lambda(\boldsymbol{\theta}) \quad (47)$$

$$J(\boldsymbol{\theta}) = H'R^{-1}H |_{\boldsymbol{\theta}=\boldsymbol{\theta}_{\text{true}}} \quad (48)$$

where H is the Jacobian matrix (34). Since $\boldsymbol{\theta}_{\text{true}}$ is not available in practice, J will be evaluated at the estimate, and, as it is shown later, the two results are practically the same.

H. Statistical Test for Efficiency with Monte Carlo Runs

Another measure of performance involves weighting the estimate error by the inverse of the covariance matrix P . The normalized estimation error squared (NEES) for the parameter $\boldsymbol{\theta}$ under the hypothesis of efficiency, i.e.,

$$P = J^{-1} \quad (49)$$

is defined as

$$\epsilon_{\theta} = (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})' P^{-1} (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) = (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})' J(\boldsymbol{\theta}) (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \quad (50)$$

and is chi-square distributed with n_{θ} (the dimension of $\boldsymbol{\theta}$) degrees of freedom, that is,

$$\epsilon_{\theta} \sim \chi_{n_{\theta}}^2 \quad (51)$$

The hypothesis test for efficiency whether (51) can be accepted, as discussed in [2] and outlined next. The NEES is used in simulations to check whether the estimator is efficient, that is, the errors are statistically consistent with the covariance given by the CRLB—this is the efficiency check. Thus the efficiency check of the estimator (in simulation—because this is the only

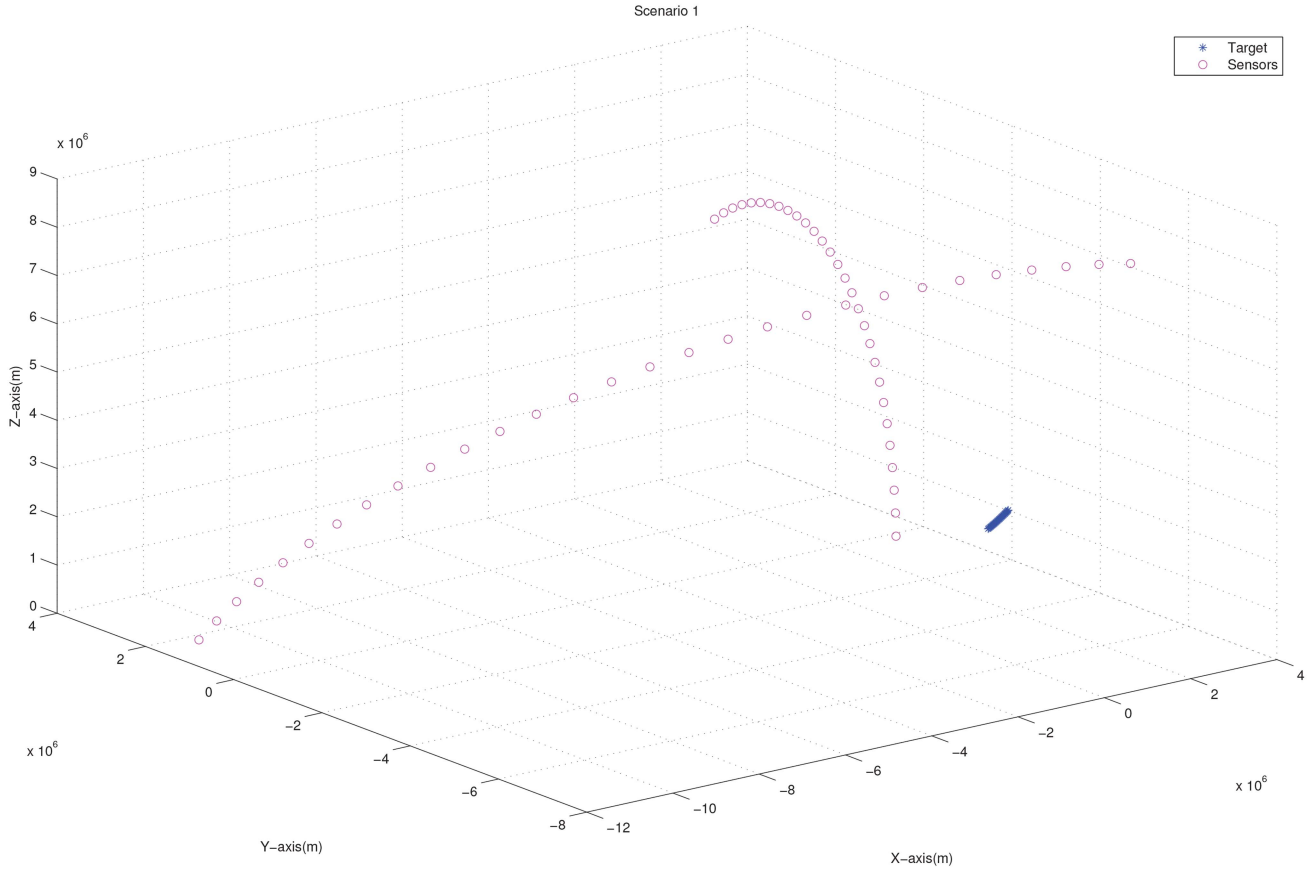


Fig. 2. Target and satellite trajectories for the two-sensor case

situation where θ is available) consists of verifying whether (51) holds. The practical procedure to check the estimator efficiency is using the sample average NEES from N independent Monte Carlo runs defined as

$$\bar{\epsilon}_\theta = \frac{1}{N} \sum_{i=1}^N \epsilon_\theta^i \quad (52)$$

The quantity $N\bar{\epsilon}_\theta$ is chi-square distributed with Nn_θ degrees of freedom.

Let Q be the type I error probability of the test. The $1 - Q$ two-sided probability region for $N\bar{\epsilon}_\theta$ is the interval $[\epsilon'_1, \epsilon'_2]$.

$$\epsilon'_1 = \chi_{Nn_\theta}^2 \left(\frac{Q}{2} \right) \quad (53)$$

$$\epsilon'_2 = \chi_{Nn_\theta}^2 \left(1 - \frac{Q}{2} \right) \quad (54)$$

where in view of the division by N in (52), one has

$$\epsilon_i = \frac{\epsilon'_i}{N} \quad (55)$$

Thus, if the estimator is efficient, one has to have

$$P\{\bar{\epsilon}_\theta \in [\epsilon'_1, \epsilon'_2]\} = 1 - Q \quad (56)$$

III. SIMULATIONS

In this paper we used a hypothetical scenario to test our new methodology. The missile and satellite trajec-

tories are generated using System Tool Kit (STK). The sensor satellites are in a circular orbits of 600 km and 700 km altitude with 0° , 60° degrees inclination, respectively. The target modeled represents a long range ballistic missile with a flight time of about 20 minutes. STK provides the target and sensor positions in three dimensional Cartesian coordinates at 1 s intervals. The measurement noise standard deviation σ_s (identical across sensors for both azimuth and elevation measurements, $\sigma_s^\alpha = \sigma_s^\epsilon = \sigma_s$) was assumed to be $30 \mu\text{rad}$. The target launch time was chosen so that the satellite sensors were able to follow the missile trajectory throughout its flight path. As shown in Figure 3, these satellite orbits enabled maximum visibility of the missile trajectory from multiple angles. The missile and satellite trajectories displayed in Figure 3 represent 5 minutes of flight time (exoatmospheric). In order to establish a baseline for evaluating the performance of our method, we also ran the simulations without biases and with biases, but without bias estimation. As discussed in the previous section, the three sensor biases were roll, pitch and yaw angle offsets. Table II summarizes the bias values (in mrad).

In order to test for the statistical efficiency of the estimate (of the 12 dimensional vector (15)), the NEES is used, with the CRLB as the covariance matrix. The sample average NEES over 100 Monte Carlo runs calculated using the FIM evaluated at the true bias values, target

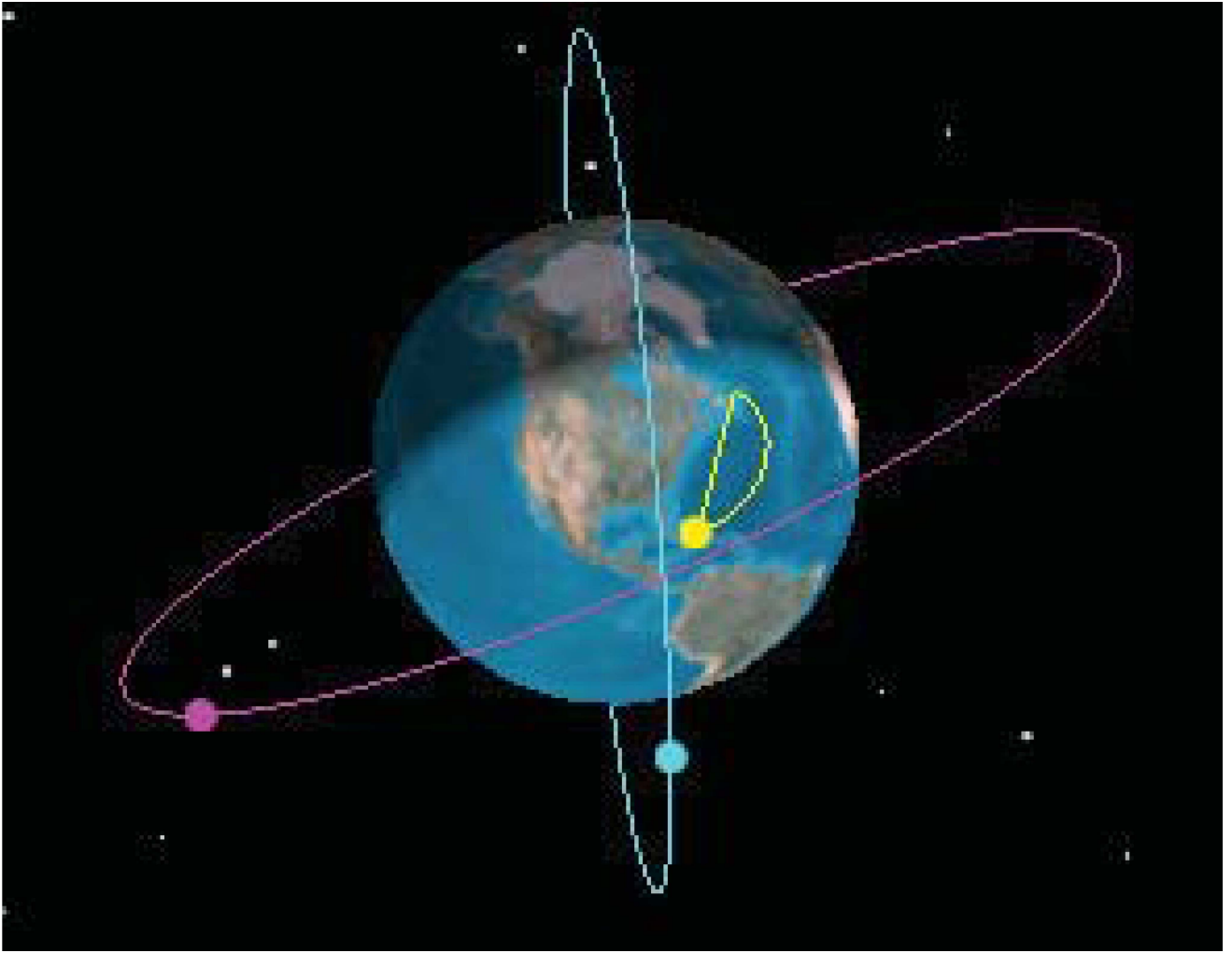


Fig. 3. Target and satellite trajectories for the two-sensor case

TABLE II
Sensor Biases (mrad).

	ψ	ρ	ϕ
Sensor 1	5.7596	4.3633	-3.8397
Sensor 2	4.8869	5.4105	-5.0615

TABLE III
Sample average bias NEES (CRLB evaluated at the estimate), for each of the 6 biases, over 100 Monte Carlo runs.

Biases	ψ_1	ρ_1	ϕ_1	ϕ_2	ψ_2	ρ_2
NEES	1.0326	0.9723	1.0239	1.0248	1.2009	0.8922

position, and velocity is approximately 11.52, and the sample average NEES calculated using the FIM evaluated at the estimated biases, target position and velocity is approximately 11.63 and both fall in the interval given below. According to the CRLB, the FIM has to be evaluated at the true parameter. Since this is not available in practice, however, it is useful to evaluate the FIM also at the estimated parameter, the only one available in real world implementations [12]. The results are practically identical regardless of which values are chosen for evaluation of the FIM. The 95% probability region for the 100 sample average NEES of the 12 dimensional parameter vector is [11.20, 12.81]. This NEES is found to be within this interval and the MLE is therefore statistically efficient. Table III shows the individual bias component NEES. The 95% probability region for the 100 sample

average single component NEES is [0.74, 1.29]. These NEES are found to be within this interval.

The RMS errors for the target position and velocity are summarized in Table IV. In this table, the first estimation scheme was established as a baseline using bias-free LOS measurements to estimate the target position and velocity. For the second scheme, we used biased LOS measurements but we only estimated target position and velocity. In the last scheme, we used biased LOS measurements and we simultaneously estimated the target position, velocity, and sensor biases. Once again, bias estimation yields significantly improved target RMS position and velocity errors in the presence of biases.

Each component of θ should also be individually consistent with its corresponding σ_{CRLB} (the square root

TABLE IV

Sample average RMSE for the target position (m) and velocity (ms^{-1}), over 100 Monte Carlo runs, for the 3 estimation schemes.

Scheme	Position RMSE	Velocity RMSE
1	107.44	5.16
2	47,161.10	25,149.32
3	494.49	19.55

TABLE V

Sample average bias (μrad) RMSE over 100 Monte Carlo runs and the corresponding bias standard deviation from the CRLB.

	RMSE	σ_{CRLB}
ψ_1	0.0326	0.0334
ρ_1	0.0239	0.0211
ϕ_1	0.0239	0.0261
ψ_2	0.0248	0.0252
ρ_2	0.0099	0.0096
ϕ_2	0.0122	0.0122

of the corresponding diagonal element of the inverse of FIM). In this case, the sample average bias RMSE over 100 Monte Carlo runs should be within 15% of its corresponding bias standard deviation from the CRLB (σ_{CRLB}) with 95% probability. The utmost limit (“existing information”) for the scenario considered is around 10–33 μrad standard deviation for the bias errors, i.e., of the order of σ_s . Table V demonstrates the efficiency of the individual bias estimates.

IV. CONCLUSIONS AND FUTURE WORK

In this paper we presented a new algorithm that uses a target of opportunity for estimation of measurement biases together with target state. The first step was formulating a general bias model for synchronized space-based optical sensors at known locations. The association of measurements is assumed to be perfect. Based on this, we used an ML approach that led to a batch nonlinear least-squares estimation problem for simultaneous estimation of the 3D Cartesian position and velocity components of the target of opportunity and the angle measurement biases of the sensors. The bias estimates, obtained via ILS, were shown to be unbiased and statistically efficient. For future work we plan to relax the no process noise assumption, reformulate the problem and again evaluate the statistical efficiency of the algorithm.

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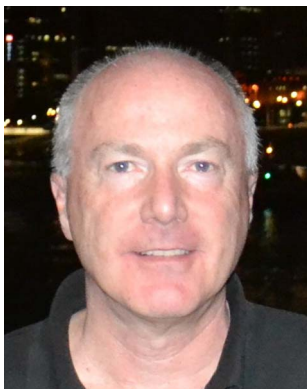
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