June 2013 Volume 8 Number 1 ISSN 1557-6418

JAIF 13

Journal of Advances in Information Fusion

A semi-annual archival publication of the International Society of Information Fusion

Regular Papers Page

From the Editor-In-Chief

Content Indexing Comes to JAIF

ISIF

www.isif.org

INTERNATIONAL SOCIETY OF INFORMATION FUSION

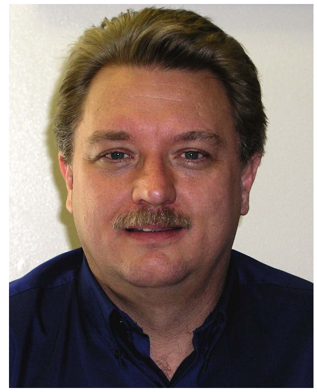
The International Society of Information Fusion (ISIF) is the premier professional society and global information resource for multidisciplinary approaches for theoretical and applied INFORMATION FUSION technologies. Technical areas of interest include target tracking, detection theory, applications for information fusion methods, image fusion, fusion systems architectures and management issues, classification, learning, data mining, Bayesian and reasoning methods.

Manuscripts are submitted at http://jaif.msubmit.net. If in doubt about the proper editorial area of a contribution, submit it under the unknown area.

Journal of Advances in Information Fusion (ISSN 1557-6418) is published semi-annually by the International Society of Information Fusion. The responsibility for the contents rests upon the authors and not upon ISIF, the Society, or its members. ISIF is a California Nonprofit Public Benefit Corporation at P.O. Box 4631, Mountain View, California 94040.

From the Editor-in-Chief:

June 2013



Content Indexing Comes to JAIF

The original vision for JAIF was to become the flagship peer-reviewed journal of the International Society for Information Fusion (ISIF). A key goal along the road to that vision involved content indexing. Content indexing of JAIF is important to our authors because other researchers can find their papers and reference them in their own publications. These references to one's publications are used to measure the impact of one's research, and the corresponding "impact factor" is often a key component of the nomination for IEEE Fellow and other research honors and evaluations for tenured faculty positions and other similar positions. Thus, content indexing of JAIF has been a focus of the editorial board of JAIF since its inception. As Editor-In-Chief (EIC) for JAIF, I am pleased to announce that Scopus accepted JAIF for content indexing as of July 29, 2013. Scopus statements accompanying their decision to accept JAIF for content indexing were a "convincing journal policy" and "fulfillment of the criteria for inclusion in Scopus." **Scopus** is a bibliographic database containing abstracts and citations for academic journal articles. It covers over 50 million entries for nearly 21,000 titles from over 5,000 publishers, of which 20,000 are peer-reviewed journals in the scientific, technical, medical, and social sciences (including arts and humanities). Therefore, authors of articles in JAIF are now assured that their research will receive broad exposure and citations as appropriate.

In preparing JAIF for content indexing, a strong editorial board of researchers from both industry and academia was organized and an operation plan that addressed the need for a rigorous and meaningful peer review process was put into place. Another important factor that contributes to the decision for content indexing is the regular and timely publication of the journal. This was a challenge in the first few issues of JAIF. As time progressed, the editorial board charged Robert "Bob" Lynch to address this weakness in the journal. Bob has led that charge for the past few years and accomplished this goal. JAIF is now a peer-reviewed journal with indexed content. ISIF certainly owes Bob Lynch a great thank you for all of this hard and successful work. *Thank you Bob.*

Well, what is the next step toward the ISIF vision for JAIF? That step involves a large increase in the number of submissions to JAIF. Without more submissions, JAIF cannot have the impact desired by ISIF and its Vice President (VP) for Publications, Yaakov Bar-Shalom. Toward that goal, the editorial board has identified three actions. First, the news of content indexing for JAIF will be broadcast broadly to the research community that includes ISIF members and others who have attended the International Conference on Information Fusion. Second, the VP for Publications, Yaakov Bar-Shalom, will contact academic departments associated with information fusion and inform them of the acceptance of JAIF into Scopus. Third, the editorial board for JAIF will take extra measures to reduce the time required for the rigorous peer review of JAIF to three months. Through these three actions, JAIF should experience an increase in the number of submissions as researchers need to publish articles in a timely manner and achieve a higher impact factor for promotions, applications for new positions, and nominations for honors such as IEEE Fellow.

William Dale Blair
Editor in Chief

Bearings-Only Localization with NLOS Reflected AoAs

XIUFENG SONG
PETER WILLETT
SHENGLI ZHOU

Bearings-only localization with light-of-sight (LOS) propagation is well understood. This paper concentrates on bearings-only localization with non-line-of-sight (NLOS) measurements, where target images arrive at a network of sensors each after a single specular reflection. The reflecting surface can be 1) flat or 2) circular (inner side of a circle), and is assumed known. In this paper, we derive the least squares (LS), Stansfield, and maximum likelihood (ML) estimators for both cases. As to the former, their estimation performances are similar to their counterparts in LOS localization: Stansfield is very close to ML, and both are usually significantly better than LS. As regards the second, since the target-sensor geometry has multiple possibilities, the ML solution is extremely intricate. However, if a concentric opaque circle (such as the earth) lies within the reflecting one, e.g. the earth within the ionospheric layer, the propagation path becomes unique; a grid search based ML is available for such a circumstance. ML is computationally intensive for a circularly reflecting surface; two suboptimal algorithms, LS and Stansfield, are developed based on small angle approximation. These algorithms perform differently from those for the flat case: ML significantly outperforms LS and Stansfield, especially for a large observation error; however, Stansfield is not necessarily better than LS.

Manuscript received November 7, 2011; revised July 19, 2012; released for publication December 12, 2012.

Refereeing of this contribution was handled by Huimin Chen.

This work was supported by the U.S. Office of Naval Research under Grants N00014-07-10429 and N00014-09-10613.

Author's addresses: X. Song, LSI Corporation, 1320 Ridder Park Drive, San Jose, CA 95131, E-mail: (xiufeng.song@gmail.com); P. Willett and S. Zhou, Department of Electrical and Computer Engineering, University of Connecticut, 371 Fairfield Way U-2157, Storrs, CT 06269, E-mail: (willett@engr.uconn.edu, and shengli@engr.uconn.edu).

1557-6418/13/$17.00

1. INTRODUCTION

1.1. Distributed Localization: from LOS to NLOS

Sensor networks enable the localization of a target (or source) of interest with spatially complementary observations. Current available physical measurements include received signal strength [1], time of arrival [11], angle of arrival (AoA) [6, 7, 10, 13, 19], and so forth. They can be individually or cooperatively utilized in target information extraction. In this paper, we concentrate on localizing a single target with distributed AoAs, specifically bearings-only localization.

Bearings-only localization infers the position of a target with multiple AoA lines, which share a unique intersection—the target location—in the absence of noise. If observation uncertainty is included, a global intersection may not exist and advanced estimators are required. Least squares (LS) is a straightforward choice if the noise distribution is unknown [6]. If noise statistics are known, maximum likelihood (ML) is an option and is popular [10, 13]. Its good performance is guaranteed at the cost of computational load. The Stansfield estimator is a kind of weighted LS for independent Gaussian noise [19]; it is a compromise between estimation performance and computation. Reference [10] shows that the root mean square errors (RMSEs) of a Stansfield estimator are not necessarily larger than those of ML in bearings-only localization. Other approaches include total least squares [7], and so forth.

The aforementioned works focus on line-of-sight (LOS) propagation, where a direct path exists between the target and sensors; nevertheless, practical problems may not necessarily have a LOS. When the wavefront (acoustic, light, or electromagnetic) of target radiation meets an interface between two media, reflection will happen [14]. The reflection is helpful for the extraction of target information in some circumstances, especially where LOS propagation is unavailable. An interesting application is over-the-horizon radar (OTHR) [9, 17] (see Fig. 1). If multiple geometrically complementary radar sensors are available, a fusion center can infer the position of the target with proper data association.[1] Instead of LOS AoAs, this paper studies the localization with non-line-of-sight (NLOS) reflection measurements, where the radiation from a target reaches a sensor after a single specular reflection.

1.2. Localization with Reflected Measurements

This NLOS based localization problem is motivated by OTHR, which utilizes ionospheric reflection to capture a target beyond the horizon [9, 14, 17]. Such a reflection based technique has been used in long-range missile and aircraft detection, and it is considered an

[1] OHTRs are not the only system utilizing electromagnetic reflection phenomenon. Some other applications such as array aperture synthesis and low elevation target extraction can be respectively found in [4] and [2].

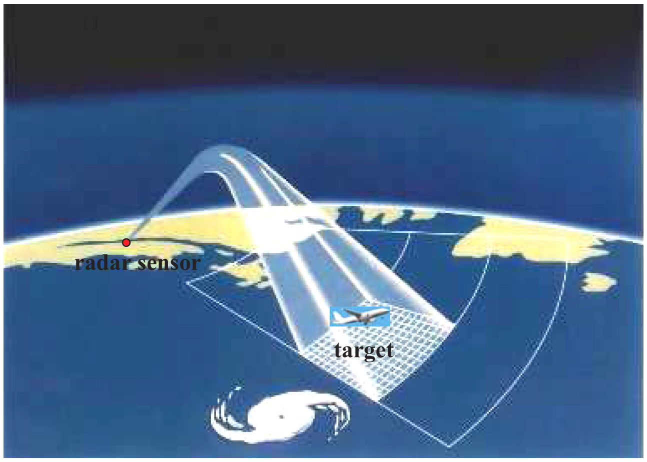


Fig. 1. An illustration of reflection based target perception with an OHTR [17].[2] The LOS path is unavailable due to the earth curvature, and the radiation of the target reaches a radar sensor after a single reflection on ionosphere. If multiple distributed radar sensors are available, one could properly associate their data to infer the location of the target.

effective means of wide-area surveillance [5]. The ionosphere has two typical reflection layers: E and F, whose characteristics are frequency dependent and resolvable [14]. In this paper, sensors are assumed to be either frequency selective or capable of resolving echo frequencies and labeling a AoA to its corresponding layer. We thus could focus on a single layer localization with multiple sensors, and the extension to double layers is straightforward. In addition, a real OTHR system works in three dimensions. This paper initiates an investigation in two dimensions—same as do [9] and [17]—where the reflection layer is the *inner* side of a circle—a slice of the ionosphere (see Fig. 3 for an immediate perception). As is the case for [9] and [17] we work in two dimensions and do not offer any discussion of incorporating the third.

As opposed to the usual LOS based localization [6, 10, 13, 19], a sensor here in the NLOS case observes the *virtual* image of a target. To understand the ramifications thoroughly, we investigate in two steps: *flat* and *circular* reflecting surfaces, where the reflection is assumed to be specular [9]. The ML, Stansfield, and LS algorithms are derived for both cases. Even though the former does not have a clear physical application, it facilitates the importation, from conventional LOS based localization [6, 10, 13, 19] ideas, to the latter situation of NLOS based localization with circular reflected AoAs.

As for a flat surface, the target-sensor reflection geometry is unique, and we show that:

- the algorithm comparison for the LOS case in [6], [10], [13], [19] still holds here: LS has the worst performance, while the RMSE of Stansfield estimator is not necessarily larger than that of ML.

[2]Reference [17] acknowledges that the picture is in turn derived from an image provided by the US National Oceanic and Atmospheric Administration (NOAA).

TABLE I
Coordinates Notions

Coordinate	Objective
(x_t, y_t)	target
(x_i^s, y_i^s)	the ith sensor
$(\bar{x}_i^t, \bar{y}_i^t)$	image of (x_t, y_t) corresponding to the ith mirror
$(\bar{x}_i^s, \bar{y}_i^s)$	image of (x_i^s, y_i^s) corresponding to the ith mirror
$(\bar{x}_i^n, \bar{y}_i^n)$	image of (x_i^s, y_i^s) corresponding to the normal line
(x_i^c, y_i^c)	intersection between circle and the ith observation line
(x_b, y_b)	boundary of blocking circle division area

The circular NLOS case has two unique properties: 1) the reflection with a circular surface is nonlinear, and the image of a single point with respect to it is not unique; 2) the spatial uncertainty of the target does not coincide with that of the AoA, since the circular reflection nonlinearly changes the noise spatial distribution around the target (a focusing effect, see Fig. 7). That is, circular/NLOS is considerably more complicated than flat/LOS, and this paper investigates it from the following perspectives:

- We give the reflection model and reveal that multiple reflection paths exist between a sensor and the target. A ML solution turns out to be quite intricate.
- If an opaque blocking circle (no propagation can pass through it) is concentric to the reflecting one, and all the sensors are deployed on the blocking circle, then the unblocked target-sensor reflection path is unique and can be found numerically. Therefore, a grid search based ML can be performed.
- The grid search is laborious; suboptimal algorithms such as LS and Stansfield are given based on a small arc approximation. Their performance are compared.

Note that the suboptimal algorithms for the circular reflection case utilize the corresponding results of the flat case.

This paper includes some material from [18], however, with significant expansions including a thorough investigation of flat reflecting surface and explicit target-sensor geometric analysis for circular reflecting surface. The rest of this paper is as follows. Section 2 studies the bearings-only localization with flat reflecting surfaces; Section 3 analyzes the reflection geometry for a circular surface, and gives the ML localization algorithm; suboptimal localization approaches for circular reflection are given in Section 4; numerical results are in Section 5, while conclusions are drawn after that.

Notation: Boldface uppercase and lowercase letters denote matrices and column vectors respectively. $\|\cdot\|$ stands for the Frobenius norm, while diag($\mathbf{a}$) denotes the diagonal matrix formed by vector $\mathbf{a}$. $(\cdot)^T$ and $(\cdot)^{-1}$, respectively, represent matrix transpose and inverse. $\overline{AB}$ and $\widehat{AB}$ respectively denote the line and arc through points A and B, while $\angle\widehat{AB}$ measures the angle of $\widehat{AB}$. $\mathcal{N}(0, \varepsilon^2)$ is a zero-mean Gaussian distribution with variance ε^2. Finally, the coordinates of different locations are collected in Table I for clarity.

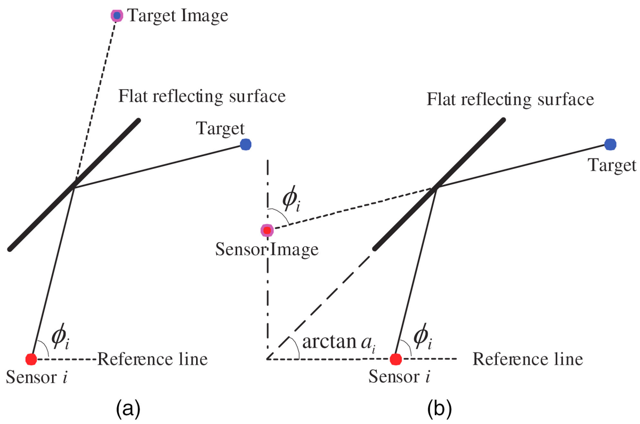


Fig. 2. Bearings-only measurement with flat reflecting surface: (a) target image based modeling, and (b) sensor image based modeling.

2. BEARINGS-ONLY LOCALIZATION WITH FLAT REFLECTING SURFACES

This section investigates NLOS localization with known *flat* reflection surfaces, where a LOS propagation path is assumed unavailable. The radiations of a target arrive at a network of passive sensors after a single specular reflection, and the AoA is recorded by an individual sensor. Suppose that proper synchronization and communication links are extant, and hence a processing center can collect the reflected AoAs to infer the location of target.

2.1. Maximum Likelihood Estimation

ML with flat reflecting surfaces has two modeling approaches: target and sensor images based (see Fig. 2). In the following, we will show their equivalence.

2.1.1. Target Image Based Modeling

If a reflecting surface is flat, the image of a target is unique. A sensor actually observes the target image instead of the target itself. Therefore, the image based modeling as shown in Fig. 2(a) is obvious.

Let the coordinates of the target be (x_t, y_t), and then the location of its image with respect to a reflecting surface $y = a_i x + b_i$ is expressed as $(\bar{x}_i^t, \bar{y}_i^t)$, where

$$\begin{aligned} \bar{x}_i^t &= \frac{1-a_i^2}{1+a_i^2}x_t + \frac{2a_i}{1+a_i^2}y_t - \frac{2a_i b_i}{1+a_i^2} \\ \bar{y}_i^t &= \frac{2a_i}{1+a_i^2}x_t + \frac{a_i^2-1}{1+a_i^2}y_t + \frac{2b_i}{1+a_i^2} \end{aligned} \tag{1}$$

based on Lemma 4 in Appendix A. As a result, the measured AoA for the ith sensor is

$$\varphi_i = \underbrace{\arctan\left(\frac{\bar{y}_i^t - y_i^s}{\bar{x}_i^t - x_i^s}\right)}_{\triangleq \phi_i'(x_t, y_t)} + n_i' \tag{2}$$

where (x_i^s, y_i^s) stands for the (known) coordinates of sensor i, and n_i' denotes its measurement noise.

2.1.2. Sensor Image Based Modeling

The measured AoA can be transformed as a function of sensor image as shown in Fig. 2(b). Let the reflecting surface be $y = a_i x + b_i$, and then the slope of the image of horizontal reference line is $\tan(2a_i)$. Therefore, the observed AoA for sensor i is written as

$$\varphi_i = \underbrace{2\arctan a_i - \arctan\left(\frac{y_t - \bar{y}_i^s}{x_t - \bar{x}_i^s}\right)}_{\triangleq \phi_i(x_t, y_t)} + n_i \tag{3}$$

where $(\bar{x}_i^s, \bar{y}_i^s)$ denotes the coordinates of the image of sensor i corresponding to $y = a_i x + b_i$,

$$\begin{aligned} \bar{x}_i^s &= \frac{1-a_i^2}{1+a_i^2}x_i^s + \frac{2a_i}{1+a_i^2}y_i^s - \frac{2a_i b_i}{1+a_i^2} \\ \bar{y}_i^s &= \frac{2a_i}{1+a_i^2}x_i^s + \frac{a_i^2-1}{1+a_i^2}y_i^s + \frac{2b_i}{1+a_i^2} \end{aligned} \tag{4}$$

and n_i represents the noise for sensor image based modeling. Note that the $(\bar{x}_i^s, \bar{y}_i^s)$s can be precalculated.

The measurement uncertainty of the second approach n_i is an imaging transformation of that for the first one n_i'. If the reflecting surface is flat, the transformation will not change the distribution. Based on Appendix B, we obtain $\phi_i(x_t, y_t) = \phi_i'(x_t, y_t)$, so the two modeling approaches are equivalent. As for the first one, the unknown parameters, x_t and y_t, are included in both the numerator and denominator of $\tan\phi_i'(x_t, y_t)$; direct optimization with it will have a nontrivial computational load. The following estimation algorithms adopt the second modeling approach.

Suppose that the measurement uncertainty of sensor i subjects to zero-mean Gaussian distribution with variance σ_i^2. Collecting the unknown parameters as $\boldsymbol{\theta} = [x_t, y_t]^T$, the conditional probability density function (PDF) of the observed AoA for the ith sensor is

$$f(\varphi_i \mid \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{|\varphi_i - \phi_i|^2}{2\sigma_i^2}\right) \tag{5}$$

where $\phi_i \triangleq \phi_i(x_t, y_t)$ for notational simplicity. Let each sensor send its bearing measurement to a central processing unit, and then the central processor utilizes all of them to estimate target location. Suppose that $f(\varphi_i \mid \boldsymbol{\theta})$s are mutually independent; the centralized ML estimator is given by

$$\begin{aligned} \hat{\boldsymbol{\theta}}_{\mathrm{ML}} &= \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{N} f(\varphi_i \mid \boldsymbol{\theta}) \\ &= \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \frac{1}{\sigma_i^2} |\varphi_i - \phi_i|^2 \end{aligned} \tag{6}$$

where N counts the number of sensors.

2.2. Stansfield Estimation

Stansfield estimator approximates ML under small observation errors [19]. It relies on *the law of small-*

angle approximation: the bearing noise n_i is small enough that

$$n_i \approx \sin n_i. \tag{7}$$

Substituting (3) into (7), we have

$$\begin{aligned}\varphi_i - \phi_i &\approx \sin(\varphi_i - \phi_i) &(8)\\ &= \sin\left(\underbrace{\varphi_i - 2\arctan a_i}_{\triangleq \gamma_i} + \arctan\left(\frac{y_t - \bar{y}_i^s}{x_t - \bar{x}_i^s}\right)\right)\\ &= \frac{(x_t - \bar{x}_i^s)\sin\gamma_i + (y_t - \bar{y}_i^s)\cos\gamma_i}{d_i(x_t, y_t)}. &(9)\end{aligned}$$

Substituting (9) into (6), the optimization is recast as

$$\hat{\boldsymbol{\theta}}_{\text{SE}} = \arg\min_{\boldsymbol{\theta}} (\mathbf{U}\boldsymbol{\theta} - \mathbf{v})^T \boldsymbol{\Lambda}^{-1}\mathbf{D}^{-1}(\mathbf{U}\boldsymbol{\theta} - \mathbf{v}) \tag{10}$$

where $\boldsymbol{\Lambda} = \text{diag}([\sigma_1^2, \ldots, \sigma_N^2])$,

$$\mathbf{U} = \begin{bmatrix} \sin\gamma_i & \cos\gamma_i \\ \vdots & \vdots \\ \sin\gamma_N & \cos\gamma_N \end{bmatrix} \tag{11}$$

is a $N \times 2$ matrix assumed with full rank,

$$\mathbf{v} = \begin{bmatrix} \bar{x}_1^s \sin\gamma_1 + \bar{y}_1^s\cos\gamma_1 \\ \vdots \\ \bar{x}_N^s \sin\gamma_N + \bar{y}_N^s \cos\gamma_N \end{bmatrix} \tag{12}$$

and

$$\mathbf{D} = \text{diag}([d_1^2(x_t, y_t), \ldots, d_N^2(x_t, y_t)]) \tag{13}$$

where $d_i(x_t, y_t)$ denotes the distance between the target and the image of the ith sensor:

$$d_i(x_t, y_t) \triangleq \sqrt{(x_t - \bar{x}_i^s)^2 + (y_t - \bar{y}_i^s)^2}. \tag{14}$$

Clearly, the distance matrix $\mathbf{D}$ depends on target location (x_t, y_t), and it is unknown.

In [19], the distance matrix $\mathbf{D}$ is assumed available from secondary observations; therefore, (10) degenerates to a standard quadratic optimization. Later, [10] shows that a rough estimate, say $\hat{\mathbf{D}}$, can be used in (10) without significantly affecting the estimation accuracy, because its objective function only weakly relies on $\mathbf{D}$. With $\hat{\mathbf{D}}$, the solution for (10) is

$$\hat{\boldsymbol{\theta}}_{\text{SE}} = (\mathbf{U}^T\boldsymbol{\Lambda}^{-1}\hat{\mathbf{D}}^{-1}\mathbf{U})^{-1}\mathbf{U}^T\boldsymbol{\Lambda}^{-1}\hat{\mathbf{D}}^{-1}\mathbf{v} \tag{15}$$

which has the form of weighted LS.

2.3. Least Squares Initialization

Both ML and Stansfield estimators require a guess of $\boldsymbol{\theta}$: the former uses it for optimization initialization, while the latter employs it to obtain $\hat{\mathbf{D}}$. This can be realized via LS.

The line through the target and the image of sensor i is

$$y - \bar{y}_i^s = \tan(\underbrace{2\arctan a_i - \varphi_i}_{=-\gamma_i})(x - \bar{x}_i^s) \tag{16}$$

equivalently written as

$$(x - \bar{x}_i^s)\sin\gamma_i + (y - \bar{y}_i^s)\cos\gamma_i = 0. \tag{17}$$

Therefore, one can minimize

$$\begin{aligned}\hat{\boldsymbol{\theta}}_{\text{LS}} &= \arg\min_{x,y}\left\{\sum_{i=1}^N |(x - \bar{x}_i^s)\sin\gamma_i + (y - \bar{y}_i^s)\cos\gamma_i|^2\right\}\\ &= \arg\min_{\boldsymbol{\theta}} \|\mathbf{U}\boldsymbol{\theta} - \mathbf{v}\|^2 &(18)\\ &= (\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T\mathbf{v} &(19)\end{aligned}$$

to get an initial guess of $\boldsymbol{\theta}$. Here $\mathbf{U}$ and $\mathbf{v}$ share the same definitions as those in the previous subsection.

2.4. Cramér-Rao Lower Bound

The Cramér-Rao lower bound (CRLB) reveals performance limitation of an unbiased estimator. For a nonrandom vector $\boldsymbol{\theta}$, its estimation covariance matrix is bounded by [15]

$$\mathbb{E}\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\} \geq \mathbf{J}_{\boldsymbol{\theta}}^{-1} \tag{20}$$

where $\mathbf{J}_{\boldsymbol{\theta}}$ is the Fisher information matrix defined as

$$\begin{aligned}\mathbf{J}_{\boldsymbol{\theta}} &= -\mathbb{E}\left\{\nabla_{\boldsymbol{\theta}}\left[\nabla_{\boldsymbol{\theta}}\log\left(\prod_{i=1}^N f(\varphi_i \mid \boldsymbol{\theta})\right)\right]^T\right\}\\ &= \sum_{i=1}^N \frac{1}{2\sigma_i^2}\mathbb{E}\{\nabla_{\boldsymbol{\theta}}(\nabla_{\boldsymbol{\theta}}\underbrace{|\varphi_i - \phi_i|^2}_{\triangleq P_i})^T\}. &(21)\end{aligned}$$

Clearly, $\mathbf{J}_{\boldsymbol{\theta}}$ is a 2×2 matrix, and we now specify it element-by-element. The first-order derivatives of P_i are

$$\frac{\partial P_i}{\partial x_t} = 2(\phi_i - \varphi_i)\frac{\partial \phi_i}{\partial x_t} \qquad \text{and} \qquad \frac{\partial P_i}{\partial y_t} = 2(\phi_i - \varphi_i)\frac{\partial \phi_i}{\partial y_t}. \tag{22}$$

Recall the definition of $d_i(x_t, y_t)$ in (14), and then $\partial\phi_i/\partial x_t$ and $\partial\phi_i/\partial y_t$ shall be specified as

$$\frac{\partial \phi_i}{\partial x_t} = \frac{y_t - \bar{y}_i^s}{d_i^2(x_t, y_t)} \qquad \text{and} \qquad \frac{\partial \phi_i}{\partial y_t} = -\frac{x_t - \bar{x}_i^s}{d_i^2(x_t, y_t)}. \tag{23}$$

Based on (22), the second order derivatives of P_i are

$$\frac{\partial^2 P_i}{\partial^2 x_t} = 2\left(\frac{\partial \phi_i}{\partial x_t}\right)^2 + 2(\phi_i - \varphi_i)\frac{\partial^2 \phi_i}{\partial^2 x_t} \tag{24}$$

$$\frac{\partial^2 P_i}{\partial^2 y_t} = 2\left(\frac{\partial \phi_i}{\partial y_t}\right)^2 + 2(\phi_i - \varphi_i)\frac{\partial^2 \phi_i}{\partial^2 y_t} \tag{25}$$

$$\frac{\partial^2 P_i}{\partial x_t \partial y_t} = \frac{\partial^2 P_i}{\partial y_t \partial x_t} = 2\frac{\partial \phi_i}{\partial x_t}\frac{\partial \phi_i}{\partial y_t} + 2(\phi_i - \varphi_i)\frac{\partial^2 \phi_i}{\partial x_t \partial y_t}. \tag{26}$$

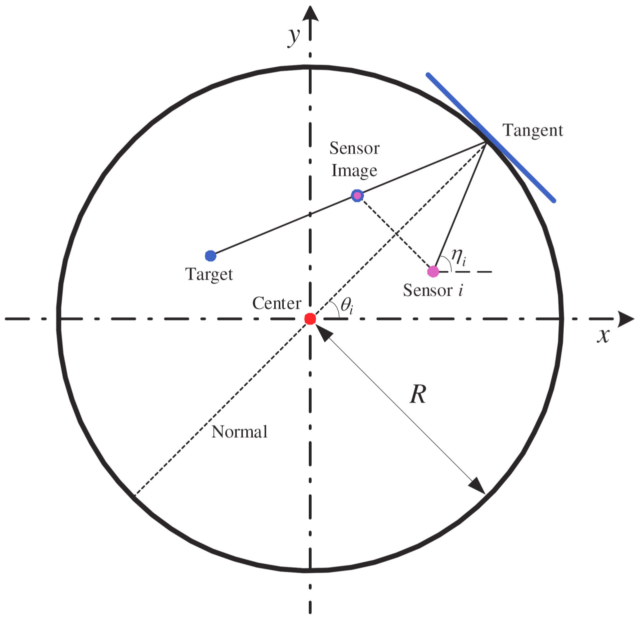


Fig. 3. AoA measurement with circular reflecting surface: the radiation of the target arrives at a sensor after a single specular reflection. The reflection geometry can be equivalently addressed as that the image of sensor with respect to the reflecting normal is exactly on the line determined by target and reflection point.

Since $\mathbb{E}\{\phi_i - \varphi_i\} = 0$, we obtain

$$[\mathbf{J}_{\boldsymbol{\theta}}]_{1,1} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left(\frac{\partial \phi_i}{\partial x_t} \right)^2 \tag{27}$$

$$[\mathbf{J}_{\boldsymbol{\theta}}]_{2,2} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left(\frac{\partial \phi_i}{\partial y_t} \right)^2 \tag{28}$$

$$[\mathbf{J}_{\boldsymbol{\theta}}]_{1,2} = [\mathbf{J}_{\boldsymbol{\theta}}]_{2,1} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left(\frac{\partial \phi_i}{\partial x_t} \cdot \frac{\partial \phi_i}{\partial y_t} \right) \tag{29}$$

where $[\mathbf{J}_{\boldsymbol{\theta}}]_{j,k}$ refers to a particular element of $\mathbf{J}_{\boldsymbol{\theta}}$.

3. BEARINGS-ONLY LOCALIZATION WITH CIRCULAR REFLECTING SURFACE

The reflecting surface was flat in the previous section. Now we will focus on bearings-only localization with a circular reflecting surface (inner side of a circle), where the target and N sensors are all within a circle as depicted in Fig. 3. The radiation from the target reaches each sensor after a single specular reflection on the circle, where the reflection is assumed to be specular [9]. A fusion center collects the noisy AoAs to infer the target position. Note that some similar problems exist in elastic collision and optical imaging [8, 12, 16]; results of this paper may be useful in their cases, especially where there is noise.

3.1. Geometric Modeling

The geometric relationships between the target and sensors are required for localization. Suppose the reflection be specular, so the angle of incidence equals the angle of reflection. Let the center of the reflecting circle be ($x_c = 0,\ y_c = 0$), and let $(R\cos\theta_i, R\sin\theta_i)$ stand for the (unknown) reflection point for sensor i, where R denotes the radius of the circle, and θ_i is an instrumental variable as shown in Fig. 3. Instead of direct application of the reflection law, we use an equivalent transformation: the image of sensor i corresponding to the normal line

$$y = x\tan\theta_i \tag{30}$$

is exactly on the line through target and reflection point as shown in Fig. 3. Mathematically, it is expressed as

$$\frac{R\cos\theta_i - \bar{x}_i^n}{R\sin\theta_i - \bar{y}_i^n} = \frac{x_t - R\cos\theta_i}{y_t - R\sin\theta_i} \tag{31}$$

where (x_t, y_t) denotes the coordinates of the target, while $(\bar{x}_i^n, \bar{y}_i^n)$ denotes the image coordinates of the ith sensor with respect to the normal line. As the coordinates of the ith sensor is (x_i^s, y_i^s), and hence $(\bar{x}_i^n, \bar{y}_i^n)$ can be written as

$$\begin{aligned} \bar{x}_i^n &= \cos(2\theta_i)x_i^s + \sin(2\theta_i)y_i^s \\ \bar{y}_i^n &= \sin(2\theta_i)x_i^s - \cos(2\theta_i)y_i^s \end{aligned} \tag{32}$$

based on Appendix A. Substituting (32) into (31), we have

$$\frac{R(y_i^s + y_t)\cos\theta_i - R(x_i^s + x_t)\sin\theta_i}{(x_i^s y_t + x_t y_i^s)\cos(2\theta_i) - (x_i^s x_t - y_i^s y_t)\sin(2\theta_i)} = 1. \tag{33}$$

Now, the coordinate connection between sensor i and target is obtained with the help of θ_i.

3.2. Maximum Likelihood Estimation

Suppose that θ_i could be expressed as a function of x_t and y_t, say $\theta_i(x_t, y_t)$, and then we can choose the reflection point $(R\cos(\theta_i(x_t,y_t)), R\sin(\theta_i(x_t,y_t)))$ as a reference and formulate the observed AoA ψ_i as

$$\psi_i = \underbrace{\arctan\left(\frac{R\sin(\theta_i(x_t,y_t)) - y_i}{R\cos(\theta_i(x_t,y_t)) - x_i} \right)}_{\triangleq \eta_i(x_t,y_t)} + w_i \tag{34}$$

where $w_i \sim \mathcal{N}(0, \varepsilon_i^2)$ denotes the Gaussian measurement noise. Let the w_is be independent, and hence the ML estimation becomes

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{N} f(\psi_i \mid \boldsymbol{\theta}), \tag{35}$$

where

$$f(\psi_i \mid \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\varepsilon_i^2}} \exp\left(-\frac{|\psi_i - \eta_i(x_t,y_t)|^2}{2\varepsilon_i^2} \right) \tag{36}$$

denotes the conditional probability density function of ψ_i.

3.3. Challenges for Maximum Likelihood

To this point all appears as the (straightforward?) LOS localization case. What is new? Three things are:

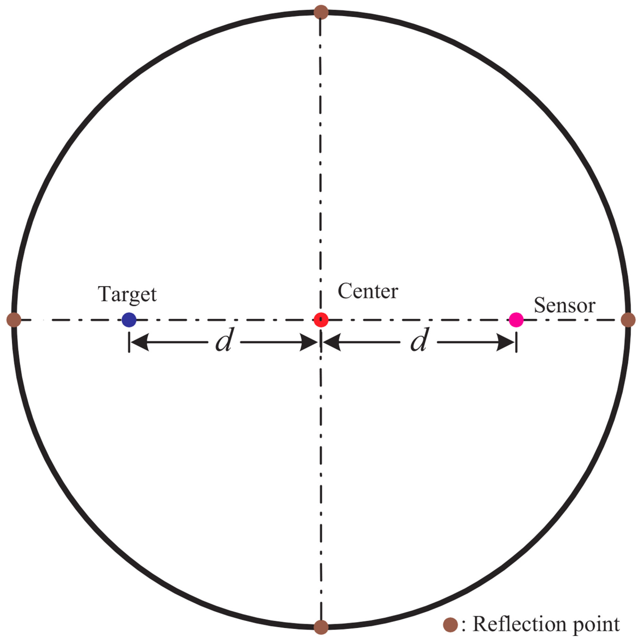


Fig. 4. An illustration of the existence of multiple solutions for (33). If a target and a sensor is symmetric about the center of the reflection circle, four reflection points can be immediately found. Note that reflection points are not necessarily as uniform as those in the figure if the sensor and the target is not centrally symmetric.

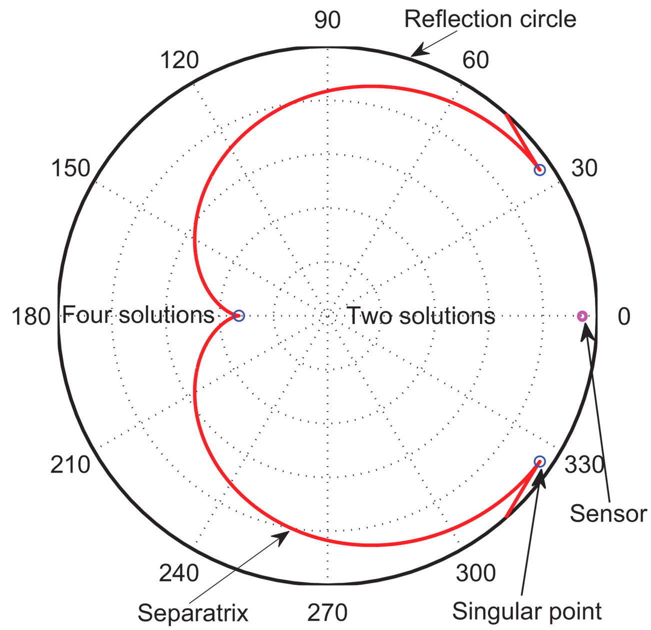


Fig. 5. Solution number distribution within a unit circle: a sensor is fixed at ($x_i^s = 0.955$, $y_i^s = 0$), while a target can be arbitrarily selected within it. If the target is located in the *two (four) solutions* region, equation (33) has two (four) possible nonoverlapping reflection paths, while if the target is on the separatrix line, equation (33) has three solutions. The sensor and the right two singular points are concentric.

1) The solution $\theta_i(x_t, y_t)$ is not unique. Therefore, the expression of (34) is not unique, neither is the likelihood equation (35). An example with a special target-sensor configuration is provided in Fig. 4 for illustration.

2) The number of $\theta_i(x_t, y_t)$ depends on geometry—the locations of target and sensor i.

3) $\theta_i(x_t, y_t)$ cannot be analytically obtained; thus, a closed-form expression of likelihood equation is unavailable.

A thorough understanding of these is important to understand the ML formulation in (35). The first two points relate to a famous geometric problem—circular billiards. Reference [8] gives a comprehensive analysis on the solution properties of (33) for a normalized circle, and we briefly summarize them below for completeness.

LEMMA 1 *Let* ($x_c = 0$, $y_c = 0$) *and* $R = 1$. *A sensor is fixed at* ($x_i^s = c$, $y_i^s = 0$), *where* $0 \le c < 1$, *while the target* (x_t, y_t) *is arbitrarily located within the circle. If* $|c| + |x_t| + |y_t| \neq 0$, *the number of solutions for* (33) *is either 2, 3, or 4 for a given* (x_t, y_t). *Furthermore, define a supplementary variable h as*

$$\begin{aligned} h = {} & (1-3c+2c^2)t^6 + 3(1-c+2c^2)t^4 \\ & + 3(1+c+2c^2)t^2 + (1+3c+2c^2) \end{aligned} \tag{37}$$

where $t \in [-\infty, +\infty]$, *and then the separatrix* $l(x,y)$

$$\begin{aligned} x &= -\frac{c}{h}[(1-c)t^6 + 3(1-3c)t^4 + 3(1+3c)t^2 + (1+c)] \\ y &= \frac{16c^2t^3}{h} \end{aligned} \tag{38}$$

divides the circle into two parts. If the target falls into the region that includes point $(0,0)$, (33) *has two solutions, while if it falls into the other one, the number of solutions for* (33) *is four. Finally, if the target falls (exactly) on* $l(x,y)$, (33) *has three solutions.*

PROOF Proof can be found in [8].

Clearly, the shape of $l(x,y)$ depends on the value of c. An illustration with $c = 0.955$ is given in Fig. 5. From this figure, we see that $l(x,y)$ is continuous but not everywhere differentiable; a nondifferentiable point is denoted singularity [8]. For $|c| \ge 1/3$, $l(x,y)$s share the similar shape as that in Fig. 5: the separatrix is open and has three singular points: ($x = -c/(1+2c)$, $y = 0$) and

$$\left(x = c(2c^2-1),\ y = \pm 2c^2\sqrt{1-c^2}\right). \tag{39}$$

Those for $|c| < 1/3$ share another shape, where the separatrix is closed, and another singular point ($x = -c/(1-2c)$, $y = 0$) is added in addition to the previous three. Note that 1) the above results are valid for $R = 1$ and $y_i^s = 0$; any other scenario with $R \neq 1$ or (and) $y_i^s \neq 0$ can be obtained via a simple scaling or (and) rotation of coordinates; 2) Lemma 1 does not include the extreme case, $c = x_t = y_t = 0$, of which the number of solutions is infinity.

The first two points are now clear. The geometrically dependent multiple-solution characteristics of circular reflection can significantly complicate ML estimation. For example, if a sensor obtains a noisy AoA, it may be responsible for up to 4 propagation paths. Since we have no knowledge to which one the AoA corresponds, a complete case-by-case enumeration is necessary in ML processing. As a result, for a system with N sensors, the total number of possible combinations can be up to 4^N. The final ML result is the best estimate from all these combinations; obviously, it is very intricate.

Regarding the third item, $\theta_i(x_t, y_t)$ cannot be analytically derived, nor can the likelihood maximization (35) be explicitly posed. As a consequence, local search algorithms such as gradient-based approaches [3] and alternating projection [21] will not in general work here. Fortunately, since the $\theta_i(x_t, y_t)$s can be numerically calculated for a given (x_t, y_t), a grid search based algorithm [20] is applicable for this problem. Specifically, uniformly divide a search area around the initial guess into a fine grid, and pick up the grid cell with the maximum ML value as an optimal estimate. This algorithm works well for low dimensions, and its performance depends on grid fineness, the search area size, and the accuracy of initialization. Generally, the larger the search area and the greater the fineness, the better performance; however, the larger the search area, the greater the computational load.

3.4. Maximum Likelihood Simplification with a Blocking Circle

In the previous subsection, target and sensors are arbitrarily located within the enclosing reflection circle; nevertheless, a real problem has more physical constraints. For example, the earth serves as a blocking (opaque) circle; no radiation can pass through it. Furthermore, the sensors are likely fixed on the earth, while the target is always within the annulus between the two circles. Suppose the reflection and blocking circles be concentric; we will show that the ML estimation could be simplified with the above constraints.

LEMMA 2 *Let the radius of the blocking circle be r, where r < R. Suppose a sensor be at A as shown in Fig. 6. Let* $\overline{AC}$ *and* $\overline{CD}$ *be two tangents of the blocking circle, and then we have that:*

- *if the target is located within region* $\mathcal{A}_1$*, enclosed by* $\overline{CD}$*,* $\widehat{DE}$*,* $\overline{EF}$*, and* $\widehat{CF}$*, it is reflectively-invisible for the sensor; or*
- *if the target is located within region* $\mathcal{A}_2$*, enclosed by* $\overline{AB}$*,* $\widehat{BC}$*,* $\overline{CD}$*, and* $\widehat{AD}$*, it is reflectively-visible for the sensor.*

PROOF Let T be an arbitrary point within $\mathcal{A}_1$, and assume $\overline{AG}$–$\overline{GT}$ be an unblocked reflection path, where G denotes the reflection point. To guarantee $\overline{AG}$ be free from blocking, G must be located on arc $\widehat{BC}$. Due to the symmetry of specular reflection, $\overline{GT}$ will have an intersection, say H, with the blocking circle. Therefore, we have $\angle\widehat{AH} = 2\angle\widehat{BG} < 2\angle\widehat{BC} = \angle\widehat{AD}$, and H is located on arc $\widehat{AD}$. As a result, line $\overline{GH}$, or, $\overline{GT}$ could not go through $\mathcal{A}_1$. Obviously, this contradicts the assumption, so an unblocked reflection path does not exist for the first case.

The proof of the second conclusion is trivial. If a reflection point, say J, continuously moves from C to B on arc $\widehat{BC}$, the intersection between the reflection line and inner circle, say K, will continuously go from D to A. Line $\overline{JK}$ will go through the entire $\mathcal{A}_2$. Therefore, every point within $\mathcal{A}_2$ is visible, and Lemma 2 holds.

A half-plane is employed in Lemma 2, and the other part is the same due to symmetry. The reflectively-visible area $\mathcal{A}_2$ includes a LOS-visible region $\mathcal{A}_3$, which is enclosed by $\overline{AB}$, $\widehat{BC}$, and $\overline{AC}$. If a target falls into $\mathcal{A}_3$ of each sensor,[3] direct arrivals will be utilized to estimate the position of the target, and hence the problem becomes a conventional LOS bearings-only localization [6, 10, 13, 19]. A mixed scenario, parts of sensors obtaining LOS while the others measuring NLOS reflective AoAs, is also physically sound. Its localization can be easily realized via a proper modification of (35), and no discussion or specific example will be given. In addition, since a real system requires a certain amount of elevation angle to avoid ground clutter, the practical reflectively- and LOS-visible areas are smaller than their theoretical results $\mathcal{A}_2$ and $\mathcal{A}_3$.

LEMMA 3 *If the sensor is deployed at* $(r, 0)$ *while the target is arbitrarily located within the reflectively-visible region* $\mathcal{A}_1$*, then the sensor-target pair has exactly one unblocked reflective path.*

PROOF Firstly, we will show that the reflectively-visible region $\mathcal{A}_1$ and the *four-solution region* shares no subarea with the help of Fig. 5 and 6. Since $\angle\widehat{AD} = \angle\widehat{BC} = 2\arccos r/R$, the coordinates of boundary point D, say (x_d, y_d), is

$$\begin{aligned} x_d &= r\cos\angle\widehat{AD} = r\left(2\frac{r^2}{R^2} - 1\right) \\ y_d &= r\sin\angle\widehat{AD} = 2\frac{r^2}{R^2}\sqrt{R^2 - r^2}. \end{aligned} \tag{40}$$

Normalize (x_d, y_d) with R, say (x'_d, y'_d), and define $c = r/R$; we see that (x'_d, y'_d) is exactly the upper singular point of the separatrix as shown in (39). For a half-plane as depicted in Fig. 6, since the four-solution region is always on the left side of line $\overline{OD}$ [8], and since the reflectively-visible region $\mathcal{A}_1$ is on its right side, their intersection is empty. In the other words, $\mathcal{A}_1$ falls into the *two-solution region* as shown in Fig. 5. Moreover, the

[3]The ionosphere is frequency selective, and only a certain frequency span is useful for beyond-horizon exploration. For a real configuration, an OTHR site is in general accompanied with another radar system equipped with a non-ionospheric-reflection frequency. Their detection results are combined to infer the target status: 1) if both of them claim a target in a certain direction, the target actually appears in $\mathcal{A}_3$; however, 2) if only the OTHR declares a detection, the target is beyond the horizon. One can properly use such information to mitigate area uncertainty before target localization.

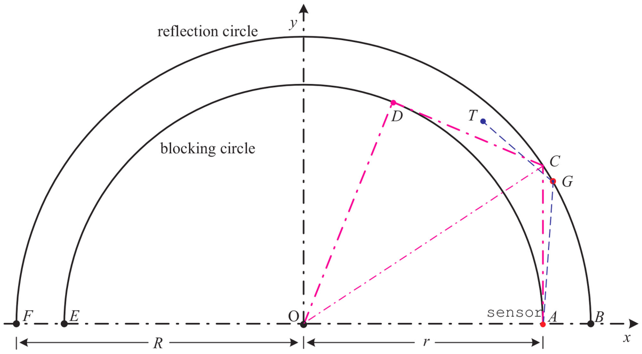


Fig. 6. Reflectively-visible and -invisible areas within in the annulus between the reflection and blocking circles. The sensor is located at $(r,0)$, while the target can be arbitrarily within the annulus. If the target falls into $\mathcal{A}_1$, the area bounded by $\overline{CD}$, $\widehat{DE}$, $\overline{EF}$, and $\widehat{CF}$, its radiation cannot arrive at sensor after one reflection. However, if it falls into $\mathcal{A}_2$, the area enclosed by $\overline{AB}$, $\widehat{BC}$, $\overline{CD}$, and $\widehat{AD}$, the target is reflectively-visible. Note that $\overline{AC}$ and $\overline{CD}$ are two tangents of the blocking circle, while $\overline{AG}$ and $\overline{GT}$ are two supplementary lines in the proof of area division.

two reflection paths are separated by the line through the sensor and target [8]; therefore, one of them will be blocked. As a result, Lemma 3 can be proven.

With a blocking circle, only one solution of (33) is valid; the likelihood equation (35) is unique for a given grid sample. As a consequence, the grid search based ML can be significantly simplified. Note that since a closed-form likelihood function is unavailable, the CRLB will not be given for the circular reflecting surface.

4. SUBOPTIMAL ESTIMATION FOR CIRCULAR REFLECTING SURFACE

Grid search based ML may obtain a globally optimal estimate of a target; however, its computational complexity is extremely high, as N calculations of $\theta_i(x_t, y_t)$ are required for even one grid point. Suboptimal algorithms with low computational load are investigated in this section based on a *small-angle approximation*. The basic idea is simple: if a sensor is close to the surface of a circle with large radius, and its observation variance is small, then the arc corresponding to the measurement uncertainty has a small angle. Approximate it as flat, and the tangent line through the intersection between the ith AoA and the reflection circle can be considered as a known flat reflecting surface.

Let the observed AoA of the ith sensor be ψ_i; its corresponding observation line, say l_i^o, can be expressed as

$$y = (x - x_i^s)\tan\psi_i + y_i^s. \tag{41}$$

The observation line l_i^o intersects the reflecting circle $x^2 + y^2 = R^2$ at two points (x_i^c, y_i^c), where

$$\begin{aligned} x_i^c &= \cos^2\psi_i[\tan\psi_i(x_i^s\tan\psi_i - y_i^s) \pm K_i] \\ y_i^c &= \cos^2\psi_i[(y_i^s - x_i^s\tan\psi_i) \pm K_i\tan\psi_i] \end{aligned} \tag{42}$$

and

$$K_i = \sqrt{R^2/\cos^2\psi_i - (y_i^s - x_i^s\tan\psi_i)^2}. \tag{43}$$

Since the observation line is a ray, one solution of (42) can easily be removed. The tangent line through the intersection (x_i^c, y_i^c), say l_i^t, is

$$y = -\frac{x_i^c}{y_i^c}(x - x_i^c) + y_i^c \tag{44}$$

and it can be regarded as a 'known' flat reflecting surface to localize a target with LS or Stansfield algorithms in Section 2 via proper slope and intercept mappings:

$$\begin{aligned} a_i &= -x_i^c/y_i^c \\ b_i &= (x_i^c)^2/y_i^c + y_i^c. \end{aligned} \tag{45}$$

Based on these, the suboptimal LS and Stansfield localization algorithms are briefly summarized below:

1) Get the AoA lines l_i^o for each sensor with (41);
2) Calculate the intersections (x_i^c, y_i^c)s between l_i^os and the reflection circle with (42);
3) Compute a_i and b_i for each tangent line l_i^t with (45);
4) Take l_i^t as a flat reflecting surface, and compute the image coordinate, say $(\bar{x}_i^s, \bar{y}_i^s)$, of sensor i with (4);
5) Substitute a_is, b_is and $(\bar{x}_i^s, \bar{y}_i^s)$s into (11) and (12) and compute $\mathbf{U}$ and $\mathbf{v}$;
6a) LS: Consider $(\bar{x}_i^s, \bar{y}_i^s)$s as a virtual sensor, and estimate $\hat{\boldsymbol{\theta}}_{\mathrm{LS}}$ with (19).
6b) Stansfield: initialize the distance matrix $\hat{\mathbf{D}}$ of the Stansfield estimator with $\hat{\boldsymbol{\theta}}_{\mathrm{LS}}$, and calculate $\hat{\boldsymbol{\theta}}_{\mathrm{SE}}$ with (15).

Note that these suboptimal algorithms can also be used in ML initialization.

The suboptimal algorithms alleviate computational burden, although they can introduce bias. The error mainly results from: 1) geometric distortion, and 2) approximating non-Gaussian with Gaussian noise. The former is straightforward. As for the latter, since a circular reflecting surface leads to nonlinear transformation, the measurement uncertainty of the target is no longer Gaussian. However, the suboptimal estimators employ sensor image based modeling, which implies that the target spatial uncertainty is Gaussian with regard to the sensor images, and that the uncertainty span linearly relates to the image-target distance $d_i(x_t, y_t)$. Actually, a circular surface may somewhat reduce this uncertainty expansion as shown in Fig. 7, and it is smaller than that of a flat surface due to *focusing*. This will be revealed via numerical simulation in Section 5.

Note that: 1) If the target is close to the reflecting surface and the ε_is are not very large, the situation that the uncertainty boundaries cross each other and sharply expand will not happen. 2) The ML in Section 3 assumes Gaussian distribution too. However, the nonlinear transformation (33) guarantees the shrink of target location uncertainty as shown in Fig. 7.

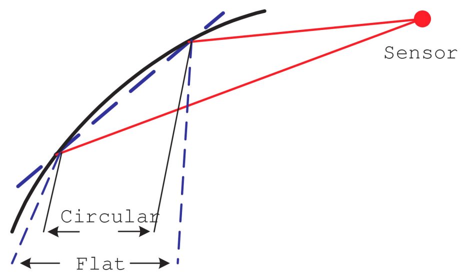


Fig. 7. Converting AoA uncertainty to target location uncertainty after flat (dashed) and circular (solid) reflections. The uncertainty span is linearly enlarged with the increase of $d_i(x_t, y_t)$ for the former, while the conical angle for the latter is somewhat reduced (focused, actually) after reflection.

TABLE II
Coordinates of Target and Sensors for Flat Reflecting Surfaces (m)

	Target	Sensor 1	Sensor 2	Sensor 3
x	10000	−3300	0	5000
y	5000	0	0	−2500

5. NUMERICAL RESULTS

5.1. Flat Reflecting Surfaces

This part compares the performance of LS, Stansfield, and ML estimators for flat reflecting surfaces. Three distributed sensors are employed in the simulation; their coordinates together with those of the target are in Table II. A LOS measurement is assumed unavailable, and three passive sensors extract AoAs of a target of interest with the help of their individual reflecting surfaces, of which the corresponding slope-intercept expressions are

$$\begin{aligned} &\text{Surface 1: } y = x/2 + 5000 \\ &\text{Surface 2: } y = -2500 \\ &\text{Surface 3: } y = x - 10000. \end{aligned} \tag{46}$$

Sensors are not perfect; measurement uncertainty (additive zero-mean Gaussian noise), exists. For simplicity of comparison, observation noises share the same variance: $\sigma_i^2 = \sigma^2$ for $\forall i$. The RMSEs versus standard variance σ for these three estimators together with CRLB are illustrated in Fig. 8. The number of Monte Carlo runs is 1000. According to Fig. 8, we observe

- The four curves linearly depend on σ;
- LS is biased and is the worst among them, while the RMSE of Stansfield is close to that of ML.

Simulation results coincide with the theoretical analysis for LOS bearings-only localization in [6], [10], [13], [19]. This is not surprising because NLOS bearing-only localization is mathematically equivalent to that of LOS.

The estimate of the Stansfield rather than the LS estimator is used to initialize the ML, as the former has slightly better localization accuracy than the latter for flat reflecting surfaces. Then, gradient-based approach is employed to search for the optimal solution of ML based on (6). The computational complexity of each method here is similar to the corresponding one for the LOS based location [6, 10, 13, 19].

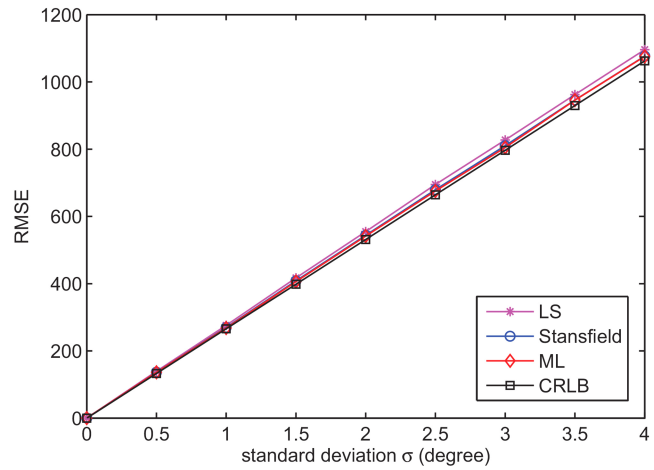


Fig. 8. Example of RMSEs versus bearing standard deviation σ for flat reflecting surfaces.

TABLE III
Polar Coordinates of the Target and Sensors for the Circular Reflecting Surface: Parameter set A

	Target	Sensor 1	Sensor 2	Sensor 3	Sensor 4
r_d (km)	$r + 50$	r	r	r	r
ϕ (π)	0	0.10	0.08	−0.09	−0.06

TABLE IV
Polar Coordinates of the Target and Sensors for the Circular Reflecting Surface: Parameter set B

	Target	Sensor 1	Sensor 2	Sensor 3
r_d (km)	$r + 50$	r	r	r
ϕ (π)	0	0.11	−0.07	−0.12

5.2. Circular Reflecting Surface

Bearings-only localization with a circular reflecting surface is investigated in this subsection. Here the reflecting and blocking circles are concentric, with center $(0, 0)$; their radii are, respectively, $R = 6671$ km and $r = 6371$ km. Two system parameter sets are used, and the coordinates of the target and sensors are collected in Tables III and IV, respectively. The coordinates are polar for simplicity, and they can be easily converted into Cartesian via $(r_d\cos(\phi), r_d\sin(\phi))$. All sensors share the same noise variance $\varepsilon_i^2 = \varepsilon^2$ as the previous case. Example of RMSEs versus ε for LS, Stansfield, and ML estimators for those two configurations are shown in Fig. 9, where the number of runs is 1000.

From those two figures, we observe:

- RMSEs for Stansfield and LS may still linearly depend on ε; however, the former is not necessarily

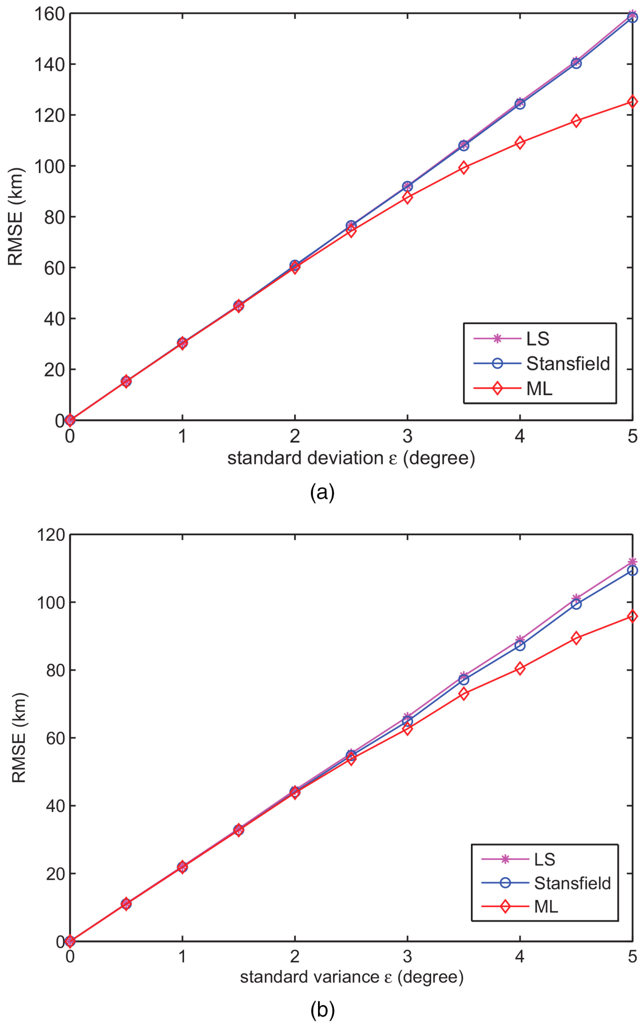


Fig. 9. Example of RMSEs versus AoA standard variance ε for a circular reflecting surface with circular block.

better than the latter. This may result from the fact that multiple approximations, such as arc, noise, and small-angle, are employed in the Stansfield estimator for the circular surface, and these may degrade its accuracy.

- ML outperforms LS and Stansfield at high ε; however, the difference is not significant for small noise levels. This is expected in that for a small variance the uncertainty area is small, and the suboptimal approximations are nearly exact.
- The RMSE for ML is not a linear function of ε, since reflection with a circular surface may shrink the uncertainty area compared to the flat one.

Apparently the Stansfield method does not significantly outperform LS in this case, so a system designer can make a choice between LS and ML trading off computation for estimation accuracy. In addition, various simulations reveal that $\varepsilon = 1.5$ would be small enough for the fearless use of suboptimal estimators in target localization.

The estimate of LS is used to initialize the ML in this part. Specifically, we firstly set the LS estimate as the center of a 160×200 km^2 rectangle, and then uniformly divide it into 1×1 km^2 grids. The ML estimate is the point with the maximal likelihood value among these 161×201 vertices. The computational complexity of the LS and Stansfield estimators are similar to those for the LOS case [6, 10, 13, 19]. However, that for the ML is much higher than its counterpart due to the grid search and solution elimination.

The received signal of an OTHR usually undergoes long distance propagation as well as frequency dependent reflection loss. Those can result in a low signal-to-noise ratio (SNR), which will cause low angular accuracy. For example, reference [17] points out that a tapered aperture of 3 km will lead to an angular resolution as large as 4 deg if the OTHR is operated at 3 MHz. Thus, a standard deviation up to 5 deg would be reasonable in simulation, even though it sounds large for traditional LOS radar systems such as phased array.

6. CONCLUSION

This paper studies bearings-only target localization with NLOS reflection measurements. Two kinds of reflecting surfaces are investigated: flat and circular. We derive the LS, Stansfield, and ML algorithms for both of them, and their performances are analyzed both via theory and algorithmically.

The reflecting surfaces are idealized and assumed known; however, the practical situation has many ingredients not discussed in this paper. For example, sensors need not be synchronized, in which case a dynamic component (tracking, or at least motion vector estimation) must be added. Multiple targets may also be present, in which case some means of data association is required. A practical system must also allow for nonidealities, such as of errors in sensor position and reflection surface, with possible extension of the circular results to the elliptical case.

Lastly, and probably most important, a real OTHR system operates in three dimensions. The estimators derived in this manuscript can easily be extended from the circular to the spherical case. But although we fully expect the existence and uniqueness statements (Lemmas 1, 2, and 3) to be extensible (and very interestingly so) to three dimensions, we doubt that this would be straightforward.

APPENDIX

6.1. Image of Specular Reflection with Flat Surface

LEMMA 4 *The image of a point* (x_0, y_0) *with respect to a flat reflecting surface* $y = ax + b$ *locates at* $(\bar{x}_0, \bar{y}_0)$ *in two dimensions, where*

$$\begin{aligned} \bar{x}_0 &= \frac{1-a^2}{1+a^2}x_0 + \frac{2a}{1+a^2}y_0 - \frac{2ab}{1+a^2} \\ \bar{y}_0 &= \frac{2a}{1+a^2}x_0 + \frac{a^2-1}{1+a^2}y_0 + \frac{2b}{1+a^2}. \end{aligned} \tag{47}$$

PROOF Proof is straightforward.

Lemma 4 is not universal: if the surface overlaps with $x = c$ (the special case is not included in $y = ax + b$), the image location is $(\bar{x}_0 = 2c - x_0, \bar{y}_0 = y_0)$. However, this special case can be easily avoided via a proper coordinate rotation, so we will no longer separately discuss it in target localization.

6.2. Equivalence: Target and Sensor Images Based Modelings

The equivalence of the target and sensor images based modelings without noise are proven via $\tan\phi_i = \tan\phi_i'$ in the following:

$$\begin{aligned}\tan\phi_i &= \frac{\tan(2\arctan a_i) - \dfrac{y_t - \bar{y}_i^s}{x_t - \bar{x}_i^s}}{1 + \tan(2\arctan a_i)\dfrac{y_t - \bar{y}_i^s}{x_t - \bar{x}_i^s}} \\ &= \frac{\dfrac{2a_i}{1-a_i^2} - \dfrac{y_t - \bar{y}_i^s}{x_t - \bar{x}_i^s}}{1 + \dfrac{2a_i}{1-a_i^2}\cdot\dfrac{y_t - \bar{y}_i^s}{x_t - \bar{x}_i^s}} \\ &= \frac{2a_i(x_t - \bar{x}_i^s) - (1-a_i^2)(y_t - \bar{y}_i^s)}{(1-a_i^2)(x_t - \bar{x}_i^s) + 2a_i(y_t - \bar{y}_i^s)} \end{aligned} \qquad (48)$$

Substitute (4) into (48), we have

$$\begin{aligned}\tan\phi_i &= \frac{2a_i x_t + (a_i^2 - 1)y_t + 2b_i - (1 + a_i^2)y_i^s}{(1-a_i^2)x_t + 2a_i y_t - 2a_i b_i - (1+a_i^2)x_i^s} \\ &= \frac{\bar{y}_i^t - y_i^s}{\bar{x}_i^t - x_i^s} = \tan\phi_i'. \end{aligned} \qquad (49)$$

REFERENCES

[1] C. R. Berger, S. Choi, S. Zhou, and P. Willett
Channel energy based estimation of target trajectories using distributed sensors with low communication rate.
IEEE Transactions on Signal Processing, **58**, 4 (Apr. 2010), 2339–2350.

[2] W. D. Blair and M. Brandt-Pearce
Statistics of monopulse measurements of Rayleigh targets in the presence of specular and diffuse multipath.
In *Proceedings of the IEEE Radar Conference*, Atlanta, GA, May 2001.

[3] S. Boyd and L. Vandenberghe
Convex Optimization.
Cambridge University Press, Cambridge, U.K., 2004.

[4] L. Chen, Q. Li, W. Guo, and Y. Zhu
One-dimensional mirrored interferometric aperture synthesis.
IEEE Geoscience and Remote Sensing Letters, **7**, 2 (Apr. 2010), 357–361.

[5] J. W. Ciboci
Over-the-horizon radar surveillance of airfields for counterdrug applications.
IEEE Aerospace and Electronic Systems Magazine, **13**, 1 (Jan. 1998), 31–34.

[6] K. Doğançay
On the bias of linear least squares algorithms for passive target localization.
Signal Processing, **84**, 3 (Mar. 2004), 475–486.

[7] K. Doğançay
Bearings-only target localization using total least squares.
Signal Processing, **85**, 9 (Sept. 2005), 1695–1710.

[8] M. Drexler and M. J. Gander
Circular billiard.
SIAM Review, **40**, 2 (1998), 315–323.

[9] R. L. Fante and S. Dhar
A model for target detection with over-the-horizon radar.
IEEE Transactions on Aerospace and Electronic Systems, **26**, 1 (Jan. 1990), 68–83.

[10] M. Gavish and A. J. Weiss
Performance analysis of bearing-only target location algorithms.
IEEE Transactions on Aerospace and Electronic Systems, **28**, 3 (Aug. 1992), 817–828.

[11] K. C. Ho, X. Lu, and L. Kovavisaruch
Source localization using TDOA and FDOA measurements in the presence of receiver location errors: Analysis and solution.
IEEE Transactions on Signal Processing, **55**, 2 (Feb. 2007), 684–696.

[12] J. E. Howard
Time-dependent circular billiard.
Mathematical Problems in Engineering, 1 (2009), 1–10.

[13] L. M. Kaplan, Q. Le, and P. Molnar
Maximum likelihood methods for bearings-only target localization.
In *Proceedings of the International Conference on Acoustics, Speech and Signal Processing*, Salt Lake City, UT, May 2001.

[14] L. F. McNamara
The Ionosphere: Communications, Surveillance, and Direction Finding.
Malabar, FL: Kreiger, 1991.

[15] N. Mukhopadhyay
Probability and Statistical Inference.
Marcel Dekker Inc., New York, 1st ed., 2000.

[16] P. M. Neumann
Reflections on reflection in a spherical mirror.
American Mathematical Monthly, **105**, 6 (1998), 523–528.

[17] R. J. Riddolls
A Canadian perspective on high frequency over-the-horizon radar.
Defence R&D Canada, Ottawa, http://cradpdf.drdc.gc.ca/PDFS/unc64/p527279.pdf.

[18] X. Song, P. Willett, and S. Zhou
Target localization with NLOS circularly reflected AoAs.
In *Proceedings of the International Conference on Acoustics, Speech and Signal Processing*, Prague, Czech, May 2011.

[19] R. G. Stansfield
Statistical theory of DF fixing.
Journal of the IEE, **14**, 15 (Dec. 1947), 762–770.

[20] P. Stoica and A. B. Gershman
Maximum-likelihood DOA estimation by data-supported grid search.
IEEE Signal Processing Letters, **6**, 10 (Oct. 1999), 273–275.

[21] I. Ziskind and M. Wax
Maximum likelihood localization of multiple sources by alternating projection.
IEEE Transactions on Signal Processing, **36**, 10 (Oct. 1988), 1553–1560.

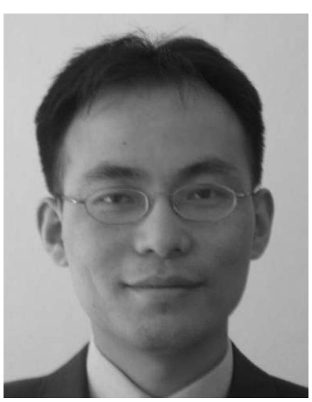
Xiufeng Song (S'08) received the B.S. degree from Xidian University, Xi'an, China, in 2005, the M.S. degree from Institute of Electronics, Chinese Academy of Sciences (CAS), Beijing, China, in 2008, and the Ph.D. degree from University of Connecticut, in 2012, all in electrical engineering.

Dr. Song is currently working at the LSI Corporation, San Jose, CA. His research interests lie in signal processing, detection, estimation, and tracking. He received the Best Student Paper Award (second place) at the 2012 IEEE Sensor Array and Multichannel Signal Processing Workshop.

Peter Willett (F'03) received his B.A.Sc. (engineering science) from the University of Toronto in 1982, and his Ph.D. degree from Princeton University in 1986.

Dr. Willett has been a faculty member at the University of Connecticut ever since, and since 1998 has been a professor. His primary areas of research have been statistical signal processing, detection, machine learning, data fusion and tracking. He has interests in and has published in the areas of change/abnormality detection, optical pattern recognition, communications and industrial/security condition monitoring.

He was editor-in-chief for *IEEE Transactions on Aerospace and Electronic Systems* from 2006–2011, and is presently the VP of Publications for the IEEE AESS. In the past he has been associate editor for three active journals—*IEEE Transactions on Aerospace and Electronic Systems* (for Data Fusion and Target Tracking) and *IEEE Transactions on Systems, Man, and Cybernetics, parts A and B*. He is also associate editor for the *IEEE AES Magazine*, associate editor for ISIF's electronic *Journal of Advances in Information Fusion*. He was General Co-Chair (with Stefano Coraluppi) for the 2006 ISIF/IEEE Fusion Conference in Florence, Italy, Executive Chair of the 2008 Fusion Conference in Cologne, and Emeritus Chair for the 2011 Fusion Conference in Chicago. He was Program Cochair (with Eugene Santos) for the 2003 IEEE Conference on Systems, Man & Cybernetics in Washington, D.C., and Program Cochair (with Pramod Varshney) for the 1999 Fusion Conference in Sunnyvale. He has been a Member of the IEEE Signal Processing Society's Sensor-Array & Multichannel (SAM) Technical Committee since 1997, and both serves on that TC's SAM Conferences' Program Committees and maintains the SAM website.



Shengli Zhou (SM'11) received the B.S. degree in 1995 and the M.Sc. degree in 1998, from the University of Science and Technology of China (USTC), Hefei, both in electrical engineering and information science. He received his Ph.D. degree in electrical engineering from the University of Minnesota (UMN), Minneapolis, in 2002.

He has been an assistant professor with the Department of Electrical and Computer Engineering at the University of Connecticut (UCONN), Storrs, 2003–2009, and now is an associate professor. He holds a United Technologies Corporation (UTC) Professorship in Engineering Innovation, 2008–2011. His general research interests lie in the areas of wireless communications and signal processing. His recent focus is on underwater acoustic communications and networking.

Dr. Zhou served as an associate editor for *IEEE Transactions on Wireless Communications*, Feb. 2005–Jan. 2007, and *IEEE Transactions on Signal Processing*, October 2008–October 2010. He is now an associate editor for *IEEE Journal of Oceanic Engineering*. He received the 2007 ONR Young Investigator award and the 2007 Presidential Early Career Award for Scientists and Engineers (PECASE).

Shooter Localization using a Wireless Sensor Network of Soldier-Worn Gunfire Detection Systems

JEMIN GEORGE
LANCE M. KAPLAN

This paper addresses the problem of shooter localization using a wireless sensor network of soldier-worn gunfire detection systems. If the sensor is within the field of view of the shockwave generated by the supersonic projectile, then using acoustic phenomena analysis, the gunfire detection system can localize the source of the incoming fire with respect to the sensor location. These relative solutions from individual gunfire detection systems are relayed to the central node, where they are fused to yield a highly accurate geo-rectified solution, which is then relayed back to the soldiers for added situational awareness. Detailed formulation of the fusion methodology presented here indicates that the multi-sensor fusion algorithm for soldier-worn gunfire detection systems is essentially a weighted nonlinear least-squares algorithm, which can easily be implemented using the Gauss-Newton method. The performance analysis of the proposed fusion algorithm through numerical simulations reveals that the fused solution is much more accurate compared to the individual best sensor solution and the simple averaged sensor solution. Since the proposed fusion algorithm requires consistent weighting of individual sensor solutions, a consistency-based weighting scheme is introduced to tackle the lack of reliability among sensor provided weights. Implementation of the proposed fusion scheme along with the consistency-based weighting scheme on experimental data further confirms the numerical results.

Manuscript received December 16, 2011; revised May 18, 2012; released for publication September 15, 2012.

Refereeing of this contribution was handled by Benjamin Slocumb.

Author's addresses: J. George and L. M. Kaplan are with the Sensors and Electron Devices Directorate, U.S. Army Research Laboratory, 2800 Powder Mill Rd., Adelphi, MD 20783, E-mail: (jemin.george.civ@mail.mil).

1557-6418/13/$17.00

1. INTRODUCTION

Highly accurate small-arms gunfire detection systems on individual soldiers are vital requirement for added battlefield situational awareness and threat assessment. Today, several acoustic shooter localization systems are commercially available [2, 7, 29]; an overview of such systems can be found in [26]. A few examples of soldier-wearable shooter localization systems include the Shoulder-Worn Acoustic Targeting System (SWATS) by QinetiQ North America, Inc., Boomerang Warrior-X by BBN Technologies, and PinPoint by BioMimetic Systems. These Soldier-wearable Gunfire Detection Systems (SW-GDSs) can provide a good level of localization accuracy as long as the soldier is at an ideal location relative to the shooter and the bullet trajectory. However, due to the dissipative nature of acoustic signals, localization systems suffer severe performance degradation as the distance to the shooter and the bullet trajectory increases [22, 23, 28]. Moreover, when a relative solution, i.e., the shooter location relative to the sensor, is transformed into a geo-rectified solution using a magnetometer and GPS, the solution often becomes unusable due to localization errors. Geo-rectified solutions are necessary when displaying hostile fire icons on a Command and Control Geographic Information System (C2 GIS) map display.

SW-GDSs use acoustic phenomena analysis of small-arms fire to localize the source of incoming fire, usually with a bearing and range relative to the user [12]. Currently, the individual SW-GDSs operate separately and are not designed to exploit the sensor network layout of all the soldiers within a Small Combat Unit (SCU) to help increase accuracy. Researchers are exploring some novel solutions that utilize the team aspect of these SCUs by exploiting all SW-GDSs in a squad/platoon to increase detection rates and localization accuracy [9, 10, 32]. Apart from soldier-wearable systems, there exist several single-microphone as well as microphone array-based sensor network approaches to shooter localization [6, 15, 16, 19, 24]. Most of the existing sensor fusion schemes for shooter localization are centralized approaches where the individual sensor measurements, such as time of arrival or angle of arrival of the muzzle blast or the shockwave are combined to yield a single estimate of the shooter position [5, 16, 19, 20, 32]. Here we consider a hierarchical approach where the relative shooter position from the individual sensors are fused to obtain a more accurate geo-rectified shooter position. The proposed approach takes full advantage of the team aspect of a SCU to provide a fused solution that would be more accurate and suitable for a C2 GIS map display than the individual soldier's solution. The objective here is to improve accuracy across an entire SCU so even soldiers in non-ideal settings (out of range, bad angle, etc.) can exploit the good solutions from their neighbors. Furthermore, the proposed hierarchical approach would allow the individual sensors to operate independently in an event of network failure.

The individual SW-GDS is composed of a passive array of microphones that is able to localize a gunfire event by measuring the direction of arrival for both the acoustic wave generated by the muzzle blast and the shockwave generated by the supersonic bullet [2, 7, 12, 23]. After detecting a gunfire, the individual sensors report their solution along with their orientation and GPS positions to a central node over a communication network. At the central node, the individual sensor solutions are fused along with the GPS positions to yield a highly accurate, geo-rectified solution, which is then relayed back to individual soldiers for added situational awareness.

This paper presents a detailed account of our continuing effort in the field of shooter localization using a wireless sensor network, where the main goal is to develop a fusion algorithm that would work well (compared to the individual sensor solutions) across all the off-the-shelf SW-GDSs and not tailored toward any particular acoustic sensor [3, 9, 10, 13, 30]. Even though the exact details of the measurement process in an acoustic GDS is sensor dependent and may considered as proprietary, a brief description of the shooter localization process is presented in Sections 2 and 3 for completeness. Sections 2 and 3 are not intended to provide a detailed and comprehensive explanation of acoustic gunfire detection process; rather, they are presented as a prologue to the fusion algorithm presented in Section 4 and to point out that even with the most simplistic measurement model, the fusion algorithm amounts to a complex nonlinear optimization problem. Readers who are interested in further details of the shooter localization process are referred to [2], [23], and the references within them.

The sensor fusion scheme presented here is a maximum likelihood approach and since here we consider additive white Gaussian noise, the maximum likelihood estimation problem can be posed as a weighted nonlinear least-squares problem. But due to the interdependence between the latent parameters and the measurement noise covariance, the weighted nonlinear least-squares problem is not readily solvable considering the practical limitations in processing time and capability. Therefore, a variance versus bias trade-off study is conducted to reduce the number of parameters in the optimization problem. Furthermore, the SW-GDSs are designed to provide confidence weights along with their individual solutions. From analyzing the experimental data, it was noticed that the weights provided by the sensors are inconsistent with the individual solution accuracy and therefore, a consistency-based weighting scheme is provided. In summary, compared to the existing literature, the four main contributions of this manuscript are:

- A detailed formulation of the multi-sensor data fusion scheme for a wireless network of SW-GDSs.
- A variance versus bias trade-off study to reduce the number of parameters in the optimization problem for the real-time implementation of the fusion algorithm.
- A consistency-based weighting scheme to tackle the lack of reliability among the sensor provided weights.
- Experimental results and an in-depth analysis of data obtained from implementing the proposed sensor fusion algorithm for realistic sensor formation.

The structure of this paper is as follows: Section 2 presents the measurement model for the soldier-wearable acoustic sensor nodes. Section 3 presents the localization algorithm that converts the sensor measurements to a shooter position estimate. Details of the central node data fusion and the corresponding nonlinear least-squares problem are given in Section 4. Section 5 presents the results from numerical simulations and Section 6 presents the results obtained from implementing the fusion algorithm on experimental data. Finally, Section 7 concludes the paper and discusses the current research challenges.

2. SENSOR MODEL

Consider a SCU consisting of n individual soldiers equipped with the SW-GDS. In order to set up the problem and develop a sensor model, consider a scenario where there is only one shooter and the SW-GDS receives both the muzzle blast and shockwave. The shooter or the target location and the soldier or the ith sensor location are defined as T and S_i, respectively. For simplicity, the problem is formulated in $\mathbb{R}^2$, i.e., $T \equiv [T_x \ \ T_y]^T \in \mathbb{R}^2$ and $S_i \equiv [S_{i_x} \ \ S_{i_y}]^T \in \mathbb{R}^2$. Now define the individual range, r_i, and bearing, ϕ_i, between the ith sensor node and the target as

$$r_i = \sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} \tag{1}$$

$$\begin{aligned}\phi_i &= \arctan\left(\frac{T_y - S_{i_y}}{T_x - S_{i_x}}\right) \pm \pi\{-1,0,1\}\\ &= 2\arctan\frac{(T_y - S_{i_y})}{\sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} + (T_x - S_{i_x})}.\end{aligned} \tag{2}$$

REMARK 1 For descriptional simplicity, we consider a constant velocity bullet model while the sensors in reality account for the decelerating bullet speed [2]. Since we are mainly interested in developing an algorithm for SW-GDS fusion as opposed to improving the individual sensor capability, the simplified sensor model is presented only for completeness.

When a gun fires, the blast from the muzzle produces a spherical acoustic wave that can be heard in any direction. The bullet travels at supersonic speeds

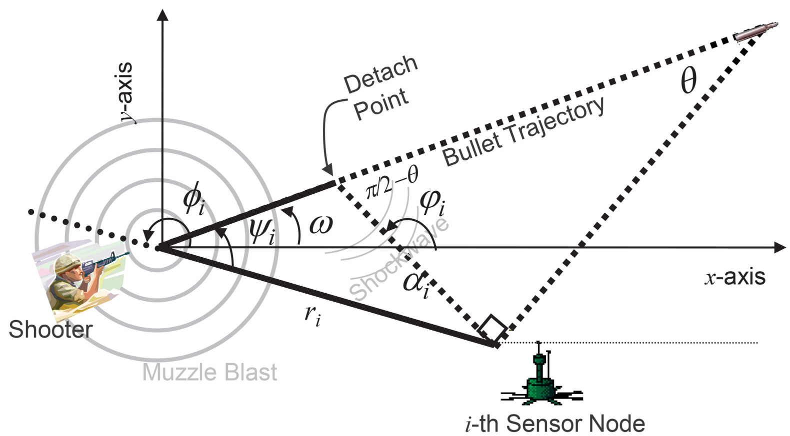


Fig. 1. Geometry of the bullet trajectory and propagation of the muzzle blast and shockwave to the sensor node.

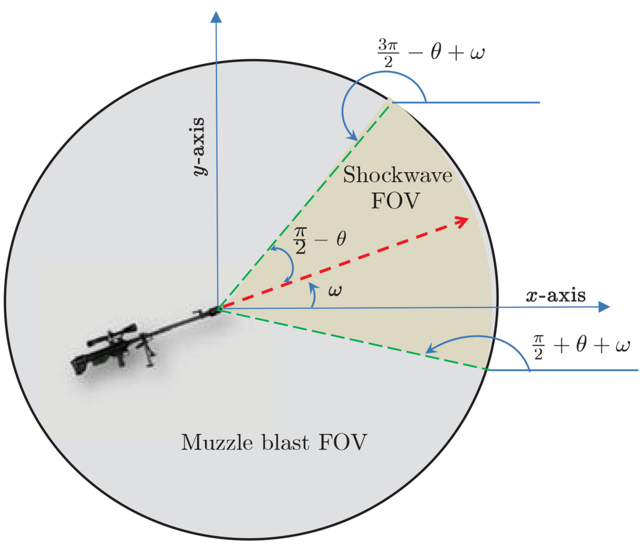


Fig. 2. Muzzle blast and shockwave field of view.

and produces an acoustic shockwave that emanates as a cone from the trajectory of the bullet. Because the bullet is traveling faster than the speed of sound, the shockwave arrives at the sensor node before the wave from the muzzle blast [19], which we simply refer to as the muzzle blast. Figure 1 illustrates the geometry of the shockwave and the muzzle blast for the ith sensor node when the orientation of the bullet trajectory is ω with respect to the horizontal axis. As the bullet pushes air, it creates an impulse wave. The wavefront is a cone whose angle θ with respect to the trajectory is

$$\theta = \arcsin\left(\frac{1}{m}\right) \tag{3}$$

where m is the Mach number [8]. The Mach number is assumed to be known since the typical Mach number for sniper ammunition is $m = 2$.[1] Since the Mach number directly influences the range (distance from the sensor to the shooter) estimates, uncertainty in bullet speed may be treated as a range estimation error.

As indicated in Fig. 1, the angle ϕ_i indicates the direction of arrival (DOA) of the muzzle blast, and φ_i indicates the DOA of the shockwave. The muzzle blast DOA[2] is measured counter-clockwise such that $0 \le \phi_i \le 2\pi$. For a more detailed description of the scenario, please refer to [12]. Figure 2 indicates the field of view (FOV) for both the muzzle blast and the shockwave. Note that the FOV of the muzzle blast is 2π, i.e., omnidirectional, and the FOV for the shockwave is $\pi - 2\theta$. SW-GDS receives the shockwave only if the muzzle blast DOA is within the bounds

$$\pi/2 + \theta + \omega < \phi_i < 3\pi/2 - \theta + \omega. \tag{4}$$

Now, the DOA angle for the shockwave can be written as

$$\varphi_i = \begin{cases} -\dfrac{\pi}{2} - \theta + \omega, & \text{if} \quad \pi + \omega < \phi_i < \dfrac{3\pi}{2} - \theta + \omega \\ \dfrac{\pi}{2} + \theta + \omega, & \text{if} \quad \dfrac{\pi}{2} + \theta + \omega < \phi_i < \pi + \omega \end{cases}. \tag{5}$$

The first case, $\pi + \omega < \phi_i < (3\pi/2) - \theta + \omega$, corresponds to the scenario where the sensor is located above the bullet trajectory and the second case, $(\pi/2) + \theta + \omega < \phi_i < \pi + \omega$, corresponds to the scenario where the sensor is located below the bullet trajectory (as shown in Fig. 1). The case where $\phi_i = \pi + \omega$ corresponds to the scenario when the sensor is located on the bullet trajectory and we do not consider such a scenario here. If ϕ_i is outside the bounds given in (4), then the sensor node only receives the muzzle blast as it is outside the FOV of the shockwave.

Under the assumptions that the bullet maintains a constant velocity over its trajectory, the time difference of arrival (TDOA) between the shockwave and the muzzle blast can be written as [2]

$$\tau_i = \frac{r_i}{c}[1 - \cos|\phi_i - \varphi_i|], \qquad \forall \phi_i \ne \varphi_i \tag{6}$$

[1] http://www.chuckhawks.com/rifle_ballistics_table.htm.

[2] Equation (2) yields $-\pi \le \phi_i \le \pi$. Thus π must be added to ϕ_i to obtain a positive ϕ_i if $\phi_i < 0$.

where c indicates the speed of sound. Utilizing (5), the bullet trajectory angle, ω, can be obtained from the shockwave DOA angle. Though this paper assumes that the bullet speed is constant over its trajectory, others have proposed localization algorithms [1], [14], [19] that employ more realistic bullet speed models at the expense of computational efficiency.

When the sensor node is within the FOV of the shockwave, the three available measurements are the two DOA angles and the TDOA between the muzzle blast and the shockwave, i.e.,

$$\hat{\phi}_i = h_1(T, S_i, \omega) + \eta_\phi \tag{7a}$$

$$\hat{\varphi}_i = h_2(T, S_i, \omega) + \eta_\varphi \tag{7b}$$

$$\hat{\tau}_i = h_3(T, S_i, \omega) + \eta_\tau \tag{7c}$$

where $h_1(\cdot)$ is given in (2), $h_2(\cdot)$ is given in (5), and $h_3(\cdot)$ is given in (6). The measurement noise is assumed to be zero mean Gaussian white noise, i.e., $\eta_\phi \sim \mathcal{N}(0, \sigma_\phi^2)$, $\eta_\varphi \sim \mathcal{N}(0, \sigma_\varphi^2)$ and $\eta_\tau \sim \mathcal{N}(0, \sigma_\tau^2)$. Actually, it is has been shown that, with the high signal-to-noise ratio, a maximum likelihood DOA estimator is unbiased and its estimates approximately follow a Gaussian distribution [21, 25]. Here (7) represents the measurement equations and after receiving these measurements, the processing capability internal to the individual SW-GDS converts these measurement into shooter location estimates. It is important to note that the typical SW-GDS is equipped with a magnetometer to obtain the orientation of the sensor and thus the DOA measurements are reported in a global reference frame as shown in Fig. 1. Thus, it is not necessary to report the individual sensor orientation to the central node, unless the DOA is given in a local sensor reference frame. Furthermore, assuming the magnetometer measurement errors are Gaussian, the uncertainty associated with the sensor orientation can be simply added to the DOA uncertainty.

3. DATA FUSION AT SENSOR NODE LEVEL

Let $\hat{Z}_i$ denote the individual sensor level estimates on the target bearing, range, and bullet trajectory, i.e., $\hat{Z}_i = [\hat{\phi}_i \ \hat{r}_i \ \hat{\omega}_i]$. Data fusion at the sensor node involves calculating these individual estimates based on the three sensor measurements.

Using (5), the bullet trajectory angle, ω, can be obtained from the shockwave DOA measurements. Thus, the observations on the trajectory angle can be written as

$$\hat{\omega}_i = \omega + \eta_\varphi. \tag{8}$$

Now the likelihood function, $p(\hat{\omega}_i \mid T, S_i, \omega)$, can be written as

$$p(\hat{\omega}_i \mid T, S_i, \omega) = \mathcal{N}(\omega, \sigma_\varphi^2).$$

From (6), the range can be written in terms of the TDOA as

$$r_i = \frac{c\tau_i}{[1 - \cos|\phi_i - \varphi_i|]}. \tag{9}$$

The observation of r_i may be written as

$$\hat{r}_i = \frac{c\hat{\tau}_i}{[1 - \cos|\hat{\phi}_i - \hat{\varphi}_i|]}. \tag{10}$$

Using the first-order Taylor series, the range measurement can be approximated as

$$\begin{aligned}\hat{r}_i &\approx \frac{c\tau_i}{[1 - \cos|\phi_i - \varphi_i|]} \\ &\quad + \begin{bmatrix} \dfrac{c}{[1 - \cos|\phi_i - \varphi_i|]} & -\dfrac{c\tau_i \sin|\phi_i - \varphi_i|}{[1 - \cos|\phi_i - \varphi_i|]^2} \end{bmatrix} \begin{bmatrix} \eta_\tau \\ \eta_{\phi\varphi} \end{bmatrix} \\ &= r_i + H(T, S_i, \omega)\eta_r \end{aligned}$$

where

$$\eta_r = \begin{bmatrix} \eta_\tau \\ \eta_{\phi\varphi} \end{bmatrix}, \qquad \eta_{\phi\varphi} \sim \mathcal{N}(0, \sigma_\phi^2 + \sigma_\varphi^2)$$

and

$$H(T, S_i, \omega) = \begin{bmatrix} \dfrac{c}{[1 - \cos|\phi_i - \varphi_i|]} & -\dfrac{c\tau_i \sin|\phi_i - \varphi_i|}{[1 - \cos|\phi_i - \varphi_i|]^2} \end{bmatrix}.$$

Now the likelihood $p(\hat{r}_i \mid T, S_i, \omega)$ can be approximated as

$$p(\hat{r}_i \mid T, S_i, \omega) \approx \mathcal{N}(r_i, \sigma_r^2(T, S_i, \omega))$$

where the variance $\sigma_r^2(T, S_i, \omega)$ can be written as

$$\sigma_r^2(T, S_i, \omega) = H(T, S_i, \omega) \begin{bmatrix} \sigma_\tau^2 & 0 \\ 0 & \sigma_\phi^2 + \sigma_\varphi^2 \end{bmatrix} H^T(T, S_i, \omega). \tag{11}$$

Thus, the likelihood function $p(\hat{Z}_i \mid T, S_i, \omega)$ can be approximated as

$$p(\hat{Z}_i \mid T, S_i, \omega) \approx \mathcal{N}(\boldsymbol{\mu}_{Z_i}, \Sigma_{Z_i}) \tag{12}$$

where

$$\boldsymbol{\mu}_{Z_i} = \begin{bmatrix} \phi_i \\ r_i \\ \omega \end{bmatrix}, \qquad \Sigma_{Z_i} = \begin{bmatrix} \sigma_\phi^2 & 0 & 0 \\ 0 & \sigma_r^2(T, S_i, \omega) & 0 \\ 0 & 0 & \sigma_\varphi^2 \end{bmatrix}.$$

It is assumed that a GPS receiver is used to obtain an accurate positioning on each sensor. Thus, the position observation on the sensors are given as

$$\hat{S}_i = \begin{bmatrix} S_{i_x} \\ S_{i_y} \end{bmatrix} + \begin{bmatrix} v_{i_x} \\ v_{i_y} \end{bmatrix} \tag{13}$$

where the noise terms are assumed to be zero mean Gaussian white, i.e., $v_{i_x} \sim \mathcal{N}(0, \sigma_{i_x}^2)$ and $v_{i_y} \sim \mathcal{N}(0, \sigma_{i_y}^2)$. Now the GPS measurement likelihood function may be written as

$$p(\hat{S}_i \mid S_i) \sim \mathcal{N}\left(\begin{bmatrix} S_{i_x} \\ S_{i_y} \end{bmatrix}, \begin{bmatrix} \sigma_{i_x}^2 & 0 \\ 0 & \sigma_{i_y}^2 \end{bmatrix} \right) \equiv \mathcal{N}(\boldsymbol{\mu}_{S_i}, \Sigma_{S_i}). \tag{14}$$

Assumption 1 Without loss of generality, it can be assumed that the GPS observations on sensor position are independent of target location, observations on target location, and the projectile trajectory information, i.e.,

$$p(\hat{S}_i \mid S_i) = p(\hat{S}_i \mid T, S_i, \omega) = p(\hat{S}_i \mid \hat{Z}_i, T, S_i, \omega).$$

Based on Assumption 1, the joint probability $p(\hat{Z}_i, \hat{S}_i \mid T, S_i, \omega)$ can be calculated as

$$p(\hat{Z}_i, \hat{S}_i \mid T, S_i, \omega) = p(\hat{S}_i \mid \hat{Z}_i, T, S_i, \omega) p(\hat{Z}_i \mid T, S_i, \omega). \tag{15}$$

Substituting (12) and (14), the above joint likelihood can be written as

$$p(\hat{Z}_i, \hat{S}_i \mid T, S_i, \omega) \approx \mathcal{N}(\boldsymbol{\mu}_{S_i}, \Sigma_{S_i}) \mathcal{N}(\boldsymbol{\mu}_{Z_i}, \Sigma_{Z_i}). \tag{16}$$

Now for a sensor located in the FOV of the shockwave, the target location can be estimated as

$$\hat{T}_{x_i} = \hat{S}_{i_x} + \hat{r}_i \cos(\hat{\phi}_i) \tag{17}$$

$$\hat{T}_{y_i} = \hat{S}_{i_y} + \hat{r}_i \sin(\hat{\phi}_i). \tag{18}$$

When the sensor is located outside the shockwave FOV, the only estimate would be the bearing angle. After individual estimates are obtained at the sensor node level, the measured information is transmitted to a central node where it is fused to obtain a more accurate estimate of shooter location.

4. DATA FUSION AT THE CENTRAL NODE

While sensors in the FOV of the muzzle blast and the shockwave yield a range, bearing, and trajectory angle estimates, the gunfire detection systems outside the FOV of the shockwave yield a muzzle blast DOA. Also, GPS measurements are available on each sensor locations. At the central node, this information from the individual sensor nodes is fused to obtain an accurate estimate of the shooter location, bullet trajectory angle, and sensor locations.

Based on Assumption 1, the joint likelihood function associated with each sensor is given in (15). Let $\mathbf{S}_{1:n} = \{S_1, S_2, \ldots, S_n\}$, $\hat{\mathbf{Z}}_{1:n} = \{\hat{Z}_1, \hat{Z}_2, \ldots, \hat{Z}_n\}$, and $\hat{\mathbf{S}}_{1:n} = \{\hat{S}_1, \hat{S}_2, \ldots, \hat{S}_n\}$, where n indicates the number of sensors. Since the measurement errors for the sensor nodes are independent of each other, the joint conditional density $p(\hat{\mathbf{Z}}_{1:n}, \hat{\mathbf{S}}_{1:n} \mid T, \mathbf{S}_{1:n}, \omega)$ can be defined as

$$p(\hat{\mathbf{Z}}_{1:n}, \hat{\mathbf{S}}_{1:n} \mid T, \mathbf{S}_{1:n}, \omega) = \prod_{i=1}^{n} p(\hat{Z}_i, \hat{S}_i \mid T, S_i, \omega). \tag{19}$$

In the maximum likelihood estimation approach considered here, estimates of the sensor locations, shooter location, and bullet trajectory angle are obtained so that the joint log-likelihood function is maximized, i.e.,

$$\max_{T, \mathbf{S}_{1:n}, \omega} \ln\{p(\hat{\mathbf{Z}}_{1:n}, \hat{\mathbf{S}}_{1:n} \mid T, \mathbf{S}_{1:n}, \omega)\}$$

$$\Rightarrow \max_{T, \mathbf{S}_{1:n}, \omega} \sum_{i=1}^{n} \ln\{p(\hat{Z}_i, \hat{S}_i \mid T, S_i, \omega)\}. \tag{20}$$

Based on the results given in the previous section, the criteria for the maximum likelihood estimation can be written as

$$\max_{T, \mathbf{S}_{1:n}, \omega} \sum_{i=1}^{n} [\ln\{\mathcal{N}(\boldsymbol{\mu}_{Z_i}, \Sigma_{Z_i})\} + \ln\{\mathcal{N}(\boldsymbol{\mu}_{S_i}, \Sigma_{S_i})\}]. \tag{21}$$

Note that the density $\mathcal{N}(\boldsymbol{\mu}_{Z_i}, \Sigma_{Z_i})$ may be written as

$$\mathcal{N}(\boldsymbol{\mu}_{Z_i}, \Sigma_{Z_i}) = \frac{1}{\sqrt{|2\pi\Sigma_{Z_i}|}} \exp\left\{ -\frac{1}{2}(\hat{Z}_i - \boldsymbol{\mu}_{Z_i})^T \Sigma_{Z_i}^{-1} (\hat{Z}_i - \boldsymbol{\mu}_{Z_i}) \right\} \tag{22}$$

where $\boldsymbol{\mu}_{Z_i}$ and Σ_{Z_i} are the same quantities given in (12) if the sensor is within the FOV of the shockwave and

$$\boldsymbol{\mu}_{Z_i} = \phi_i = h_1(T, S_i, \omega), \qquad \Sigma_{Z_i} = \sigma_\phi^2$$

if the sensor is outside the FOV of the shockwave. The density $\mathcal{N}(\boldsymbol{\mu}_{S_i}, \Sigma_{S_i})$ is given as

$$\mathcal{N}(\boldsymbol{\mu}_{S_i}, \Sigma_{S_i}) = \frac{1}{\sqrt{|2\pi\Sigma_{S_i}|}} \exp\left\{ -\frac{1}{2}(\hat{S}_i - \boldsymbol{\mu}_{S_i})^T \Sigma_{S_i}^{-1} (\hat{S}_i - \boldsymbol{\mu}_{S_i}) \right\} \tag{23}$$

where

$$\boldsymbol{\mu}_{S_i} = \begin{bmatrix} S_{i_x} \\ S_{i_y} \end{bmatrix}, \qquad \Sigma_{S_i} = \begin{bmatrix} \sigma_{i_x}^2 & 0 \\ 0 & \sigma_{i_y}^2 \end{bmatrix}.$$

After substituting (22) and (23) into (21), the maximum likelihood criteria may be written as

$$\begin{aligned} \min_{T, \mathbf{S}_{1:n}, \omega} \sum_{i=1}^{n} \Big[& \tfrac{1}{2}(\hat{Z}_i - \boldsymbol{\mu}_{Z_i})^T \Sigma_{Z_i}^{-1} (\hat{Z}_i - \boldsymbol{\mu}_{Z_i}) \\ & + \tfrac{1}{2}(\hat{S}_i - \boldsymbol{\mu}_{S_i})^T \Sigma_{S_i}^{-1} (\hat{S}_i - \boldsymbol{\mu}_{S_i}) \\ & + \ln\left\{\sqrt{|2\pi\Sigma_{Z_i}|}\right\} + \ln\left\{\sqrt{|2\pi\Sigma_{S_i}|}\right\} \Big]. \end{aligned} \tag{24}$$

Note that the term, $\ln\{\sqrt{|2\pi\Sigma_{Z_i}|}\}$, in above equation is present due to the fact that Σ_{Z_i} is a function of T, $\mathbf{S}$, and ω. The last term, $\ln\{\sqrt{|2\pi\Sigma_{S_i}|}\}$ can be ignored since Σ_{S_i} is a known constant matrix. Since Σ_{Z_i} is assumed to be a diagonal matrix, (24) can be rewritten as

$$\begin{aligned} \min_{T, \mathbf{S}_{1:n}, \omega} \sum_{i=1}^{n} [& \ln(\sigma_{r_i}) + \tfrac{1}{2}(\hat{Z}_i - \boldsymbol{\mu}_{Z_i})^T \Sigma_{Z_i}^{-1} (\hat{Z}_i - \boldsymbol{\mu}_{Z_i}) \\ & + \tfrac{1}{2}(\hat{S}_i - \boldsymbol{\mu}_{S_i})^T \Sigma_{S_i}^{-1} (\hat{S}_i - \boldsymbol{\mu}_{S_i})]. \end{aligned} \tag{25}$$

Apart from the initial term, $\ln(\sigma_r)$, the optimization problem given in (25) is similar to that used in the weighted nonlinear least-squares. Thus, the maximum likelihood approach presented here is similar to the weighted nonlinear least-squares estimation.

There exists no closed form solution to the nonlinear least-squares optimization problem given in (25) and therefore a numerical approach must be used. A few common approaches to solve the nonlinear least-squares problem include the Gauss-Newton method, Nelder-Mead simplex method, and Levenberg-Marquardt method [4]. Almost all these approaches are iterative methods that require an initial approximation to the unknown parameters and provide successively better approximations. The iterative process is repeated until the parameters do not change to within specified limits. Here we mainly utilize the Gauss-Newton method for solving the nonlinear least-squares problem given in (25). The main advantage of the Gauss-Newton method is that it exhibits a "quadratic convergence," which, simply put, means that the uncertainty in the parameters after $p+1$ iterations is proportional to the square of the uncertainty after p iterations. Once these uncertainties begin to get small, they decrease quite rapidly. An additional advantage of the Gauss-Newton method is that it only requires calculating the first-order derivatives. Even though one of the major problems with the Gauss-Newton method is that it sometimes diverges if the initial approximation is too far from truth, in the sensor fusion applications, the Gauss-Newton method can be easily initialized using the median of the individual sensor solutions.

4.1. Parameter Reduction

One of the major problems with the real-time implementation of the proposed fusion scheme is that it is a $(2n+3)-D$ problem and its dimensionality increases as the number of sensors increases. Given in this subsection is an analysis that will help to reduce the dimensionality of the optimization problem.

Most of the SW-GDSs currently available are designed so that they provide the shooter location relative to the sensor location. Moreover, some sensors also provide the weights or the confidence numbers that indicate the estimated accuracy level of the relative solution. These confidence numbers can be used to weight the measurements in the nonlinear least-squares estimation problem given in (25). Thus, (25) can be rewritten as

$$\min_{T,\mathbf{S}_{1:n}} \sum_{i=1}^{n} [\tfrac{1}{2}(\hat{Z}_i - \boldsymbol{\mu}_{Z_i})^T W_i (\hat{Z}_i - \boldsymbol{\mu}_{Z_i}) + \tfrac{1}{2}(\hat{S}_i - \boldsymbol{\mu}_{S_i})^T \Sigma_{S_i}^{-1} (\hat{S}_i - \boldsymbol{\mu}_{S_i})] \tag{26}$$

where

$$\hat{Z}_i = \begin{bmatrix} \hat{\phi}_i \\ \hat{r}_i \end{bmatrix}$$

$$\boldsymbol{\mu}_{Z_i} = \begin{bmatrix} 2\arctan \dfrac{(T_y - S_{i_y})}{\sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} + (T_x - S_{i_x})} \\ \sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} \end{bmatrix}$$

$$W_i = \begin{bmatrix} \dfrac{1}{\sigma_{\phi_i}^2} & 0 \\ 0 & \dfrac{1}{\sigma_{r_i}^2} \end{bmatrix}$$

$$\hat{S}_i = \begin{bmatrix} \hat{S}_{i_x} \\ \hat{S}_{i_y} \end{bmatrix}$$

$$\boldsymbol{\mu}_{S_i} = \begin{bmatrix} S_{i_x} \\ S_{i_y} \end{bmatrix}.$$

Since the SW-GDS do not report the bullet trajectory, $\hat{\omega}_i$ is not included in $\hat{Z}_i$. Also,

$$\Sigma_{S_i} = \begin{bmatrix} \sigma_{i_x}^2 & 0 \\ 0 & \sigma_{i_y}^2 \end{bmatrix}$$

is assumed to be a known matrix and W_is indicate the weights reported by the sensors. The nonlinear least-squares problem given in (26) is of dimension $2n+2$. If the sensor reported GPS positions are taken as absolute truth, then the nonlinear least-squares problem given in (26) becomes two dimensional and it may be rewritten as

$$\min_{T} \sum_{i=1}^{n} [\tfrac{1}{2}(\hat{Z}_i - \boldsymbol{\mu}_{Z_i})^T W_i (\hat{Z}_i - \boldsymbol{\mu}_{Z_i})]. \tag{27}$$

Note that the problem given in (26) involves estimating more parameters compared to the problem in (27). Thus, based on the arguments given in [11], it can be shown that the Cramér-Rao lower bound for the latter is always less than the lower bound for the former, i.e., the problem in (26) yields higher variance for the shooter location compared to the problem in (27). On the other hand, the low dimensional problem in (27) yields biased estimates since it considers the GPS measurements as absolute truth. This bias grows as GPS errors increases. For small errors, the bias is small so that (27) is more accurate than (26) due to the lower variance. Once the GPS errors exceed a threshold, the bias dominates and (26) becomes more accurate. Simulations in the next section help to determine this threshold.

4.2. Weighting Scheme

It is well known that the performance of the least-squares problems given in (26) and (27) depends on the weights associated with each measurements. The fusion scheme presented earlier assumes that the sensors are designed to provide these weights along with its relative shooter position estimates. These weights indicate the estimated accuracy level of the calculated range and bearing. From analyzing the experimental data, it was noticed that the weights provided by the sensors are inconsistent with the relative solution accuracy. This inconsistency is particularly visible in the case of outliers. Using these inconsistent weights in the fusion process would bias the fused solution toward an outlier. Thus, we provide an ad hoc weighting scheme, which is based on a consistency check, i.e., the weight is selected based on how consistent a particular sensor solution is to the rest of the relative solutions. Recently, several consistency-function-based source localization algorithms have been proposed, which can provide accurate solutions even if a large number of independent outliers are present in a measurement set [17, 18, 31]. Here, the consistency check is conducted by comparing the individual sensor solution to the fused solution obtained by combining the remaining individual sensor measurements. To this end, we first consider the entire n measurement set and remove the particular sensor measurement for which we would like to generate the weight. Let $\hat{\mathbf{Z}}_{1:n}$ indicate the set of all n sensor measurements and $\hat{\mathbf{Z}}_{1:n}^{\{j\}}$ indicate the set of measurements excluding the jth sensor measurement. Now we obtain a fused solution, $T^{\{j\}}$, by combining the remaining $n-1$ measurements, $\hat{\mathbf{Z}}_{1:n}^{\{j\}}$, after equally weighting them, i.e.,[3]

$$T^{\{j\}} = \min_{T} \sum_{i=1\, i\neq j}^{n} \frac{1}{2} \left(\begin{bmatrix} 2\arctan \dfrac{(T_y - S_{i_y})}{\sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} + (T_x - S_{i_x})} \\ \sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} \end{bmatrix} - \begin{bmatrix} \hat{\phi}_i \\ \hat{r}_i \end{bmatrix} \right)^T \begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix}$$
$$\times \left(\begin{bmatrix} 2\arctan \dfrac{(T_y - S_{i_y})}{\sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} + (T_x - S_{i_x})} \\ \sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} \end{bmatrix} - \begin{bmatrix} \hat{\phi}_i \\ \hat{r}_i \end{bmatrix} \right) \tag{28}$$

where

$$W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix}$$

is the weight matrix. After obtaining the fused solution, it is then converted into relative range and bearing solutions, $r^{\{j\}}$ and $\phi^{\{j\}}$, using the sensor GPS measurements.

$$\begin{bmatrix} \phi^{\{j\}} \\ r^{\{j\}} \end{bmatrix} = \begin{bmatrix} 2\arctan \dfrac{(T_y - S_{i_y})}{\sqrt{(T_x - S_{i_x})^2 + (T_y - S_{i_y})^2} + (T_x - S_{i_x})} \\ \sqrt{(T_x^{\{j\}} - S_{i_x})^2 + (T_y^{\{j\}} - S_{i_y})^2} \end{bmatrix}. \tag{29}$$

Now, the difference between the fused relative solution and the measured relative solution is calculated.

$$E_r^{\{j\}} = (r^{\{j\}} - \hat{r}_j)^2 \tag{30}$$

$$E_\phi^{\{j\}} = (\phi^{\{j\}} - \hat{\phi}_j)^2. \tag{31}$$

If the individual solution is very close to the fused solution, then it is of high consistency and a large weight is selected. Conversely, if the individual solution is far from the fused solution, then it is of low consistency and a low weight is selected. Thus, the weight are obtained as

$$W_j = \begin{bmatrix} \dfrac{1}{E_r^{\{j\}}} & 0 \\ 0 & \dfrac{1}{E_\phi^{\{j\}}} \end{bmatrix}. \tag{32}$$

This procedure is repeated n time so that a consistency-based weight is obtained for the entire n-sensor measurements.

5. NUMERICAL SIMULATIONS

This section presents numerical simulations to assess the localization improvement due to the proposed fusion algorithm. For the simulation scenario considered here, we assume that there are five sensor nodes and the node locations in meters are

$$\mathbf{S} = \begin{bmatrix} 127 & 20 & 90 & 136 & 182 \\ 107 & 22 & 0 & 68 & 59 \end{bmatrix}.$$

For simplicity, we assume a constant velocity model for the bullet. Thus, the Mach number is selected to be $m = 2$ and the speed of sound is selected to be $c = 342$ m/sec. The measurement noise models are selected as $\sigma_{i_x} = \sigma_{i_y} = 5$ m, $\sigma_\phi = \sigma_\varphi = 4°$, and $\sigma_\tau = 1$ msec. Since there exist several approaches to solve the nonlinear least-squares problem, two different methods are used to obtain solutions for both simulation scenarios. In the first method, the optimization problem is solved using the Gauss-Newton method [4] mentioned in the previous section. The second approach uses the Nelder-Simplex algorithm [27], i.e., the *fminsearch* function in Matlab. Both algorithms are initialized using the median of the sensor-reported shooter location.

For simulation, the shooter is assumed to be located at $T = [50 \text{ m} \;\; 50 \text{ m}]^T$ and we select the bullet trajectory to be $\omega = 30°$. Figure 3 shows the first simulation scenario. Due to the sensor locations, the second and the third sensors do not receive the shockwave.

5.1. Simulation Results I

The simulation results presented in this subsection corresponds to the results obtained from solving the full dimensional problem given in (25), where the bullet trajectory as well as the sensor locations are estimated

[3]The arctangent formulation given in (28) is equivalent to the atan 2 function in Matlab and it has a range of $[-\pi, \pi]$.

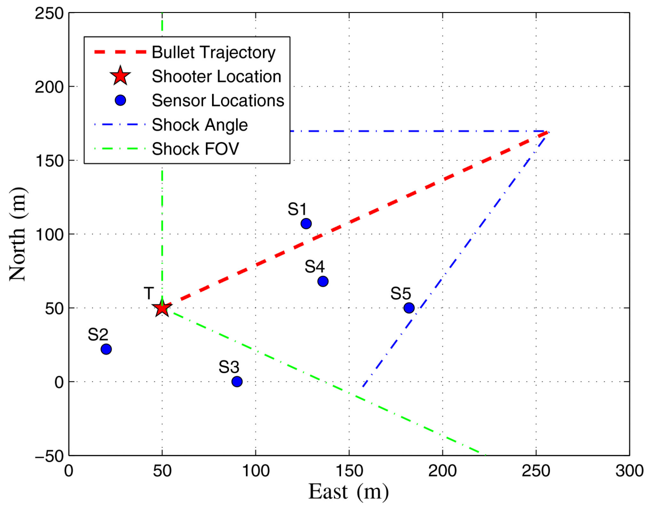


Fig. 3. Simulation I Scenario.

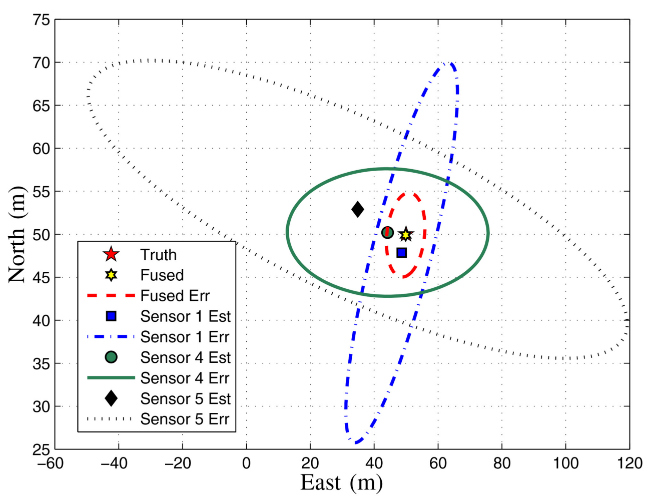


Fig. 4. Simulation result I: Mean results from Monte Carlo runs.

along with the shooter location. In order to evaluate the system performance, 1000 Monte Carlo simulations are conducted for both the Gauss-Newton method and the simplex algorithm. The mean shooter locations and the associated error ellipses obtained from the Monte Carlo simulations using the Gauss-Newton method are given in Fig. 4. A separate plot is not provided for the results obtained using the simplex algorithm since they are very similar to those obtained for the Gauss-Newton method. Figure 4 indicates that sensor five performs the worst out of the three sensors within the shockwave FOV; this is due to the fact that the localization accuracy is inversely proportional to the miss distance. Figure 4 also indicates that the fused estimate is superior to the individual sensor estimates, and the uncertainty associated with the fused estimates is much less than the uncertainty associated with the individual sensor estimates. It seems that the orientation of the error ellipse depends on what side of the trajectory the sensor is located. In addition, the orientation of the error ellipse indicates that the estimation error along the x and y directions varies with the sensor location.

TABLE I
Simulation Result I: Shooter Location

	T_x (m)	T_y (m)	RMSE (m)
Truth	50	50	—
Sensor 1	48.3513	47.2948	23.2870
Sensor 2	—	—	—
Sensor 3	—	—	—
Sensor 4	42.9248	50.2141	31.1132
Sensor 5	37.1197	52.0782	65.6542
Average	42.7986	49.8623	25.9660
Gauss-Newton	49.9066	49.9134	6.8639
Nedler-Simplex	50.0493	50.0588	6.9972

TABLE II
Simulation Result I: Bullet Trajectory

	ω (deg)	RMSE (deg)
Truth	30	—
Sensor 1	30.0641	3.9690
Sensor 2	—	—
Sensor 3	—	—
Sensor 4	30.3402	3.9970
Sensor 5	29.9591	3.9029
Average	30.1211	2.2128
Gauss-Newton	30.1211	2.2128
Nedler-Simplex	30.1999	2.4674

TABLE III
Simulation Result I: Sensor Location RMSE

	GPS (m)	Gauss-Newton (m)	Nedler-Simplex (m)
Sensor 1	7.0215	6.5453	6.5938
Sensor 2	7.0002	6.3195	6.3530
Sensor 3	7.0028	6.6513	6.6770
Sensor 4	7.1509	6.5259	6.6201
Sensor 5	7.0223	6.7883	6.8731

Table I summarizes the mean shooter location estimate of the individual sensors and the fusion algorithms over the Monte Carlo runs. The "average" estimate presented in Table I indicates the estimate obtained by simply averaging the individual target estimates from sensors one, four, and five. Table I also contains the root-mean-square error (RMSE) associated with each estimate. Based on the RMSE presented in Table I, one can conclude that that fused estimates outperform the individual sensors and the simple average estimate.

Table II contains the mean bullet trajectory angle estimate obtained from the individual sensors and the fusion algorithms over the Monte Carlo runs. Table II also contains the RMSE associated with each trajectory angle estimate. Note that the fused trajectory estimate is simply the average of the individual sensor estimates due to the way in which ω appears in (25).

Table III contains RMSE associated with the sensor location estimates. The performance improvement in sensor location estimate accuracy is moderate compared

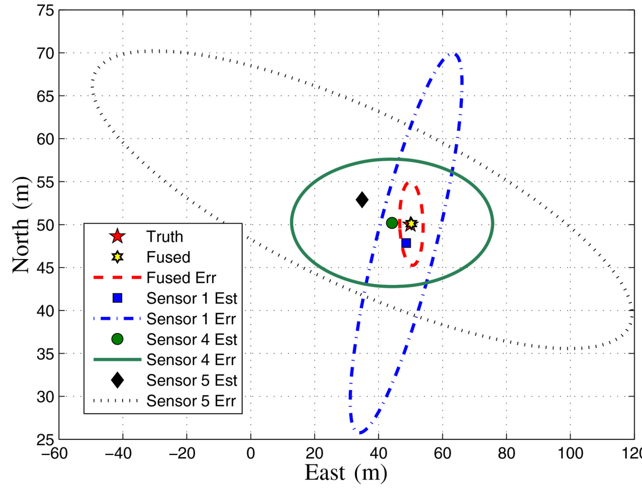


Fig. 5. Simulation result II: Mean results from Monte Carlo runs.

to the shooter location estimate accuracy since the GPS measurements are fairly accurate to begin with. Also note that the RMSE associated with the sensor location estimate given in Table III is similar to that of the RMSE associated with the fused shooter position estimate. Based on the RMSE presented in Tables I, II, and III, one can conclude that that fused estimates outperform the individual sensors.

5.2. Simulation Result II

The simulation results presented in this subsection corresponds to the results obtained from solving the two-dimensional problem given in (27), where only the shooter location is estimated. The mean shooter locations and the associated error ellipses obtained from the Monte Carlo simulation using the Gauss-Newton method are given in Fig. 5. Figure 5 indicates that the error ellipse obtained for the second simulation is smaller compared to that obtained for the first simulation. Also note that the increase in estimation accuracy is mostly along the x-direction, i.e., east. This is due to the fact that the initial error in x-direction is much larger compared to that in y-direction (north). The RMSE associated with the fused result in Fig. 5 is approximately 5.1771 m.

This performance improvement in the low-dimensional problem is due to the very low GPS bias compared to the shooter location estimation error. It can be shown that, as the GPS accuracy decreases, the performance degradation of the 2-D problem is much larger compared to that of the full-dimensional problem. Figure 6 compares the RMSE for the shooter location for both the 2-D problem given in (27) and the $(2n+2)-D$ problem given in (26). This particular result is obtained for the simulation scenario given in Fig. 3 using additive Gaussian white noise for measurement noise. Figure 6 indicates that for low GPS error of $\sigma_{x,y} \leq 7$ m, the 2-D problem yields better accuracy compared to the $(2n+2)-D$ problem. Moreover, for high GPS error of

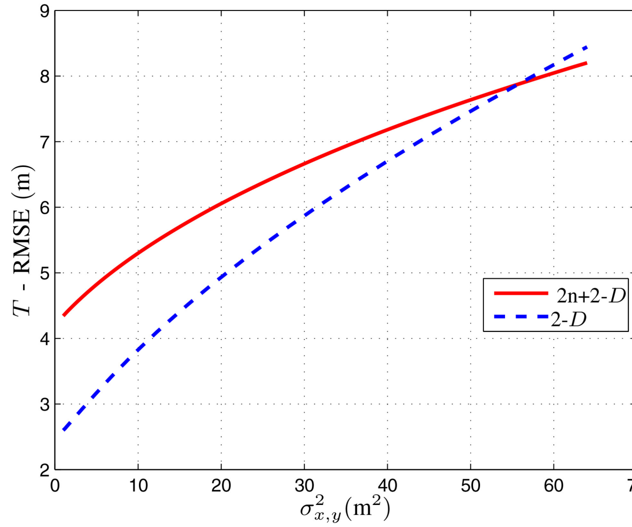


Fig. 6. RMSE sensitivity plot for simulation one scenario.

$\sigma_{x,y} \geq 7.5$ m, taking the GPS measurements as absolute truth and not accounting for the GPS error degrades the shooter location accuracy.

6. EXPERIMENTAL RESULTS

This section presents the experimental results obtained by implementing the fusion algorithm on gunfire detection data, but first, the experimental setup used for data collection is briefly explained. Experimental data were obtained using several gunfire detection systems provided by BioMimetic Systems.[4] For data collection, we used three soldier-wearable (SW) systems, three unattended ground sensors (UGSs), and three vehicle-mounted (VM) systems.

Each sensor unit had an interface unit attached, consisting of an Atom processor netbook, an Enhanced Position Location and Reporting System (EPLRS) radio, a GPS system, and an Li-145 battery. The netbook was interfaced to the sensor through a custom driver, using serial communication over USB. A standard USB to USB mini cable was used as the interface cable. The netbook was used as a stand-in for the soldier computer; the netbook has the same processor and was an inexpensive substitute for testing. At the central node, the fusion processor receives the solutions from individual sensors via EPLRS radio. The central processor is also an Atom processor netbook where the fusion algorithm combines the individual solutions to obtain a fused solution. The fused solution is then relayed back to individual sensors via EPLRS radio. At the individual sensor nodes, GIS map display is used to display the geo-rectified fused solution.

Experiments were conducted for two sensor formations, the quad symmetric formation and the wedge flank formation. Figure 7 contains the sensor layout for both scenarios. The test pattern includes nine sensors,

[4]www.biomimetic-systems.com.

(a)

(b)

Fig. 7. Sensor formation. (a) Quad symmetric formation. (b) Wedge flank formation.

TABLE IV
Shooter Locations

Shooter Position	GPS-East (m)	GPS-North (m)
Shooter Position 1	283309	4709539
Shooter Position 2	283270	4709567
Shooter Position 3	283337	4709632

TABLE V
Ammunition

Weapon	Caliber	Weight (g)	Muzzle Velocity (m/sec)	Velocity at 183 m (m/sec)
Weapon 1	7.62×39 mm	124	721	543
Weapon 2	5.56×45 mm	55	988	702
Weapon 3	7.62×54 mm	181	823	668

three VM sensors (VM-blue), three SW sensors (SW-red), and three UGSs (UGS-green). The sensor pattern is an aggregate distribution of squad-level soldiers while on patrol, it spreads over 25 m front to back. The shooter position is marked by a red human figure, and the shot line is marked by a translucent red line.

For both sensor layouts, shots were fired from three different positions using three different weapons. Figure 7 also shows the three different shooter positions used for the experiment. As Fig. 7 indicates, shooter positions one and two are approximately 200 m from the sensor formation and shooter position three is about 300 m from the sensor formation. The GPS locations of the three shooter positions are given in Table IV. The three different weapons used for the experiment and details of the ammunition used in the weapons are given in Table V.[5] For each scenario/shooter position, 10 shots were fired using each weapon. Thus, a total of 180 shots were fired, 60 shots per weapon.

6.1. Results

This subsection presents the summary of experimental results obtained by implementing the fusion algorithm on the gunfire detection data. Before we proceed further, it is important to note that the sensor GPS accuracy level is much higher than the fused solution accuracy, and estimating the sensor position along with

[5] http://www.chuckhawks.com/rifle_ballistics_table.htm.

the shooter location and bullet trajectory does not improve the fused solution accuracy. This is clearly visible in both simulations presented in the previous section, where the fused solution accuracy is very close to the GPS measurement accuracy for the first simulation and the fused solution accuracy is much lower than the GPS accuracy for the second simulation. Besides, including the sensor location as well as the bullet trajectory within the fusion algorithm significantly increases the problem dimensionality and thus contributes to the computational cost. Therefore, the fusion approach used for the experiment does not try to estimate the bullet trajectory and sensor locations based on the results presented in Section 4.1. The sensor locations reported by the sensor GPS is taken as the absolute truth. Thus, the 2-*D* nonlinear least-squares problem associated with the sensor fusion is similar to that given in (27).

As mentioned earlier, the sensors are designed to provide these weights along with its relative shooter position estimates. These weights indicate the estimated accuracy level of the calculated range and bearing. From analyzing the experimental data, it was noticed that the weights provided by the sensors are inconsistent with the relative solution accuracy. This inconsistency is particularly visible in the case of outliers. Using these inconsistent weights in the fusion process would bias the fused solution toward an outlier. Thus, we implemented the fusion algorithm using three different weighting schemes. The first weighting scheme simply uses the weights provided by the sensors; this fusion scheme is denoted as "Fusion-SW (Fusion-Sensor Weights)." For the particular sensor under consideration, the sensor provided weights are obtained based on the signal-to-noise ratio.

The second weighting scheme involves calculating the weights based on the true error associated with the range and bearing estimates; this fusion scheme is denoted as "Fusion-TE (True Error)." For this weighing scheme, the difference between the measured range/bearing and the ground truth are first calculated. The square of these errors are then taken as the weight associated with the range and the bearing measurements. Note that this weighting scheme is not practical in reality since the ground truth is unknown. We use this weighting scheme strictly for comparative purposes. The third weighting scheme is the consistency-based weighing scheme presented in Subsection 4.2 and is denoted as "Fusion-CW (Fusion-Consistency Weights)."

Given next are the results obtained from implementing the fusion algorithm on experimental data. Five different fused solutions are presented per scenario/shooter location. These fused solutions correspond to i) individual best solution, ii) individual average solution, iii) Fusion-SW solution, iv) Fusion-TE solution, and v) Fusion-CW solution. Individual best and individual average solutions are obtained by selecting the best sensor solution or simply averaging the individual solutions across the nine sensors.

TABLE VI
Sensor Locations and Heading for Wedge Flank Formation

Sensor	GPS-East (m)	GPS-North (m)	Heading (deg)
SW1	283147	4709413	35
SW2	283134	4709443	40
SW3	283165	4709401	31
UGS1	283133	4709431	39
UGS2	283195	4709396	26
UGS3	283156	4709413	34
VM1	283127	4709432	40
VM2	283182	4709394	28
VM3	283184	4709384	26

6.1.1. Scenario 1: Wedge flank formation

The sensor locations and headings corresponding to the wedge flank formation are given in Table VI. After receiving the shot data, each sensor estimates the shooter location relative to its position. This relative solution, in terms of range and bearing, is then relayed to the central node along with the GPS measurements of the sensor locations and the sensor heading (see Table VI). Sensors also provide weights, which indicate the estimated accuracy level of the relative solution, along with its relative solution estimates. After receiving the measurements from the sensors, the central node combines the individual solutions to yield the fused solution.

Figure 8 shows the relative performance between the fusion schemes using the different weighting schemes mentioned previously. In Fig. 8(a), the fusion results obtained from consistency-based weighting scheme (Fusion-CW) is compared against the fusion results obtained from sensor-provided weighting scheme (Fusion-SW) and the individual average. Individual average is the simplest form of fusion, where the fused result is obtained by simply averaging the individual solutions. Figure 8(a) indicates that the fusion results obtained from consistency-based weighting scheme are within the 20 m error circle while the fusion results obtained from sensor-provided weighting scheme and the individual average are mostly outside the 20 m error circle. Figure 8(a) also indicates that the individual average estimates are strongly biased with a T_x-error of 20 m and a T_y-error of 10 m. This bias is clearly visible in the Fusion-SW and Fusion-CW results.

Figure 8(b) contains the histogram of the fusion error for scenario one, shooter position one. Besides the fusion results obtained using the three different weighting schemes mentioned earlier, Fig. 8(b) also contains the results from individual average and individual best. In the individual best approach, the fused solution is the one with the least error, i.e., most accurate. Note that this approach requires knowing the true shooter position a priori and thus it is not feasible in reality. It is important to note that the fusion results obtained from true error based weighting scheme (Fusion-TE) is more accurate than the individual best sensor as shown in Fig. 8(b).

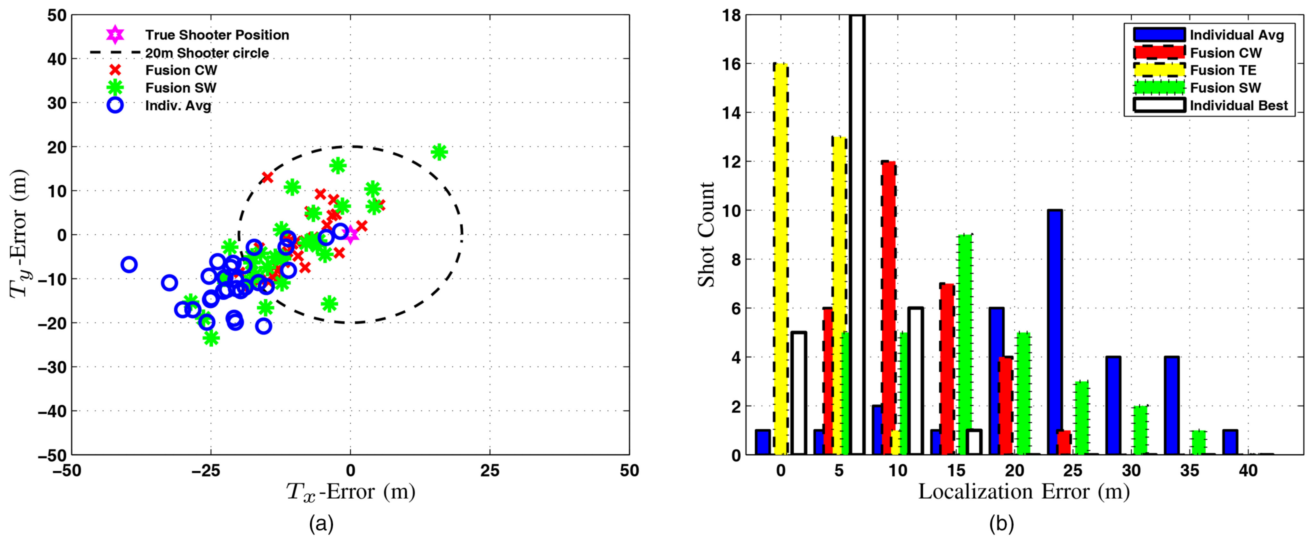


Fig. 8. Fusion result: Scenario 1, shooter position 1. (a) Fusion error. (b) Fusion error histogram.

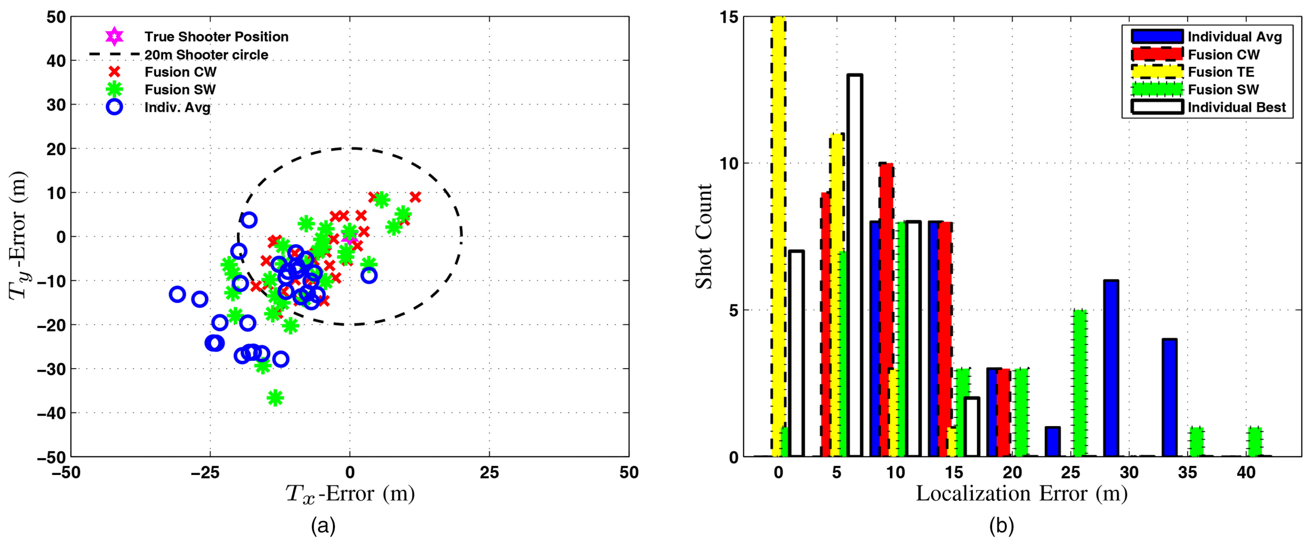


Fig. 9. Fusion result: Scenario 1, shooter position 2. (a) Fusion error. (b) Fusion error histogram.

Fused solution obtained from Fusion-TE yields a zero estimation error 16 out of 30 times while the individual best only has 5 out of 30 solutions with a zero estimation error. The fused solution obtained from individual average is the lest accurate with 20 out of 30 solutions with an estimation error of 25 m or higher. Compared to the individual average, the Fusion-SW yields a more accurate solution. In contrast to the results obtained for the numerical simulation, estimation errors are not Gaussian as indicated by Fig. 8(b) except for the error obtained from Fusion-TE.

Figure 9 shows the relative performance across the different fusion schemes for the scenario one, shooter position two. In Fig. 9(a), the fusion results obtained from Fusion-CW is compared against the results obtained from Fusion-SW and the individual average. Compared to shooter position one, these results are less biased, as indicated in Fig. 9(a). As shown in Fig. 9(a), the individual average is biased with a T_x and T_y-errors of approximately 8 m. Figure 9(a) also indicates that the fusion results obtained from Fusion-CW is within the 20 m error circle while the results obtained from Fusion-SW and the individual average are mostly outside the 20 m error circle. Figure 9(b) contains the histogram of the fusion error for scenario one, shooter position two. Figure 9(b) indicates that the fusion results obtained from Fusion-TE yields a perfect localization 50% of time, i.e., 15 shots out of 30 result in a fused solution with zero error. The fused results obtained from Fusion-SW contains two solutions with errors of 35 and 40 m. Clearly, the fusion results obtained from Fusion-TE is more accurate than the rest of the solutions, as shown in Fig. 9(b).

Figure 10 shows the relative performance across the different fusion schemes for the scenario one, shooter position three. Compared to previous two shooter positions, shooter position three yields the least accurate measurements due to the increased firing dis-

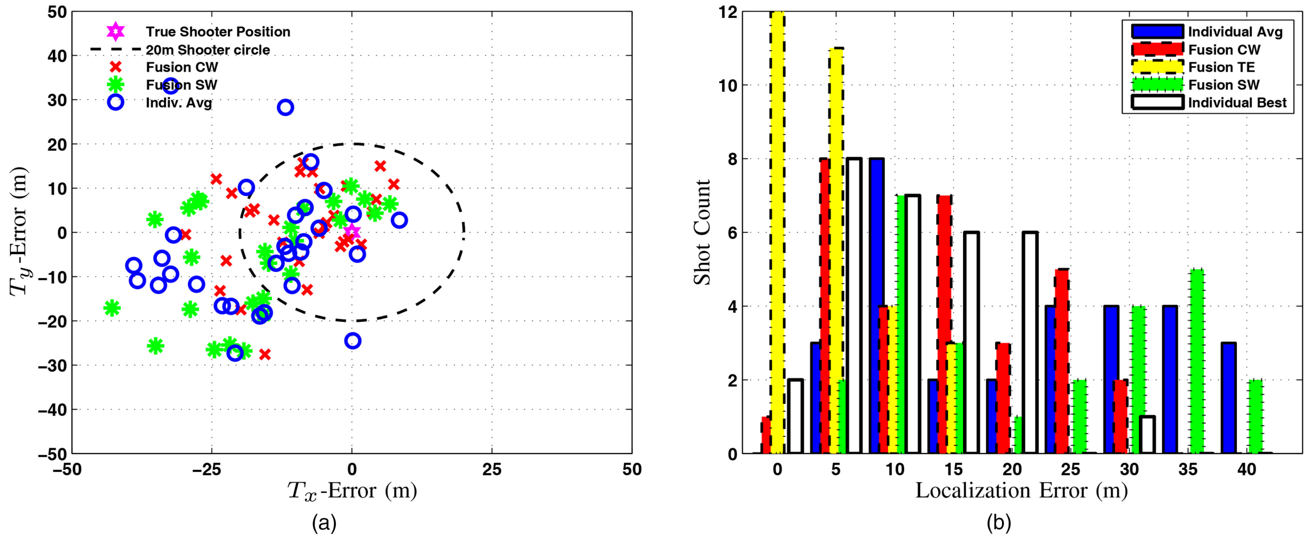


Fig. 10. Fusion result: Scenario 1, shooter position 3. (a) Fusion error. (b) Fusion error histogram.

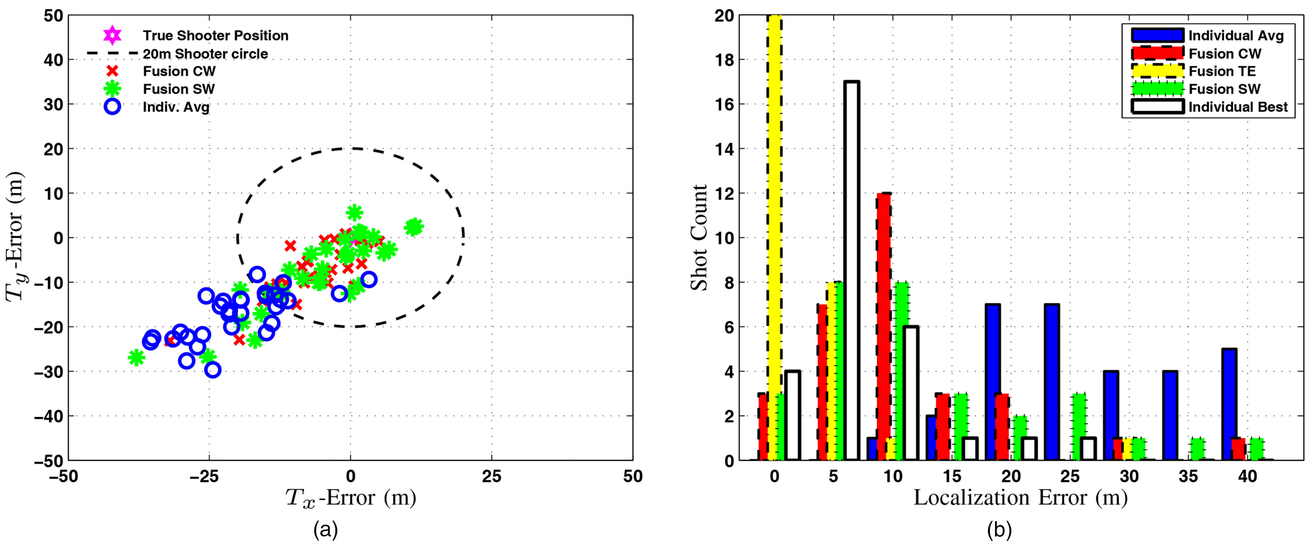


Fig. 11. Fusion result: Scenario 2, shooter position 1. (a) Fusion error. (b) Fusion error histogram.

tance of 300 m. Figure 10(a) compares the fusion results obtained from Fusion-CW against the results obtained from Fusion-SW and the individual average. Figure 10(a) indicates that the majority of fusion results obtained from Fusion-CW, as well as the results obtained from Fusion-SW and the individual average are outside the 20 m error circle. This degradation in performance compared to the previous two shooter positions might be due to the increased firing distance. Figure 10(b) contains the histogram of the fusion error for scenario one, shooter position three. Here also, the fusion results obtained from Fusion-TE is more accurate than the individual best sensor as shown in Fig. 10(b). Finally, note that the accuracy of the results from Fusion-SW is greatly influenced by the individual outliers while the results from Fusion-TE are insensitive to the outliers.

6.1.2. Scenario 2: Quad symmetric formation

This subsection presents the results obtained from scenario two, the quad symmetric sensor formation. Compared to previous scenario, the sensors are more clustered together in this scenario, and therefore, there is a higher level of consistency between the sensors. This higher consistency results in better localization accuracy, as indicated here. The sensor locations and headings correspond to the quad symmetric formation are given in Table VII. Here also, 30 shots were fired for each shooter position, 10 shots per weapon.

Figure 11 shows the relative performance across the fusion schemes using the different weighting schemes. In Fig. 11(a), the fusion results obtained from Fusion-CW are compared against the fusion results obtained from Fusion-SW and the individual average. Figure 11(a) indicates that the fusion results obtained from Fusion-CW and Fusion-SW are mostly within the 20 m

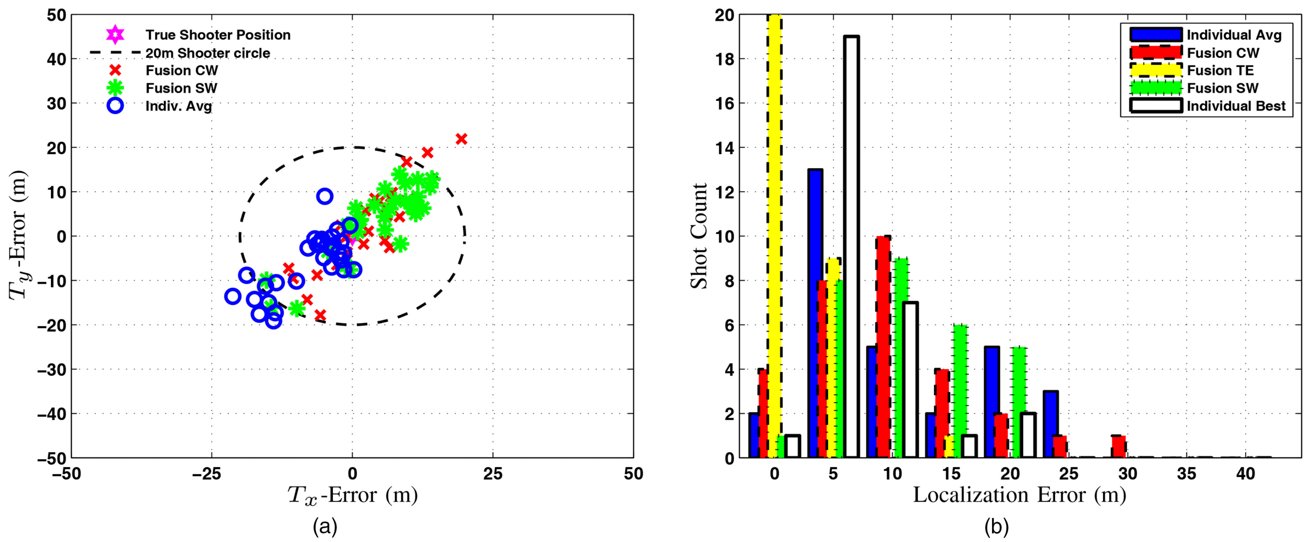


Fig. 12. Fusion result: Scenario 2, shooter position 2. (a) Fusion error. (b) Fusion error histogram.

TABLE VII
Sensor Locations and Heading for Quad Symmetric Formation

Sensor	GPS-East (m)	GPS-North (m)	Heading (deg)
SW1	283130	4709427	40
SW2	283129	4709434	39
SW3	283165	4709401	31
UGS1	283133	4709431	39
UGS2	283169	4709398	30
UGS3	283168	4709405	31
VM1	283127	4709431	40
VM2	283172	4709402	30
VM3	283177	4709395	29

error circle and they are more accurate compared to the individual average. Also note that Fig. 11(a) does not display the strong bias we observed in Fig. 8(a) and the majority of the fused results obtained from Fusion-CW and Fusion-SW shows a less than 10 m error.

Figure 11(b) contains the histogram of the fusion error for scenario two, shooter position one. Besides the fusion results obtained using the three different weighting scheme mentioned earlier, Fig. 11(b) also contains the results from individual average and individual best. Figure 11(b) indicates that the fusion results obtained from Fusion-TE yields a perfect localization two out of three time, i.e., 20 shots out of 30 shots results in a fused solution with zero error. Clearly, the fusion results obtained from Fusion-TE is more accurate than the individual best sensor as shown in Fig. 11(b). Also, note that the results obtained from Fusion-CW are more accurate compared to Fusion-SW, and both Fusion-CW and Fusion-SW yield better results compared to the individual average. Comparing Figs. 8(b) and 11(b) clearly indicates that the results obtained for the quad formation yield better results.

Figure 12 shows the relative performance across the different fusion schemes for the scenario two, shooter position two. In Fig. 12(a), the fusion results obtained from Fusion-CW are compared against the results obtained from Fusion-SW and the individual average. Figure 12(a) indicates that the fusion results obtained from Fusion-SW, Fusion-CW, and the individual average are mostly within the 20 m error circle or within the close proximity of the error circle. Figure 12(b) contains the histogram of the fusion error for scenario two, shooter position two. Here also, the histogram indicates that the fusion results obtained from Fusion-TE yields a perfect localization two out of three times, i.e., 20 shots out of 30 shots result in a fused solution with zero error. Clearly, the fusion results shown in Fig. 12 are more accurate compared to rest of the results presented here. This high level of accuracy is due to two reasons: i) the clustered quad symmetric sensor formation and ii) the bullet trajectory with sensors distributed on both sides of the trajectory to reduce the miss-distance.

Figure 13 shows the relative performance across the different fusion schemes for the scenario two, shooter position three. Figure 13(a), compares the fusion results obtained from Fusion-CW against the results obtained from Fusion-SW and the individual average. Figure 13(a) indicates that the fusion results obtained from Fusion-CW are mostly within or around the vicinity of the 20 m error circle while the results obtained from Fusion-SW and the individual average are outside the 20 m error circle. Figure 13(b) contains the histogram of the fusion error for scenario two, shooter position three. Note that the fusion results obtained from Fusion-TE are perfect more than 50% of the time and they are more accurate than the individual best sensor, as shown in Fig. 13(b). The performance degradation shown in Fig. 13 is similar to that obtained in Fig. 10 and is due to the increased firing distance compared to the previous two shooter positions. Also note that the performance degradation in Fig. 13 is slightly less than the one observed in Fig. 10 due to the quad symmetric sensor formation.

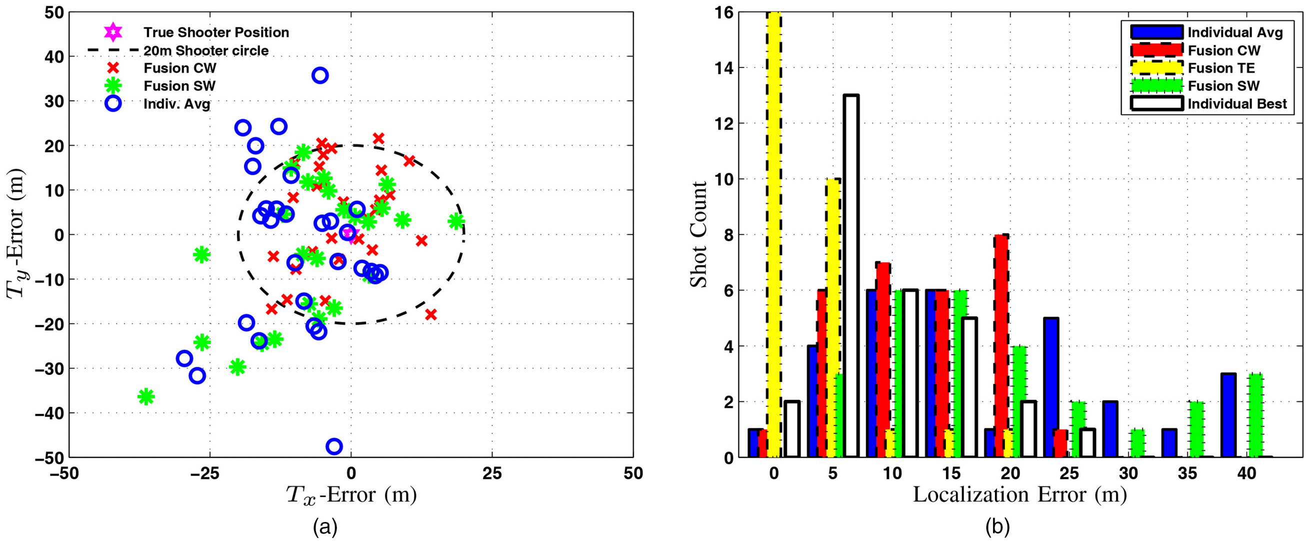


Fig. 13. Fusion result: Scenario 2, shooter position 3. (a) Fusion error. (b) Fusion error histogram.

TABLE VIII
Summary of Fusion Results

Scenario & Shooter Position	Fusion-TE Error (m)	Fusion-CW Error (m)	Fusion-SW Error (m)	Indiv. Avg Error (m)	Indiv. Best Error (m)
Scenario 1 Shooter 1	2.9	12.1	16.3	23.3	6.0
Scenario 1 Shooter 2	3.5	10.7	14.5	20.6	6.0
Scenario 1 Shooter 3	5.0	14.2	21.9	21.9	11.8
Scenario 2 Shooter 1	3.3	11.0	13.2	26.9	6.6
Scenario 2 Shooter 2	2.7	9.7	11.1	10.8	6.7
Scenario 2 Shooter 3	3.4	13.1	20.6	19.1	9.4

Given in Table VIII is the summary of average (averaged across 30 shots) localization error obtained for six different experiments using the five different fusion schemes explained earlier. As expected, the results obtained from Fusion-TE outperform the individual best, and on average the Fusion-CW yields better results compared to Fusion-SW. Also note that the Fusion-CW and Fusion-SW yield better results compared to the individual average except for scenario two, shooter positions two and three. For scenario two, shooter positions two and three, the results obtained from individual average are slightly better than Fusion-SW. This is due to the fact that the clustered sensors within the quad symmetric formation yield consistent measurements, which are equally distributed around the truth, and weighting them equally yields better results compared to using inconsistent weights. The consistency-based weighting scheme presented here is just one of the ad hoc approaches to develop synthetic weights. Numerous other schemes exist based on the consistency test that we are currently pursuing in an attempt to achieve the performance of Fusion-TE.

7. CONCLUSIONS

The shooter localization problem using a network of soldier-worn gunfire detection systems is considered here. This paper presents a fusion algorithm that utilizes the benefits of the sensor network layout of all the sensors within a small combat unit to help refine shooter localization accuracy. Main contributions of this work include (i) a detailed formulation of the fusion methodology and its performance analysis through numerical simulations; (ii) parameter reduction of the optimization problem and a consistency-based weighting scheme for the real-time implementation of the fusion algorithm; and (iii) detailed experimental results and the analysis of data. It is shown that the multi-sensor fusion algorithm for soldier-worn gunfire detection systems is essentially the weighted nonlinear least-squares algorithm, which can be easily implemented using the Gauss-Newton method. Since the GPS accuracy of the sensors is much higher compared to the shooter localization accuracy, it is also shown that accepting the GPS measurements as ground truth for the sensor locations and simply estimating the shooter location greatly reduce the dimensionality of the optimization problem and thus decrease the computational cost without sacrificing performance accuracy. The numerical results given in Section 5 indicate that the fusion algorithm is able to improve the localization accuracy by a factor of four compared to the simple averaged solution, if the underlying assumptions are valid and the weights associated with individual sensor locations are consistent. Despite the lack of consistency in the weights provided by the sensors, the fusion algorithm along with the proposed consistency-based weighting scheme is able to produce a fused solution twice as accurate as the simple individual average solution.

Though the proposed fusion approach was able to yield desirable results, there are several aspects of the proposed approach that can be further improved. Few of those features are (i) an improved weighting scheme that would yield a fused solution that approaches the accuracy obtained from the true error based weighting scheme, (ii) a mathematically rigorous method to quantify the uncertainties associated with the maximum likelihood estimates, and (iii) an investigation of the performance gain in fusing raw sensor measurements, such as the two direction of arrival angles and the time difference of arrival between the muzzle blast and the shockwave versus the relative shooter position.

ACKNOWLEDGMENT

This work is conducted in collaboration with the U.S. Army Natick Soldier Research Development & Engineering Center (NSRDEC) and the U.S. Army Armament Research, Development and Engineering Center (ARDEC). Authors would like to acknowledge Bruce Buckland from NSRDEC, Sachi Desai and George Cakiades from ARDEC for their support.

REFERENCES

[1] J. Ash, G. Whipps, and R. Kozick
Performance of shockwave-based shooter localization under model misspecification.
In *IEEE International Conference on Acoustics Speech and Signal Processing* (ICASSP), Dallas, TX, Mar. 2010, pp. 2694–2697.

[2] J. Bédard and S. Paré
Ferret: A small arms' fire detection system: Localization concepts.
In *Proceedings of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, vol. 5071, Orlando, FL, Apr. 2003, pp. 497–509.

[3] G. Cakiades, S. Desai, S. Deligeorges, B. E. Buckland, and J. George
Fusion solution for soldier wearable gunfire detection systems.
In *Proceedings of SPIE: Defence, Security, and Sensing*, Baltimore, MD, Apr. 23–27, 2012.

[4] J. L. Crassidis and J. L. Junkins
Optimal Estimation of Dynamic System.
Boca Raton, FL: Chapman & Hall/CRC, 2004, ch. 1.

[5] T. Damarla, L. Kaplan, and G. Whipps
Sniper localization using acoustic asynchronous sensors.
Sensors Journal, IEEE, **10**, 9 (Sept. 2010), 1469–1478.

[6] E. Danicki
The shock wave-based acoustic sniper localization.
Nonlinear Analysis: Theory, Methods & Applications, **65**, 5 (2006), 956–962.

[7] G. L. Duckworth, D. C. Gilbert, and J. E. Barger
Acoustic counter-sniper system.
In *Proceedings of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, vol. 2938, Boston, MA, Nov. 1997, pp. 262–275.

[8] P. Fishbane, S. Gasiorowicz, and S. Thornton
Physics for Scientists and Engineers, (2nd ed.).
Upper Saddle River, NJ: Prentice Hall, 1996, vol. 1, ch. 14, pp. 398–399.

[9] J. George, L. M. Kaplan, R. Kozick, and S. Deligeorges
Multi-sensor data fusion using soldier-worn gunfire detection systems.
In *Proceedings of MSS Battlespace Acoustic and Magnetic Sensors* (BAMS), Washington, D.C., Oct. 2011.

[10] J. George and L. Kaplan
Shooter localization using soldier-worn gunfire detection systems.
In *Proceedings of the 12th International Conference on Information Fusion* (FUSION), Chicago, IL, July 2011, pp. 1–8.

[11] L. M. Kaplan and Q. Le
On exploiting propagation delays for passive target localization using bearings-only measurements.
Journal of the Franklin Institute, **342**, 2 (Mar. 2005), 193–211.

[12] L. Kaplan, T. Damarla, and T. Pham
QoI for passive acoustic gunfire localization.
In *5th IEEE International Conference on Mobile Ad Hoc and Sensor Systems* (MASS), Atlanta GA, Sept.–Oct. 2008, pp. 754–759.

[13] R. Kozick, J. George, L. M. Kaplan, and S. Deligeorges
Centralized fusion of acoustic sensor data for gunfire localization.
In *Proceedings of MSS Battlespace Acoustic and Magnetic Sensors* (BAMS), Washington, D.C., Oct. 2011.

[14] R. Kozick, G. Whipps, and B. Sadler
Accuracy and tradeoff analysis of sniper localization systems with a network of acoustic sensors.
In *Proceedings MSS Battlespace Acoustics & Seismic Sensing, Magnetic & Electric Field Sensors*, Laurel, MD, Aug. 2009.

[15] P. Kuckertz, J. Ansari, J. Riihijarvi, and P. Mahonen
Sniper fire localization using wireless sensor networks and genetic algorithm based data fusion.
In *IEEE Military Communications Conference* (MILCOM 2007), Orlando, FL, Oct. 2007, pp. 1–8.

[16] A. Ledeczi, P. Volgyesi, M. Maroti, G. Simon, G. Balogh, A. Nadas, B. Kusy, S. Dora, and G. Pap
Multiple simultaneous acoustic source localization in urban terrain.
In *Fourth International Symposium on Information Processing in Sensor Networks* (IPSN), Los Angeles, CA, Apr. 2005, pp. 491–496.

[17] A. Ledeczi, A. Nadas, P. Volgyesi, G. Balogh, B. Kusy, J. Sallai, G. Pap, S. Dora, K. Molnar, M. Maroti, and G. Simon
Countersniper system for urban warfare.
ACM Transactions on Sensor Networks, **1** (Nov. 2005), 153–177.

[18] P. Li and X. Ma
Robust acoustic source localization with tdoa based ransac algorithm.
In *Emerging Intelligent Computing Technology and Applications*, ser. Lecture Notes in Computer Science, Springer Berlin/Heidelberg, 2009, vol. 5754, pp. 222–227.

[19] D. Lindgren, O. Wilsson, F. Gustafsson, and H. Habberstad
Shooter localization in wireless sensor networks.
In *Proceedings of the 12th International Conference on Information Fusion* (FUSION), Seattle, WA, July 2009, pp. 404–411.

[20] D. Lindgren, O. Wilsson, F. Gustafsson, and H. Habberstad
Shooter localization in wireless microphone networks.
EURASIP Journal on Advances in Signal Processing, **2010**, 6 (Feb. 2010), 1–25.

[21] J. Liu, J. Reich, and F. Zhao
Collaborative in-network processing for target tracking.
EURASIP Journal on Applied Signal Processing, **2003**, 4 (Jan. 2003), 378–391.

[22] R. Maher
Modeling and signal processing of acoustic gunshot recordings.
In *Digital Signal Processing Workshop, 12th-Signal Processing Education Workshop*, 4th, Teton National Park, WY, Sept. 2006, pp. 257–261.

[23] T. Makinen and P. Pertila
Shooter localization and bullet trajectory, caliber, and speed estimation based on detected firing sounds.
Applied Acoustics, **71**, 10 (2010), 902–913.

[24] M. Maroti, G. Simon, A. Ledeczi, and J. Sztipanovits
Shooter localization in urban terrain.
Computer, **37**, 8 (Aug. 2004), 60–61.

[25] J. M. Mendel
Lessons in Digital Estimation Theory.
Englewood Cliffs, NJ: Prentice-Hall, Inc., 1987.

[26] J. Millet and B. Balingand
Latest achievements in gunfire detection systems.
In *Meeting Proceedings RTO-MP-SET-107, Battlefield Acoustic Sensing for ISR Applications*, NeuillysurSeine, France, Oct. 2006, pp. 1–14.

[27] J. A. Nelder and R. Mead
A simplex method for function minimization.
The Computer Journal, **7**, 4 (1965), 308–313.

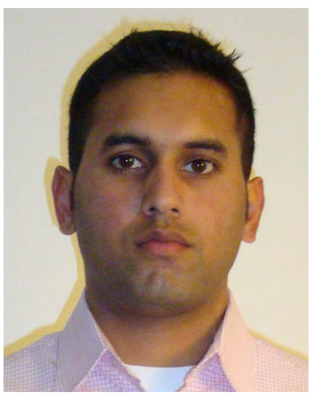
[28] R. Stoughton
Measurements of small-caliber ballistic shock waves in air.
The Journal of the Acoustical Society of America, **102**, 2 (Aug. 1997), 781–787.

[29] R. B. Stoughton
SAIC SENTINEL acoustic counter-sniper system.
In *Proceedings of Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, vol. 2938, Boston, MA, Nov. 1997, pp. 276–284.

[30] R. Osborne, III, Y. Barshalom, J. George, and L. Kaplan
Statistical efficiency of simultaneous target and sensors location with position dependent noise.
In *Proceedings of SPIE: Defence, Security, and Sensing*, Baltimore, MD, Apr. 23–27, 2012.

[31] G. Vakulya and G. Simon
Fast adaptive acoustic localization for sensor networks.
IEEE Transactions on Instrumentation and Measurement, **60**, 5 (May 2011), 1820–1829.

[32] P. Volgyesi, G. Balogh, A. Nadas, C. B. Nash, and A. Ledeczi
Shooter localization and weapon classification with soldier-wearable networked sensors.
In *Proceedings of the 5th International Conference on Mobile Systems, Applications and Services*, New York, NY, June 2007, pp. 113–126.



Jemin George received his B.S. (2005), M.S. (2007), and Ph.D. (2010) in aerospace engineering from the State University of New York at Buffalo.

In 2008, he was a summer research scholar with the U.S. Air Force Research Laboratory's Space Vehicles Directorate at Kirtland Air Force Base in Albuquerque, NM. He was a National Aeronautics and Space Administration Langley Aerospace Research Summer Scholar with the Langley Research Center in 2009. From 2009–2010 he was a research fellow with the Stochastic Research Group, Department of Mathematics, Technische Universitat Darmstadt, Darmstadt, Germany. He is currently with the Networked Sensing & Fusion Branch of the U.S. Army Research Laboratory. His principal research interests include stochastic systems, control theory, nonlinear filtering, information fusion, and target tracking.

Lance M. Kaplan received his B.S. degree with distinction from Duke University, Durham, NC, in 1989 and his M.S. and Ph.D. degrees from the University of Southern California, Los Angeles, in 1991 and 1994, respectively, all in electrical engineering.

From 1987–1990, he worked as a technical assistant at the Georgia Tech Research Institute. He held a National Science Foundation Graduate Fellowship and a USC Dean's Merit Fellowship from 1990–1993, and worked as a research assistant in the Signal and Image Processing Institute at the University of Southern California from 1993–1994. Then, he worked on staff in the Reconnaissance Systems Department of the Hughes Aircraft Company from 1994–1996. From 1996–2004, he was a member of the faculty in the Department of Engineering and a senior investigator in the Center of Theoretical Studies of Physical Systems (CTSPS) at Clark Atlanta University (CAU), Atlanta, GA. Currently, he is a researcher in the Networked Sensing and Fusion branch of the U.S. Army Research Laboratory. Dr. Kaplan serves as editor-in-chief for the *IEEE Transactions on Aerospace and Electronic Systems* (AES). In addition, he also serves on the Board of Governors of the IEEE AES Society and on the Board of Directors of the International Society of Information Fusion. He is a three time recipient of the Clark Atlanta University Electrical Engineering Instructional Excellence Award from 1999–2001. His current research interests include signal and image processing, automatic target recognition, information/data fusion, and resource management.

High-level Information Fusion: An Overview

PEK HUI FOO
GEE WAH NG

Data and information fusion (DIF) involves a process of combining data and information from multiple inputs. The purpose is to derive enriched information compared to that obtained from each individual input. DIF techniques were first introduced to the research community in the 1970s. The scope of applications that use DIF techniques for problem-solving has extended tremendously from the military arena at the initial stage to many non-military sectors at present. The Joint Directors of Laboratories data fusion (JDL DF) model is possibly the most widely used model for data fusion. In this functional model, the hierarchical process of data and information fusion comprises two stages, the low-level fusion processes and the high-level fusion processes. After years of intensive research that is mainly focused on low-level information fusion (IF), the focus is currently shifting towards high-level information fusion. Compared to the increasingly mature field of low-level IF, theoretical and practical challenges posed by high-level IF are more difficult to handle. Contributing factors include the lack of: well-defined spatio-temporal constraints on relevant evidence, well-defined ontological constraints on relevant evidence and suitable models for causality. In this survey paper, we first review process models proposed for data and information fusion over the past few decades. Next, we present an overview of existing work on high-level information fusion, based on the fusion levels of the current JDL DF model. Finally, we discuss relevant application areas and topics with potential for further research.

Manuscript received January 06, 2012; revised June 22, 2012; released for publication September 17, 2012.

Refereeing of this contribution was handled by Benjamin Slocumb.

Authors' address: DSO National Laboratories, 20 Science Park Drive, Singapore 118230, Singapore.

1557-6418/13/$17.00

1. INTRODUCTION

Data and information fusion (DIF) involves a process of combining data from multiple inputs (from one or more sources such as sensors and textual reports).[1] The aim is to obtain information that is better (more useful and meaningful) than that would be derived from each of the sources individually (that is, without fusing). DIF is emerging as an important field of multidisciplinary study [113, 316]. This is due to increase in data and information flow, as well as improvement in communication, computing and sensor technology. The first applications of DIF techniques were in the military arena [177, 179, 455]. The use of DIF techniques for problem-solving has extended to many non-military applications in the commercial and industrial sectors [177, 179, 199, 228].

In general, data and information fusion can provide enhancement to the outcomes of processes for solving various application problems. Some advantages of carrying out DIF include [316]:

- improvement in the accuracy of data, as well as reduction in uncertainty and ambiguity within data, and
- improvement in situation awareness (SAW) and inference that lead to better decision making.

The main objective of this paper is to provide a useful aid to researchers in the field of data and information fusion, through an extensive (albeit non-exhaustive) literature survey. We review existing models for DIF, point to salient publications, and discuss relevant application domains and topics for further research. It is not our intention (and hence, beyond the scope of this paper) to critique or evaluate (a) the research topics presented, or (b) research in the field.

1.1. Structure of the Paper

The remainder of this paper is as follows. In Section 2, we review process models proposed for data and information fusion over the past few decades. Section 3 presents a discussion on the Joint Directors of Laboratories data fusion (JDL DF) model, one of the most widely used models to define the levels of the hierarchical process of data and information fusion. The JDL DF model has been revised and extended several times since it was first proposed. In the current version, the data fusion process comprises five levels, which are categorized into two stages, the low-level fusion processes and the high-level fusion processes. The low-level fusion processes support data pre-processing, target discrimination and target tracking. The high-level fusion processes support

[1] Generally, data entities (for example, raw sensor observations) have limited predefined attributes; information entities have assigned attributes with some logical relationships between them. Here, the terms "data fusion" and "information fusion" are used interchangeably. The term "sensor fusion" refers to the specific case of DIF in which each data/information source is a sensor.

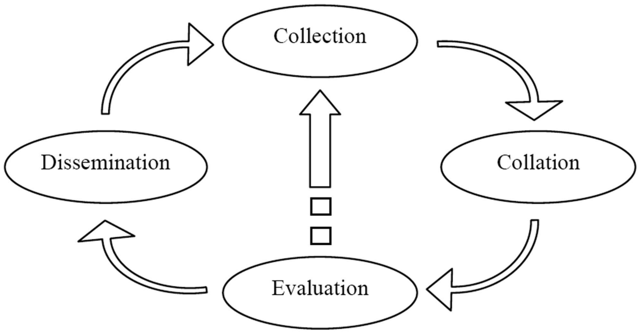


Fig. 1. The Intelligence Cycle [27].

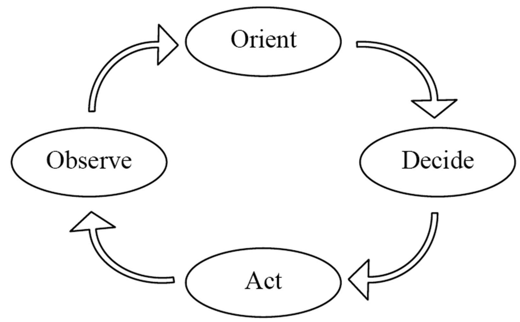


Fig. 2. The OODA Loop [325].

situation assessment, threat (or impact) assessment and process refinement [232]. Section 4 focuses on high-level information fusion, a field that is gaining much interest within the DIF research community in the recent years. An overview of some existing literature pertaining to the higher levels of fusion in the JDL DF model is also provided. Section 5 presents some application areas of high-level information fusion. Section 6 summarizes this work and considers potential topics for further research.

2. REVIEW OF DATA FUSION MODELS

Over the last few decades, many process models have been proposed for DIF [179, 325]. Some of the data fusion models introduced over the years are briefly reviewed in the following subsections. More details on these models are found in the respective sources and the cited references therein.

2.1. Data Fusion Models Introduced in the 1980s

In the 1980s, the Intelligence Cycle [27, 145], the Boyd Control Loop [106, 325, 346] and the Joint Directors of Laboratories data fusion model [59, 176, 280, 416, 422, 423] were developed.

2.1.1. The Intelligence Cycle

In the Intelligence Cycle [27, 145], the intelligence process is described as a cycle applicable for modeling the data fusion process. This model consists of four phases (shown in Fig. 1): *collection* (deployment of assets such as electronic sensors or human derived sources to obtain raw intelligence data, which is usually presented in the form of an intelligence report with a high abstraction level); *collation* (analysis, comparison and correlation of associated intelligence reports); *evaluation* (fusion and analysis of collated intelligence reports) and *dissemination* (distribution of the fused intelligence to users who use the information for decision making).

2.1.2. The Boyd Control Loop

The Boyd Control Loop [106, 325, 346], also known as the Observe, Orient, Decide, and Act (OODA) Loop, was first proposed to model the military command and control (C2) process. It comprises four phases (see Fig. 2): *Observe* (gather information from the environment); *Orient* (gain situation awareness and perform situation/threat assessment based on the information gathered); *Decide* (respond to situation and work out follow-up actions) and *Act* (execute the planned response/action). The emphasis is placed on shortening the cycle to perform the Observe to Act loop, to the extent that the opponent cannot respond in time to carry out countermeasure, thus gaining superiority in the battlespace. This model is well received by military commanders and decision makers.

2.1.3. The JDL Data Fusion Model

The commonly used JDL DF model was proposed for categorizing data fusion related functions. A detailed discussion on this model is given in Section 3.

2.2. Data Fusion Models Introduced in the 1990s

During the 1990s, the Waterfall model [132, 145], the Dasarathy model [110, 111], the Visual Data-Fusion (VDF) model [59, 227], the Omnibus model [27] and the Endsley model [59, 127, 128] were proposed.

2.2.1. The Waterfall Model

The Waterfall model [132, 145] consists of three levels of representation (shown in Fig. 3):

- Level 1 (sensing, signal processing)—proper transformation of raw data is carried out to provide necessary information about the surroundings, via the use of models (based on experimental analysis or on physical laws) of the sensors and where possible, of the measured phenomena;
- Level 2 (feature extraction, pattern processing)—with the aim of minimizing the data content and maximizing the information delivered, feature extraction and fusion are done to produce a list of estimates and their associated probabilities (and beliefs), which provide a symbolic level of inference about the data;
- Level 3 (situation assessment, decision making)—relationships are established between objects and events; based on the repository of information available and the human interaction, possible routes of action are assembled.

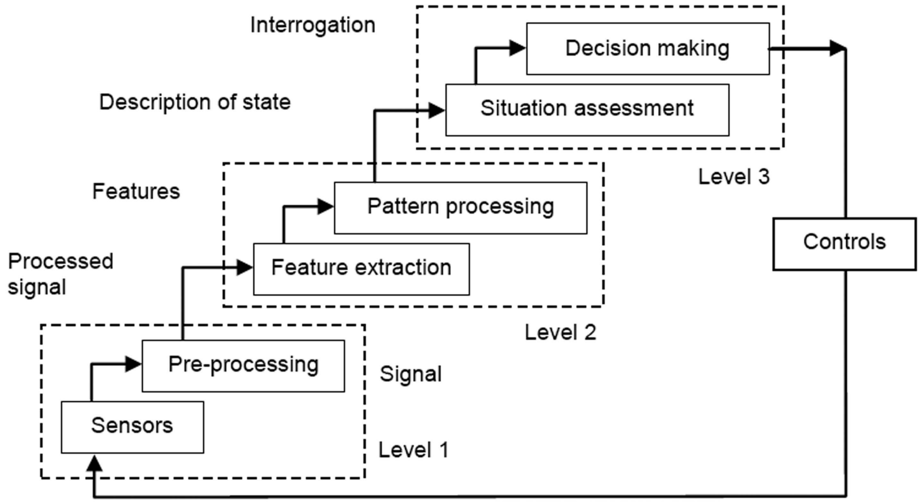


Fig. 3. The Waterfall model [132].

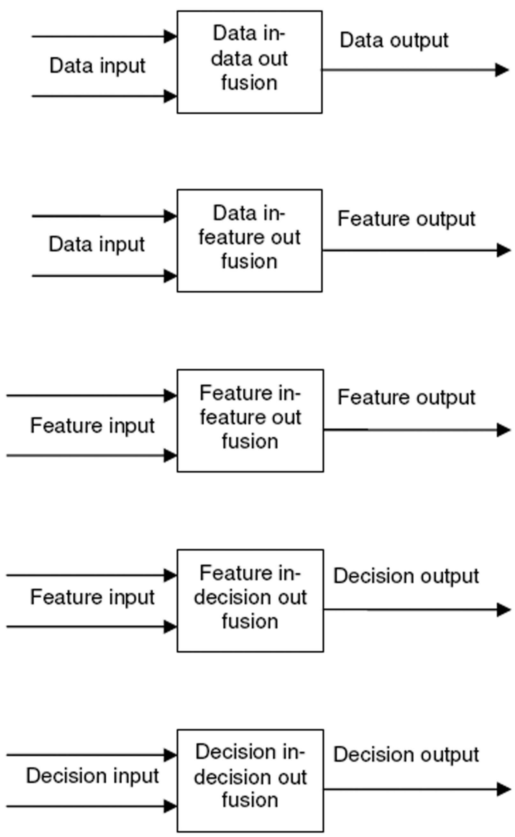


Fig. 4. The Dasarathy model [110].

The focus is on the processing functions at the lower levels. The lack of explicit depiction of the feedback appears to be the major limitation of this model.

2.2.2. The Dasarathy Model

The data fusion process has been commonly identified as a hierarchy with three general levels of abstraction: *data* (more specifically, sensor data), *features* (intermediate-level information) and *decisions* (symbols or belief values). Dasarathy [110, 111] pointed out that fusion may occur both within and across these levels. The Dasarathy model was proposed to expand the preceding hierarchy of fusion into five categories of input-output based fusion (corresponding analogues stated within parentheses): *Data In-Data Out* fusion (data-level fusion); *Data In-Feature Out* fusion (feature selection and feature extraction); *Feature In-Feature Out* fusion (feature-level fusion); *Feature In-Decision Out* fusion (pattern recognition and pattern processing) and *Decision In-Decision Out* fusion (decision-level fusion). This model is based on data fusion functions (illustrated in Fig. 4) instead of tasks and may be incorporated in each of the fusion activities.

2.2.3. The Visual Data-Fusion Model

The Visual Data-Fusion model (see Fig. 5) was proposed by Karakowski [59, 227] as an extension of the JDL DF model, with a human participant added integrally. It has the following advantages [59]:

- maximization of relevant information with minimal display of information;
- ability to provide increasingly sophisticated problem queries, in addition to tailor information fusion (IF) system capabilities for use by all skill levels of users;
- problem-driven system that relates to user's needs directly, through response to his personal perception of the problem situation.

The following premises are embodied in the VDF model [59]:

- the human is a central participant in information fusion, a creative problem-solving process;
- information derived from the fusion process that is visualized by the human is primarily used to help him gain fuller perception, as well as possible approaches towards solving the problem;
- imagery is used as the perceptual transport for user visualization, in order to minimize the amount of information required by the human to solve the problem.

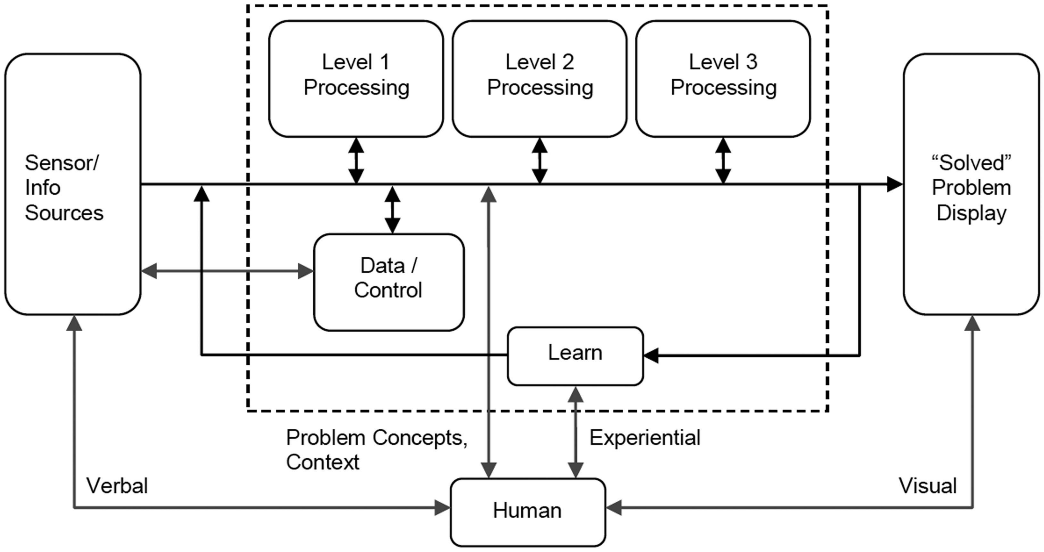


Fig. 5. The Visual Data-Fusion model [59].

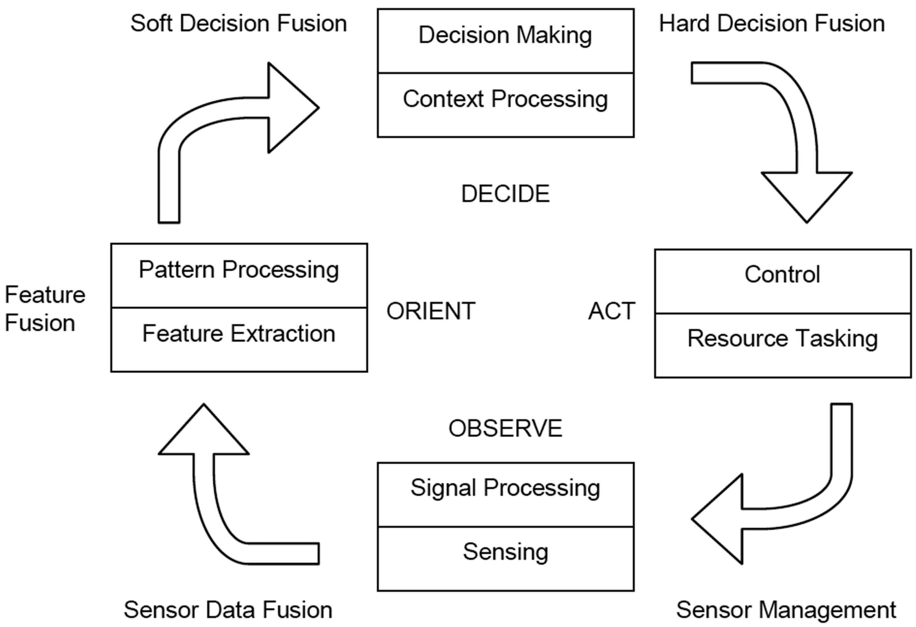


Fig. 6. The Omnibus model [179].

Basic VDF models are used as building-block elements for visual situation awareness and distributed VDF processes. More details on these research topics can be found in [59].

2.2.4. The Omnibus Model

The Omnibus model was proposed by Bedworth and O'Brien [27] as a unification of the Intelligence Cycle, the JDL DF model, the OODA Loop, the Dasarathy model and the Waterfall model. Properties of this model include: explicit feedback; acknowledgement of the *loop within loop* concept; retention of the general structure of the OODA Loop; incorporation of the fidelity of representation expressed by the Waterfall model into each of its four main modules and explicit indication of points in the processes where fusion may take place. Figure 6 presents the layout of this model.

2.2.5. The Endsley Model

The Endsley model [59, 127, 128] (shown in Fig. 7) is widely used to model situation awareness (see Section 4.2.1). It is a cognitive model and uses a general definition of situation awareness that is applicable across many domains: "Situation awareness is the perception of the elements in the environment within a volume of time and space, the comprehension of their meaning, and the projection of their status in the near future." The three hierarchical phases of the definition

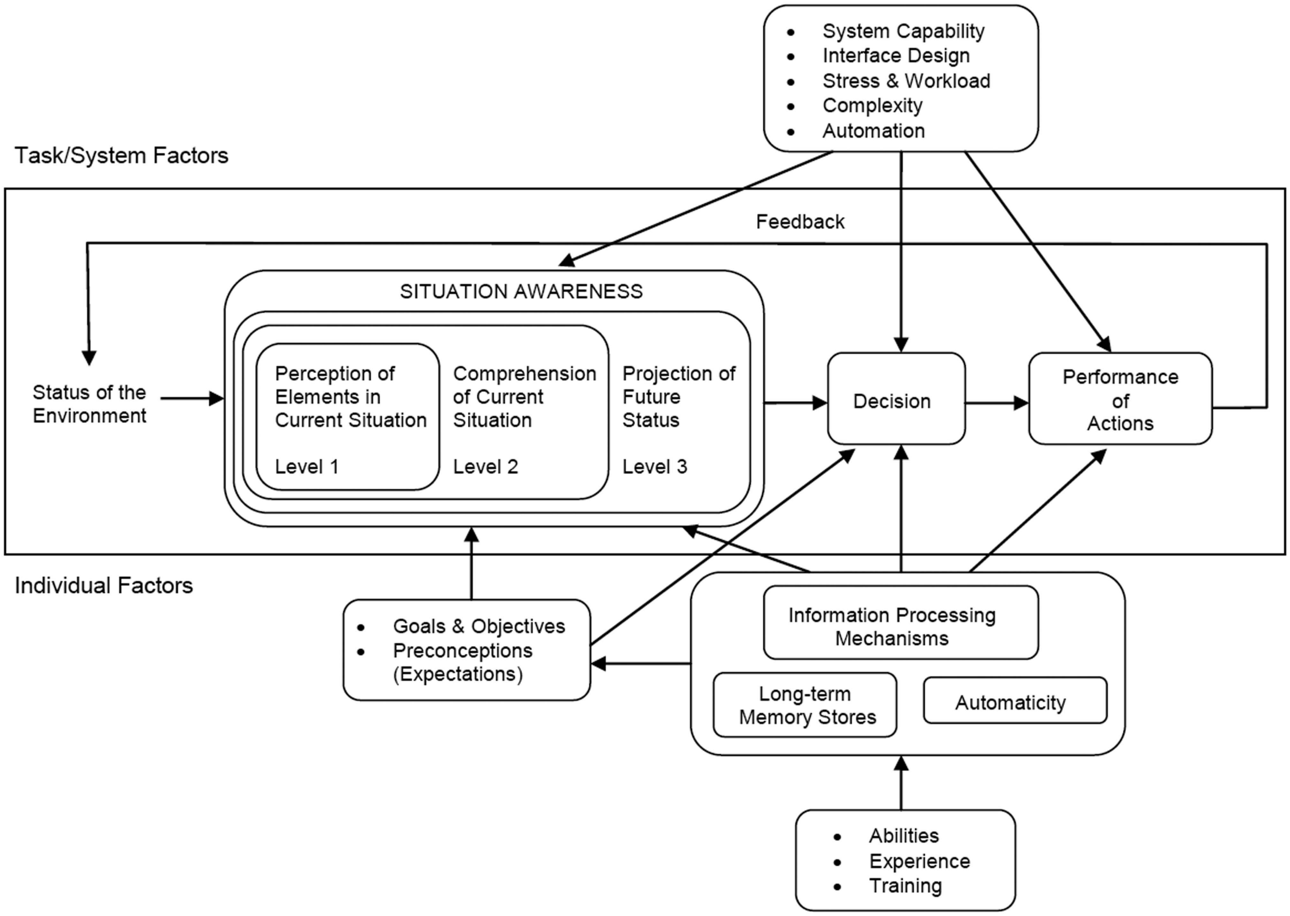


Fig. 7. Endsley's SAW model [127, 128].

are [127, 128]:

- Level 1 SAW (*Perception* of the elements in the environment)—perceive status, attributes and dynamics of relevant elements in the environment;
- Level 2 SAW (*Comprehension* of the current situation)—based on a synthesis of disjoint Level 1 elements, includes perceiving and attending to information, as well as integrating multiple pieces of information and a determination of their relevance to the operator goals;
- Level 3 SAW (*Projection* of future status)—ability to forecast/anticipate future situation events and dynamics, which is achieved through knowledge of status and dynamics of the elements and comprehension of the situation (both Levels 1 and 2 SAW), allows for timely decision making.

2.3. Data Fusion Models Introduced in the 2000s

The following data fusion models have been proposed in the 2000s:

- the Object-Centered information fusion model [236],
- the Extended OODA model [399],
- the Transformation of Requirements for the Information Process (TRIP) model [179, 272],
- the Unified data fusion (λJDL) model [59, 257],
- the Dynamic OODA Loop [65],
- the JDL-User model [48].

2.3.1. The Object-Centered Information Fusion Model

Kokar, et al. [236] introduced a fusion process reference model based on object-oriented design principles. The proposed model addressed essential issues on the design of data fusion systems with a top-down approach. Formal methods were adopted for model analysis at the design stage. They also discussed the need to develop psychological theories related to human-computer interaction (HCI). Research in this area was required for facilitating the proper integration of human and computer objects by fusion system designs based on the proposed object-oriented model.

2.3.2. The Extended OODA Model

Shahbazian, et al. [399] proposed the Extended OODA model which enables multiple concurrent and potentially interacting data fusion processes. This model can be applied to obtain a high-level functional decomposition of a system that uses data fusion for decision making. Each high-level function is examined in terms of the OODA decision loop and can be further decomposed and evaluated with respect to each OODA phase.

The Extended OODA model (see Fig. 8) has some properties that are consistent with those of several pre-

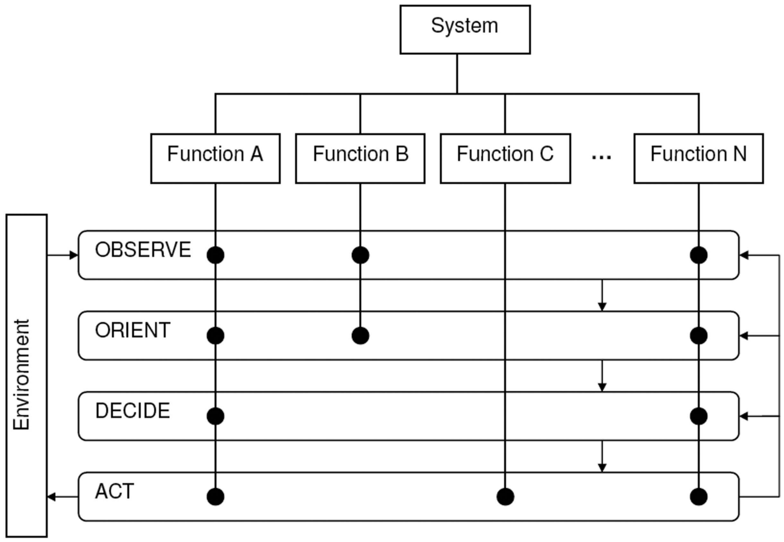


Fig. 8. The Extended OODA model [399].

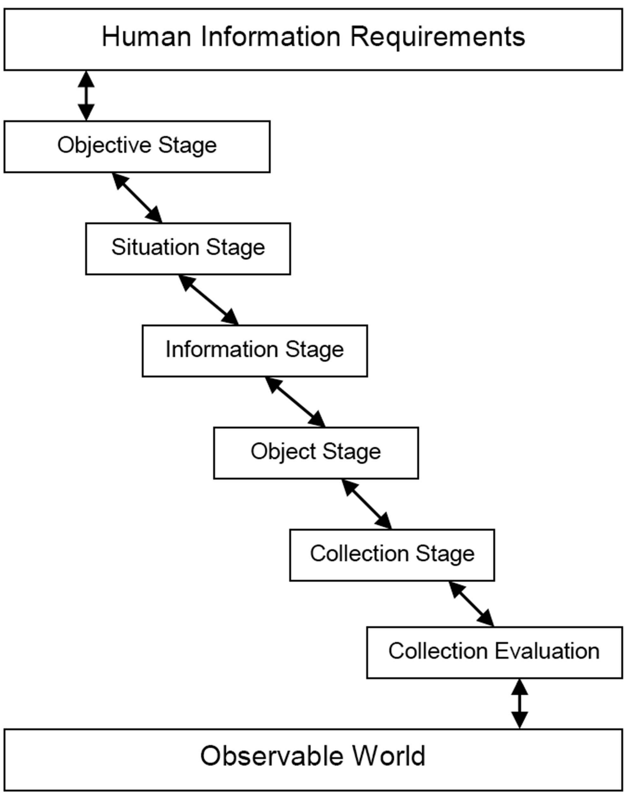


Fig. 9. The TRIP model [179].

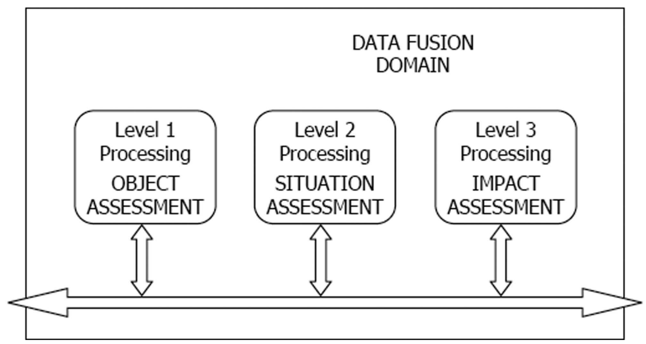


Fig. 10. The λJDL model [257].

ceding models (stated within parentheses): closes the loop between the decision making and its surroundings (OODA Loop); has increasing level of abstraction for information processing in each level (JDL DF model) and provides the *loop within loop* capability (Omnibus model).

2.3.3. The TRIP Model

The TRIP model [179, 272] (depicted in Fig. 9) was developed with the purpose of understanding a tactical commander's transformation of information needs to task assignment of sensor resources. The developers stated the following goals that they aimed to accomplish with this model [179]:

- describe the process for developing collection tasks from information requirements;
- understand relationships between collection management and the situation estimation process;
- understand where the *human in the loop* is required;
- understand the internal and external drivers for the intelligence, surveillance, and reconnaissance process.

Identification of processing functions and the detailed information interfaces between them was attempted. A link between human information requirements and data collection was provided by this model.

2.3.4. The Unified Data Fusion (λJDL) Model

The λJDL model [59, 257] (also known as the deconstructed JDL DF model), a revision of the JDL DF model (the version proposed in [423]), used the following definitions for its fusion levels (see Fig. 10):

- Level 1 (identification of objects from their properties)—*object fusion*: process of utilizing one or more data sources over time to assemble a representation of objects of interest in an environment;

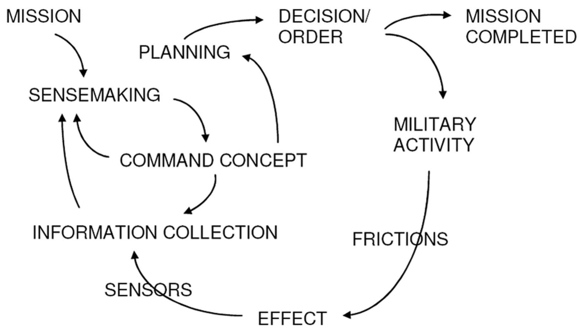


Fig. 11. The Dynamic OODA Loop [65].

object assessment: stored representation of objects obtained through object fusion;

- Level 2 (identification of relations between these objects)—*situation fusion*: process of utilizing one or more data sources over time to assemble a representation of relations of interest between objects of interest in an environment;
  situation assessment: stored representation of relations between objects obtained through situation fusion;
- Level 3 (identification of the effects of these relationships between these objects)—*impact fusion*: process of utilizing one or more data sources over time to assemble a representation of effects of situations in an environment, relative to user intentions;
  impact assessment: stored representation of effects of situations obtained through impact fusion.

The model was proposed for the development of a data fusion system for fusing three distinct types of processes that involved both humans and machines:

- psychological processes (human-related),
- technological processes (machine-related),
- integration processes (interaction between the psychological and technological processes).

The model could be applied to different aspects of the data fusion problem, depending on the different interpretations of the model components (object, situation, impact) obtained from the different combinations of the above processes.

2.3.5. The Dynamic OODA Loop

There exist criticisms that the OODA Loop fails to capture the dynamic nature of decision making in the military command and control process, as it has a limited focus on faster decisions [65]. The Dynamic OODA Loop (shown in Fig. 11) was proposed as a generic model of military command and control, based on concepts from the OODA Loop and cybernetic models of C2.

This model provides the identification of functions essential for effective C2. The problem of handling delays in C2, a form of dynamic decision making, is also dealt with. The required functions are: *sensemaking* (understanding of the current mission/situation in terms of what can be done); *command concept* (commander's overall concept of the operation); *planning* (translation of the command concept into decisions/orders); *information collection* (guided by the command concept) and *decision* (commitment to a course of action (COA)).

Other modifications of the OODA Loop include the M-OODA Loop [370] and the C-OODA Loop [66].

2.3.6. The JDL-User Model

Discussion on the JDL-User model, which was proposed to extend the JDL DF model to support a *human-in-the-loop* decision process, is deferred to Section 4.4.

3. THE JDL DATA FUSION MODEL

The original JDL DF model (shown in Fig. 12) was created by the JDL Data Fusion Group of the United States Department of Defense [176]. It is a functional model developed with the aim of facilitating communication, comprehension, coordination and cooperation among diverse data fusion communities to identify and solve problems to which data fusion can be applied.

The first revision of the initial JDL DF model was proposed by Steinberg, et al. [423]. They broadened the definitions of fusion concepts and functions beyond the original focus on military and intelligence problems, as well as described the need for an approach to the standardization of an engineering design methodology for fusion processes. They also proposed to refine definitions for the fusion "levels" characterized in the original JDL DF model as follows [423]:

- Level 0 (Sub-Object Data Assessment)—estimation and prediction of observable states of signals or features;
- Level 1 (Object Assessment)—estimation and prediction of entity states based on data association, as well as continuous and discrete state estimation;
- Level 2 (Situation Assessment)—estimation and prediction of relationships among entities;
- Level 3 (Impact Assessment)—estimation and prediction of effects of entities' actions on goals/missions;
- Level 4 (Process Refinement)—an element of Resource Management that encompasses adaptivity in the data collection and fusion processes to support mission objectives.

Figure 13 shows this revised version of the JDL DF model, which included the introduction of a "Level 0" to the original model. The five fusion levels were categorized into the low-level fusion process (Levels 0 and 1) and the high-level fusion process (Levels 2 to 4) [232, 316].

Other recent revisions/variants of the JDL DF model include the State Transition Data Fusion (STDF) model [258–260], the ProFusion2 (PF2) model [347] and the Ground C4-ISR Assessment Model (GCAM) [306].

3.1. Proposed Extension/Revision

The JDL DF model accounts for automatic machine processing, but not for human processing. To address

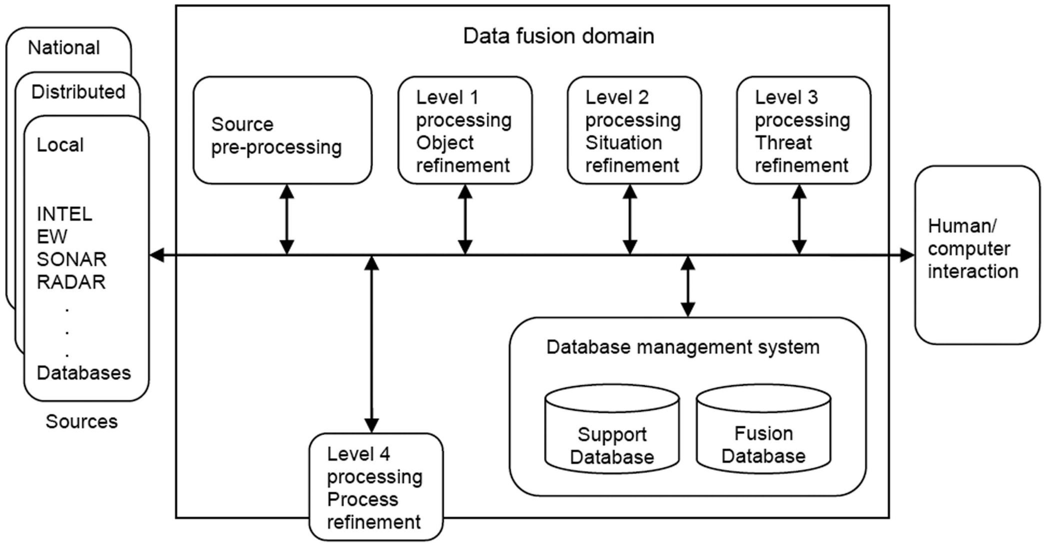


Fig. 12. JDL DF model [176].

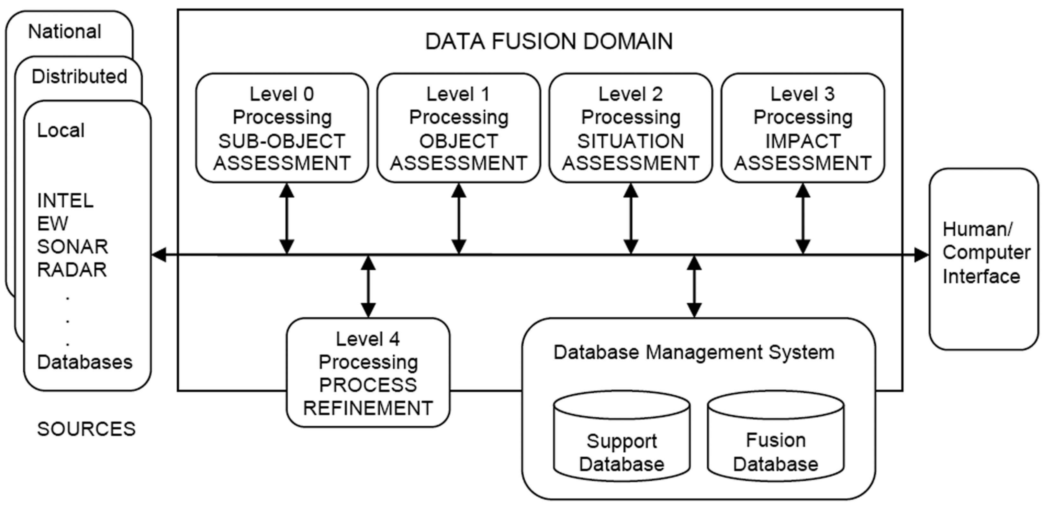


Fig. 13. Revised JDL DF model [423].

issues related to extending the human capabilities within the fusion process, the concept of Level 5 data fusion process was first introduced by Hall, et al. [181] and subsequently, in an independent work by Blasch and Plano [48]. In both works, the authors asserted the need to acknowledge functions necessary for supporting a human-in-the-loop decision process. More details on Level 5 processing are discussed in Section 4.4.

More recently, another revision to the JDL DF model (illustrated in Fig. 14) was suggested by Steinberg and Bowman [422]. The refinement involved a reexamination of the JDL DF level structure. The data fusion levels were extended to a newly introduced set of dual resource management levels (encompassed functions include signal/signature management, individual resource management, coordinated resource management, goal management and system engineering). Based on the entities of interest to information users, revision of the definitions for data fusion functional levels were suggested as follows [280, 416, 422]:

- Level 0 (Signal/Feature Assessment)—estimation and prediction of states of signals or features;
- Level 1 (Entity Assessment)—estimation and prediction of parametric and attributive states of entities;
- Level 2 (Situation Assessment)—estimation and prediction of relational/situational states of entities;
- Level 3 (Impact Assessment)—estimation and prediction of effects of fused entity/situation states on mission objectives;
- Level 4 (Performance/Process Assessment)—estimation and prediction of a system's measures of performance and measures of effectiveness based on given desired system states and/or responses.

In the proposed revision of the JDL DF model [280, 422], the Level 4 (Process Refinement) function [423] was categorized as being within the Resource Management model levels, while the proposed Level 5

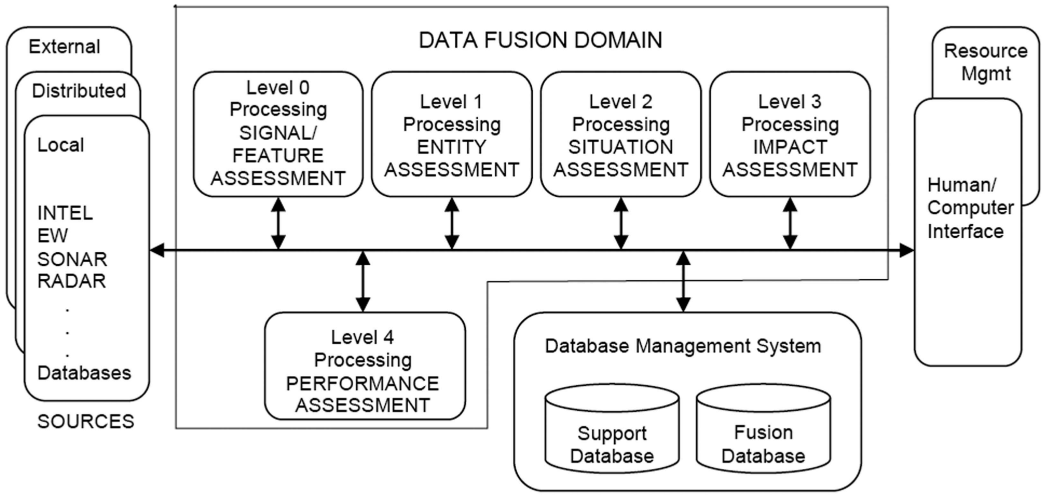


Fig. 14. Proposed revision of the JDL DF model [280].

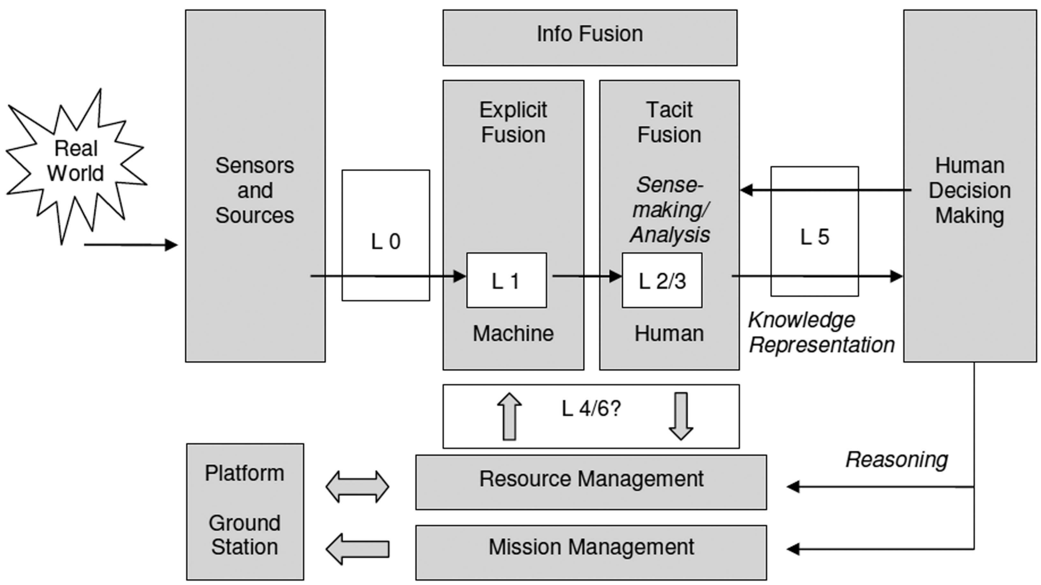


Fig. 15. Proposed DFIG 2004 model [40, 51].

[48, 181, 422] was subsumed as an element of Knowledge Management within Resource Management.

A further upgrade/revision to the JDL DF model (see Fig. 15) was proposed and assessed by the Data Fusion Information Group (DFIG) [40, 51]. The aim was to separate the information fusion and management functions. A detailed explanation on the model can be found in [41]. The definitions for this model, based on the version of the JDL DF model proposed in [423], were:

- Level 0 (Data Assessment)—estimation and prediction of observable states of signals or features;
- Level 1 (Object Assessment)—estimation and prediction of entity states based on data association, as well as continuous and discrete state estimation;
- Level 2 (Situation Assessment)—estimation and prediction of relationships among entities;
- Level 3 (Impact Assessment)—estimation and prediction of effects of entities' actions on goals/missions;
- Level 4 (Process Refinement)—an element of Resource Management that encompasses adaptivity in the data collection and fusion processes to support mission objectives;
- Level 5 (User Refinement)—an element of Knowledge Management that encompasses adaptivity in the determination of user query and access to information, as well as adaptivity in the retrieval and display of data, to support cognitive decision making and actions;
- Level 6 (Mission Management)—an element of Platform Management that encompasses adaptivity in the determination of spatial-temporal asset control, as well as route planning and goal determination to support team decision making and actions.

4. RESEARCH IN HIGH-LEVEL DATA AND INFORMATION FUSION

4.1. Shift of Research Focus from Low-level Fusion towards High-Level Fusion

After many years of intensive research, low-level fusion has become a relatively mature field [409]. The research focus is currently shifting towards fusion at higher levels. The significant amount of interest in high-level information fusion is evident from the related research activities that have been carried out in the recent years.

North Atlantic Treaty Organization Research and Technology Organisation Information Systems Technology Panel held a symposium on "Military Data and Information Fusion" in October 2003 [327] and a specialists' meeting on "Information Fusion for Command Support" in November 2005 [328] to discuss high-level fusion research and technology in the military domain. Panel discussion sessions have been dedicated to address high-level fusion research issues at the International Conference on Information Fusion (FUSION):

- 2004—Challenges in Higher Level Fusion: Unsolved, Difficult, and Misunderstood Problems/Approaches in Levels 2–4 Fusion Research [223];
- 2005—Issues and Challenges of Knowledge Representation and Reasoning Methods in Situation Assessment (Level 2 Fusion) [46];
- 2006—Issues and Challenges in Resource Management and Its Interaction with Level 2/3 Fusion with Applications to Real-World Problems [45];
- 2007—Results from Levels 2/3 Fusion Implementations: Issues, Challenges, Retrospectives and Perspectives for the Future [222];
  Agent Based Information Fusion [109];
- 2008—High-level Information Fusion: Challenges to the Academic Community [241];
- 2009—Issues and Challenges in Higher Level Fusion: Threat/Impact Assessment [221];
  Directions for Higher-Level Fusion Research: Needs and Capabilities [445];
  A Coalition Approach to Higher-Level Fusion [261];
- 2010—Issues and Challenges in Higher Level Fusion: Threat/Impact Assessment and Intent Modelling [379];
  High Level Information Fusion Developments, Issues and Grand Challenges [47];
- 2011—Social, Cultural, and Cognitive Aspects of Situation Management: Issues and Challenges [378];
- 2012—Multi-Level Fusion: Issues in Bridging the Gap between High and Low Level Fusion [233];
  Uncertainty Evaluation: Current Status and Major Challenges [98];
  Issues of Uncertainty Analysis in High-Level Information Fusion [43].

High-level information fusion topics have been gaining considerable presence among the technical sessions at the recent FUSION conferences (see Fig. 16). For example, at the 13th International Conference on Information Fusion held in July 2010, technical sessions on Advances in High-level Information Fusion Design were conducted to discuss research advances and developments in the area of high-level fusion. Areas of interest included modeling, representations, systems design and evaluation [53, 101, 155, 300, 301, 338, 368, 373, 447, 448].

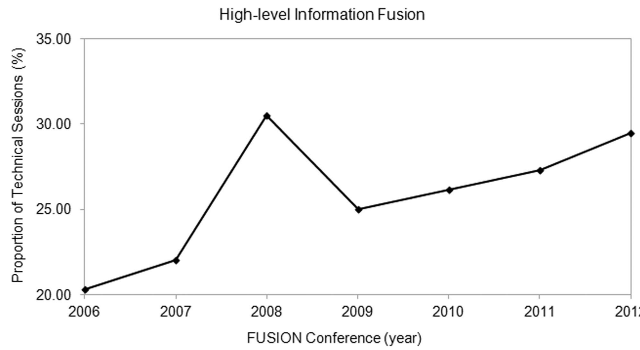


Fig. 16. Technical sessions on high-level IF topics at FUSION conferences.

The journal Information Fusion published a special issue on high-level information fusion and situation awareness [237, 240, 260, 276, 304, 344, 437, 472]. Das [106] authored a book with focus on fusion at Levels 2 and 3. Steinberg [421] provided a detailed study on principles and techniques related to situation and impact/threat assessment.

4.2. Situation and Impact Assessment

4.2.1. Situation Assessment

Level 2 fusion, also known as *Situation Assessment* (SA), is concerned with the determination and interpretation of relationships among objects and of estimation or prediction of situations; that is, of structures in the world. The objectives at this level include the derivation of high-level inference and the identification of meaningful events and activities [316, 421]. *Situation Awareness* (SAW) involves the identification and monitoring of various relationships among Level 1 physical and abstract entities, as well as various relations among them. Situation assessment is regarded as the process of achieving, acquiring or maintaining situation awareness [377]. Models for automated situation assessment tools include the JDL DF model (see Section 3) and the Endsley's Situation Awareness model [127, 128] (see Section 2.2.5).

General issues and challenges in situation assessment and situation awareness have been addressed by different researchers with various perspectives and approaches [46].

- Gorodetsky, et al. [162] did an analysis of formal frameworks proposed for specification of the situation models. Their focus was on approaches and algorithms for on-line update of situation assessment, on the generic architecture of the situation assessment systems.

- Jones, et al. [217] described the use of fuzzy cognitive maps in the development of a data fusion model to support situation awareness and human cognition, based on the Goal-Directed Task Analysis methodology.
- Kadar [220] addressed issues in situation assessment and associated Knowledge Representation and Reasoning models, with focus on a human perceptual reasoning-based model framework.
- Kokar [234, 235] identified and discussed problems pertaining to automatic situation assessment/awareness. Approaches for solving these identified problems were proposed and compared.
- A detailed discussion on developing a conceptual framework for situation assessment and awareness was given by Salerno [376]. He also addressed issues and perspectives on high-level information fusion processing.
- Salerno, et al. [377] explored various techniques believed to be necessary for providing situation awareness. They also investigated how those techniques could be bound together to form an overall system architecture, as well as how various sources of information contributed to the problem of maintaining constant awareness of the environment one was in.
- Qureshi and Urlings [356] proposed an operator assistant with a flexible concept of automation, with the objective of enhancing situation awareness.
- Settembre, et al. [398] designed a multi-agent architecture for situation assessment. The system utilized Web Ontology Language-based reasoning for high-level situation classification and analysis, and provided distributed assessment via the solution of disagreements that might exist among different agent conclusions. Experimental results from a real maritime surveillance scenario showed that the proposed approach had the capability to achieve performance similar to that of a centralized architecture. In addition, the method preserved the independency of decision makers and significantly reduced the amount of communication required.
- Smart, et al. [405] investigated knowledge-based approaches to improving situation awareness in humanitarian operational deployment. A tool for intelligent information fusion, Technical Demonstrator System, was developed for the situation awareness enhancement task. A functional overview of the system with respect to several capability areas was presented.
- Steinberg [420] described an adaptive evidence-accrual inference method for selecting context variables based on their usefulness in the refinement of explicit variables in problems of interest; the probability of obtaining these variables with predetermined amount of accuracy, given candidate system actions such as data collection, mining or processing; as well as the cost of the aforementioned actions.

4.2.2. Impact Assessment

Level 3 fusion, known as *Threat Assessment* in the original JDL DF model, was redefined as *Impact Assessment* to accommodate expansion in the concept of Level 3 fusion [421, 423]. Impact Assessment deals with the determination of the effect of current situational states on user objectives. It involves the prediction of the intent (alternative courses of action) for entities, as well as the estimation of the degree or severity with which impending (possibly adversarial) events may occur.

Broadly speaking, Level 3 fusion involves the estimation of contingent (for example, possible future) states and of their cost/benefit impacts [421]. As such Level 3 fusion can be perceived as a subset of Level 2 fusion, due to the broad definition for the latter [423]. Assignment at Level 3 is usually inferred from Level 2 associations, although processing at the fusion levels need not be performed in order [280]. In addition, given corresponding inputs, any one level can be processed on its own. Table I displays some methods that are applied to different problems on situation and impact assessment [79, 106, 421].

4.3. Process Refinement

Level 4 fusion is known as *Process Refinement* in the early versions of the JDL DF model [423]. The process involves resource management to improve the results obtained at the lower levels of data fusion [316]. In the recent proposed revision of the JDL DF model [280], the data fusion levels were extended to their dual resource management levels. In addition, a new Level 4 of data fusion and its corresponding dual Level 4 of resource management were introduced. A redefinition Level 4 (*Performance Assessment* (PA), also known as *Performance Evaluation* (PE)) was proposed with the existing Level 4 (Process Refinement) function [423] categorized as being within the resource management model levels. Based on a given desired set of system states and/or responses, the Level 4 data fusion functions combined information to estimate a system's measures of performances and measures of effectiveness. It was proposed that the purpose of the existing JDL DF levels would be preserved by these new data fusion and resource management levels.

This section gives some instances of research work that discuss PA/PE methodologies for data fusion processes, as well as issues on data/information fusion and resource management (subjects of management include signals/signatures, individual resources, coordinated resources, goals/mission objectives, system engineering and operational configuration) [45].

4.3.1. Performance Assessment/Evaluation Methodologies

A literature analysis of twenty-four journal articles and twenty-eight conference papers on the topic of performance evaluation was carried out by van Laere [450].

TABLE I
Situation and Impact Assessment: Issues and Approaches

Application Domain	Approach/Technique	Reference
Data association/correlation	Ontology	[239, 242, 243, 292–294]
	Mathematics-based metrics	[428–430]
Semantic Knowledge	Ontology	[173, 266, 330]
Tactical defense	Kohonen's self-organizing maps	[7]
—Air defense	Neural networks	[8, 195]
—Asymmetric warfare	Ontology	[17, 100, 238, 293–297, 374]
—C4ISR	Hidden Markov models and time series	[21]
—Enemy courses of action	Bayesian inference/network/theory	[26, 107, 108, 141, 195, 251, 262, 285, 285, 331, 335, 336]
—Ground battlespace	Evidential theory/networks	[32, 33, 203, 392]
—Information warfare	Fuzzy logic/Fuzzy set theory	[34, 80, 139, 140, 195, 217, 285, 321, 343, 355, 382]
—Interoperability	Support measures/functionalities	[37, 129, 142, 160, 349, 374]
—Maintenance of consistency	Knowledge-based approaches	[57, 93, 195]
in intelligence database	Contextual information, target behavior	[62, 63, 149, 334, 420]
—Maritime surveillance	extraction/classification	
—NBD/NCW	Axiomatic approach	[102]
—Threat analysis	Genetic algorithms	[157, 158, 195]
—Threat stabilization	Self-organizing peer-to-peer SAW system	[194]
—Video/visual surveillance	Real-time automated rule-based system	[200]
	Modified probabilistic neural network	[208]
	Situation, ontology, estimation theory	[216, 417, 418]
	Uncertainty propagation for dynamical systems	[245, 440]
	Asset profiling	[262]
	Team SAW measurement techniques	[263, 380, 402]
	Statistical density estimation	[267]
	Cognitive system engineering	[341]
	Information theory	[343]
	Multiple attribute decision making	[355]
	Graph-based tools	[384, 385, 425]
	Multi-agent system	[398]
	Centralised intelligence fusion	[414]

The objective was to identify the extent to which information fusion researchers were aware of the problematic nature of performance evaluation in practice, as well as problems and related known solutions. He proposed there was a need to define and study a set of comprehensive performance measures which were adaptable to domain or situation context and changing circumstances over time. He also asserted the need for incorporation of optimality checks.

Table II shows some approaches to performance assessment/evaluation for data fusion systems in various application domains.

4.3.2. Data Fusion/Information Fusion and Resource Management

Blackman and Popoli [37, Chap. 15] discussed principles and techniques for sensor management (SM). The main issues of interest were: the necessity to include sensor management in the design of a modern sensor tracking system, the understanding of the aspects of sensor operation that required management and the figures of merit (metrics for the overall performance of an entire sensor tracking system) to be optimized by that management, as well as the approaches to accomplish sensor management.

Ng and Ng [318] studied the roles of sensor management, the motivation to use SM and presented a framework for a generic SM. Ng [316, Chap. 9] discussed classification and roles of SM and carried out simulation studies to demonstrate roles of SM as a controller.

Multi-sensor management deals with the control of environment perception activities by the management or coordination of multiple sensor resource usage. It is an emerging research area and has become increasingly important in the research and development of modern multi-sensor systems for both military and civilian applications. Xiong and Svensson [464] provided a review of multi-sensor management in relation to multi-sensor information fusion. The work done included description of the role of multi-sensor management in the larger context, generalization of main problems from existing application needs and discussion on problem solving methodologies. In addition, many useful related works were cited.

A stochastic dynamic programming based approach to solving sensor resource management problems was described by Washburn, et al. [457]. The sensor resource management problem was formulated as a stochastic scheduling problem and approximate solutions based on the Gittins index rule were developed.

TABLE II
Performance Assessment/Evaluation for Data Fusion Systems

Application Domain	Approach/Technique	Reference
General	Formal definition of validation (references a standard fusion device)	[248]
	Local evaluation measures for image interpretation	[256]
	Measures of input scenario complexity and output quality	[322, 439]
	Rule-based expert system	[337]
	Data association, metrics estimation, Statistical DOE, ANOVA	[382]
Multi-source fusion	Bayesian inference	[76]
	Distributed fusion track-to-truth association, distributed fusion track-to-track association	[117, 381]
	Correlation effect, best linear unbiased estimation criteria	[19, 473]
Target tracking	Measures for assessing track detection performance, accuracy, quality and data association	[52, 95, 161, 273, 307, 393, 427, 471]
—Automatic target recognition	Track-centric metrics	[71]
—Classification, estimation and filtering	Information theoretic measures	[87]
	Context metrics that characterize problem difficulty	[148]
—Decentralized estimation	Optimal subpattern assignment-based metrics	[171, 314, 367, 395]
—Moving target identification	Multi-channel signal subspace methodology	[224]
—Multiple target tracking	Optimization-based hierarchical PE system, Statistical DOE, ANOVA	[283, 360]
	Error bounds	[400, 413]

High-level information is playing an increasingly important role in research on sensor management. There is concern about the appropriateness in using the term *Sensor Management* to encompass the functions on the information level. In view of the necessity of using intelligent agents to perceive the environment to take suitable actions, Johansson and Xiong [214] proposed a generic concept of *Perception Management*, without having to be particular about concrete sensor device details. The concept referred to controlling the data acquisition process from the external world to enhance the perception outcomes. Two different possible interrelations between sensor management and perception management were considered and discussed: either sensor management is encompassed in perception management or sensor management is separate from and independent of perception management.

Bradley [61] gave a discussion on sensor tasking capability pertaining to a resource allocation manager which integrated command, control and communications functions within various types of sensor platforms and had significant contributions to multi-platform interoperability and situation awareness operations. He gave an overview of the fusion architecture and tracking system in which a resource allocation manager was integrated. Performance analysis on the resource allocation manager was done based on measured and modeled data.

Table III provides a summary of some problems and techniques for data fusion/information fusion and resource management.

4.4. Cognitive Refinement

Information representation and human-computer interaction are important for most data fusion systems. For example, it has been noted that the efficacy of the HCI had a significant influence on the overall performance and effectiveness of a data fusion system [455]. On the other hand, the Object-Centered information fusion model [236] (see Section 2.3.1) took into consideration the role of a human for decision making.

The concept of Level 5 (*Cognitive Refinement*) processing in the original JDL DF model was introduced by Hall, et al. [181] to account for functions associated with human-computer interaction explicitly. It involved the development of functions to support a human user in a collaborative human-computer environment. The categories of functions associated with Level 5 processing included [179]: HCI utilities, dialogue and transaction management and cognitive aids. Figure 17 shows the resultant augmented JDL DF model proposed. More discussion on various issues of cognitive refinement and human-computer interaction can be found in [179, Chap. 9].

In an independent work, Blasch and Plano [48] introduced Level 5 (*User* (or *Human*) *Refinement*, an element of Knowledge Management) with the purpose of supporting cognitive workload, trust, attention and situation awareness. In addition, the JDL-User model (shown in Fig. 18) was proposed to extend the JDL DF model [423] via the incorporation of the suggested Level 5. Further issues related to User Refinement were explored in [40–42, 49–51, 54].

More related research has been done recently. Hall, et al. [180] discussed the development of a set of tools to support *whole-brain* information analysis (combines visually-oriented analysis of images with language-based analysis of text and related information). Nilsson and Ziemke [326] suggested adopting a distributed cog-

TABLE III
Data/Information Fusion and Resource Management: Problems and Techniques

Application Domain	Approach/Technique	Reference
Multi-source fusion	Market-based architecture	[16]
	Probabilistic sensor placement algorithm coverage optimization	[122]
	Shannon's entropy-based probabilistic fusion of multiple information sources	[135]
	Sensor subset selection	[151]
	Distributed Bayesian inference and reinforcement learning	[165]
	Sensor scheduling (distributed greedy/myopic algorithms, feedback control theory)	[201, 465]
	Mathematical and statistical analysis	[387]
	Unified sensor performance modeling	[451]
	Hierarchically networked agent architecture	[479]
Tactical defense	Genetic algorithm	[68]
—C4ISR	Intelligent multi-agent based sensor resource management structure	[88]
—Maritime operations	Bayesian belief networks	[159]
—Military mission planning	Fuzzy logic	[159, 305]
—NBD/NCW	Stochastic dynamic programming	[213]
	Distributed fusion on multiple platforms	[271]
	Random sets and equivalence classes of multi-target paths	[291]
	Object classification/detection	[391, 452]
	Simulation-based tool and mixed-initiative interaction	[434]
Target tracking	Sensor selection	[55, 357, 359]
—Attack-avoidance	Bayesian technique-based approach	[81, 193, 362]
—Ground target tracking	Hierarchical dynamic optimal control methods	[92]
and classification	Algebraic framework	[104]
—Multi function radar tracking	Fuzzy logic, neural network system	[244]
—Multiple target tracking	Combine invariance, robustness and self-refusal	[250]
—Target detection	Reinforcement learning	[252]
	Machine learning (active sensing)	[253]
	Game theory (linear quadratic, geometric feature-aided)	[268, 269]
	Optimization-based dynamic algorithm (utilizes Markov models, decision trees)	[348]
	Clustering techniques	[390]
	Mathematical programming-based sensor allocation and management	[443]
	Geometric factors, information and measures of merit	[469, 470]
	Quadratic programming (numerical solver for constrained minimization problem)	[480]

nition perspective to complement existing approaches to understanding and modeling information fusion.

4.5. An Area with Increasing Interest: Hard and Soft Data/Information Fusion

In a decision making task, accurate information is essential for the decision makers concerned to make precise assessment of the situation and possible impact, and subsequently, appropriate and timely decisions. The derivation of relevant information generally involves a fusion process that combines and integrates data/information from multiple sources. Data/information can be classified into two categories, namely, "hard" and "soft."

"Hard information" refers to information from traditional physical sources such as radar and acoustic sensors. Such information usually includes kinematic data on the entities of interest. "Soft information" refers to information from human-based sources such as conversations, documents, newspapers and internet web sites. Such information can include possible location and identity information, as well as activities, intent and relationships among the entities of interest.

Hard and soft data generally contain complementary information, so it is necessary for data and information fusion practitioners to develop automated tools for effective fusion of these data. The disparate characteristics of hard and soft data result in many technical challenges for hard/soft data fusion. For example, hard data is usually structured and can be modeled mathematically. On the other hand, soft data is generally unstructured and inconsistent, and hence difficult to study with a mathematical model.

The DIF community recognizes the importance of hard and soft data/information fusion, and has increasing interest in this research area [99, 229, 282, 466, 467]. In the past few years, technical sessions have been held at the International Conference on Information Fusion to discuss research and development issues related to hard and soft data/information fusion:

- 2008—Hard/Soft Information Fusion [172, 178, 218, 351, 462];
  Challenges of and Methods for Information Fusion of Soft Data [10, 14, 36, 137, 174, 277, 309, 383, 411, 424];

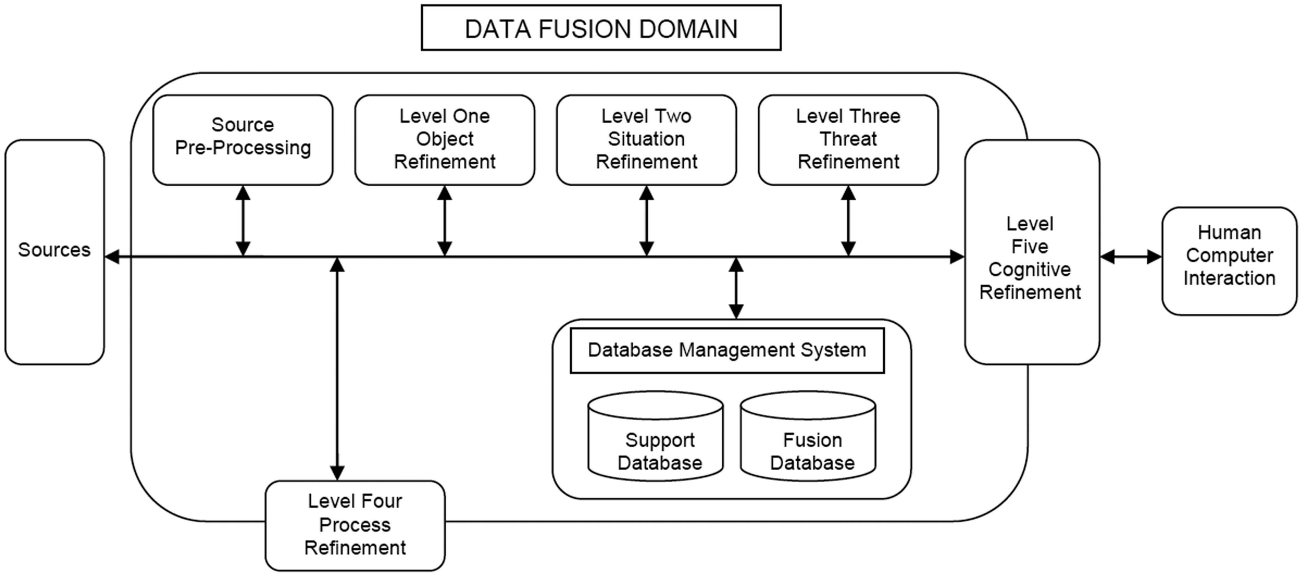


Fig. 17. Augmented JDL DF model [179].

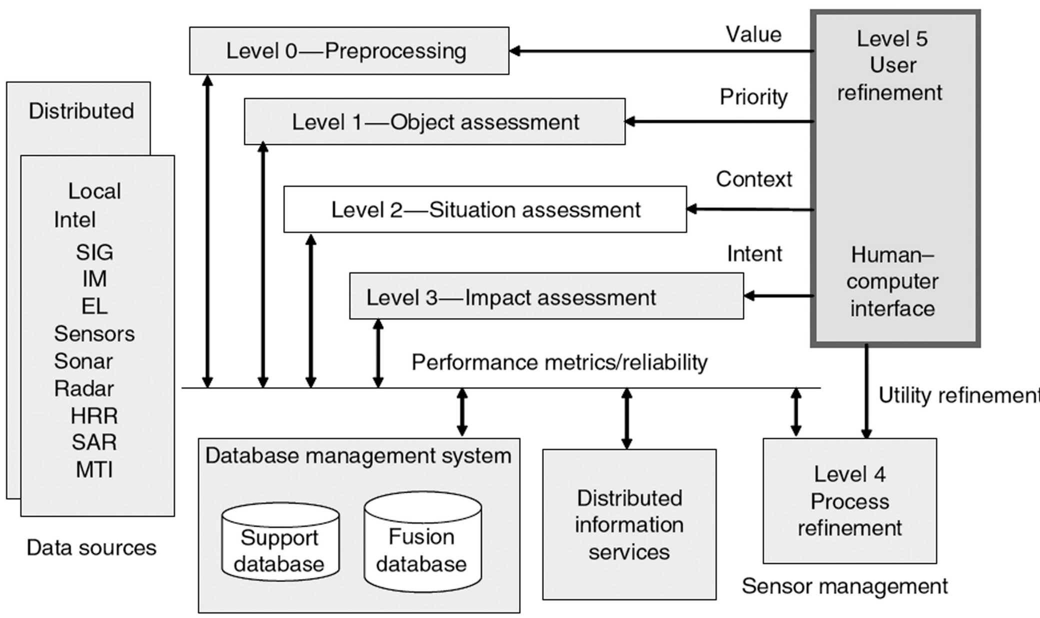


Fig. 18. JDL-User model [48].

- 2009—Fusion of Hard and Soft Information for Asymmetric, Urban Operations [156, 219, 339, 350, 352];
- 2010—Multidisciplinary Research in Hard and Soft Information Fusion [44, 168, 175, 187, 282];
- 2011—Human-based Sensing: From Passive Searching to Active Participation [186, 225, 302, 303, 353]; Hard/Soft Information Fusion: New Data Sets and Innovative Architectures [2, 20, 164, 169, 209];
- 2012—Hard/Soft Fusion [1, 89, 120, 121, 170, 183, 184, 313, 366, 404, 406, 407, 461, 463].

Many applications involve the extraction of information through processing and/or fusing huge quantities of data from multiple sources. Topics for exploration in the relatively immature research area of hard and soft data/information fusion can therefore be expected to continue to increase and evolve.

5. APPLICATIONS

Since the introduction of data and information fusion techniques to the research community in the 1970s, the scope of application areas for DIF has widened significantly. Some of the applications that involve high-level DIF (situation/impact assessment, resource management, and so on) are discussed in the following subsections. Table IV shows a summary of the techniques applied to the problems discussed.

5.1. Strategic/Tactical Defense

Data and information fusion was first used in military defense research related problems. After several

TABLE IV
Problems and Techniques in Various Application Areas that Involve High-Level DIF

Application Domain	Approach/Technique	Reference
Strategic/tactical defense		
—Biosurveillance	Information retrieval and dynamic Bayesian networks	[205]
—Drug interdiction	Multiple platform distributed fusion	[94, 270]
—Homeland security	Analytic network process	[478]
—Maritime surveillance	Hybrid fusion (interaction with data fusion processes at different information levels)	[145]
—NBD C4ISR	Dempster-Shafer clustering and template matching, particle filtering and finite set statistics	[4]
—Undersea warfare	Network-centric theatre undersea warfare architecture	[5]
Computer/information security		
—Dishonest behavior detection	Probabilistic, scalable distributed approach	[96]
	Integration of rule-based filtering, Dempster-Shafer theory and Bayesian learning	[340]
—Intrusion detection	Logic-based data model	[308]
	Fuzzy set theory	[286]
	Adaptive non-stationary autoregressive model	[454]
	Probabilistic inference	[436]
—Threat evaluation	Multiple behavior information fusion based on Markov models and Dempster-Shafer evidential reasoning	[90]
	Modeling and simulation, and risk analysis/assessment	[324]
Post-disaster management		
—Casualty mitigation operations	Cognitive work analysis, ontological analysis	[369]
—Data fusion visualization	Integrated graphical user interface framework	[290]
—Decision making	Bayesian networks, Dempster-Shafer theory, fuzzy logic, neural networks	[279]
—Dynamic situation assessment	Ontology meta-model	[274]
Engine/machinery fault diagnosis	Hybrid system parameter estimation and change detection	[22, 23]
	Dempster-Shafer evidence theory-based multi-source IF	[24, 133, 134]
Biomedical Applications		
—Data exploration/analysis	Multidimensional analysis, self-organizing map clustering algorithm	[146]
	Fuzzy logic, multiple classifier network, decision level data fusion	[477]
—Medical/clinical diagnosis	Dempster-Shafer framework	[311]
	Fuzzy logic	[130, 226]
—Patient monitoring	Dynamic Bayesian network	[29]
Environment		
—Ecological evaluation of urban biotopes	Spatial and statistical analyses of airborne hyperspectral data	[188]
—Fire detection	Formal theory of perception	[397]
	Dempster-Shafer theory	[476]
—Irrigation system management	Genetic algorithm, agrohydrological model	[85]
—Land monitoring and projection	Dempster-Shafer theory of evidence	[202]
—Soil moisture estimation	Support vector machines, relevance vector machines	[230]
Industrial applications		
—Agricultural product quality control	Bayesian inference	[371]
	Neural network training	[289]
—Decision support in manufacturing	Modeling, resource simulation and databases	[118, 119]
—Dislocation detection in construction materials	Belief function theory	[361]
—Information system deployment	Document object model for data fusion and aggregation	[354]
—System monitoring	Hierarchical, multi-layered fusion architecture	[154]
—Unmanned vehicle guidance	Information-oriented perception management	[333]

decades of development, DIF techniques are now being developed and applied in diverse non-military research areas as well. Nevertheless, military defense research remains a very prominent application area for DIF [58, 70, 131, 166, 212, 433]. Here, some research works from various defense applications are summarized.

Liggins, II, et al. [94, 270] developed distributed architectures to support relevant fusion technologies such as multi-source fusion and sensor resource management. The technologies were applied to problems in defense and drug interdiction.

Gad and Farooq [145] discussed various data fusion architectures for maritime surveillance and developed a system that interacted with the data fusion processes at different information levels. This proposed data fusion architecture was shown to perform well when employed to support the maritime surveillance for a typical maritime tactical scenario.

Aldinger and Kao [5] discussed the challenges faced in undersea warfare and some research work done on developing data fusion technology and other techniques to enhance the capabilities of the undersea warfare community.

Ahlberg, et al. [4] developed a concept demonstrator, the Information Fusion Demonstrator 2003 (IFD03), to demonstrate information fusion methodology expected to be suitable for a future network-based defense command, control, communications, computers, intelligence, surveillance and reconnaissance (C4ISR) system. The focus of IFD03 was on real-time intelligence processing in a tactical level ground warfare scenario. The architecture, methodology and user interface of the software system were described. The system was applied to a concrete scenario and related fusion results were discussed.

Introne, et al. [205] developed a novel application that employed a two-level fusion architecture to address the problem of biosurveillance.[2] Feasibility of the approach was demonstrated via simulated outbreak events on a simulation platform.

Zhang, et al. [478] applied a strictly quantitative analysis-based analytic network process to model elicitation in large-scale nation-building simulation models. The proposed approach could be used to study the significance of different kinds of factors and the interdependencies among them. This approach could circumvent the problem of possibly conflicting human expertise, which was encountered by many traditional expert knowledge-based analytic network process methods. Numerical results demonstrated that the proposed methodology could provide good approximate solutions to the nation-building simulation problems. The amount of computational time required for nation-building model analysis was also significantly less than that required for multiple replications of traditional discrete-event simulations.

5.2. Computer/Information Security

In the present age, where the use of information technology is ubiquitous, computer and information security issues are of great importance to both system administrators and general users. Information system issues such as intrusion detection in distributed communication and computer networks are receiving increasing amount of attention. Dasarathy [115] presented a general overview on research work done on intrusion detection.

[2]Biosurveillance: detection of attacks with unknown bioagents, also known as *syndromic surveillance*.

Stein, et al. [415] presented an outline of emerging concepts that were expected to guide future operations of joint military operations, and also explained the achievement of information superiority via the use of network-centric computing. Experimental tests showed the effect of employing information superiority on the approach to fighting battles.

Browne [67] proposed that new approaches to command, control, communications, computers and intelligence (C4I) defensive architecture be developed to defend against multi-mode attacks, which were enemy strategies using clever combinations of conventional and non-conventional warfare. Criticism was made on some popular existing C4I defense technologies that were considered to be vulnerable against multi-mode attacks. A speculative discussion was presented on new C4I defense technologies and policy issues regarding information superiority that were believed to be inadequately addressed in existing literature.

A model based on multiple behavior information fusion was developed for quantitative evaluation of network security threat by Chen, et al. [90]. The proposed method was used for tests in a real network environment and was shown to be a reasonable and feasible tool for its system administrators.

Nicol [324] gave a discussion on using simulation to evaluate computer security in areas such as impact assessment (determine how security measures affect system and application performance) and emulation (combine real and virtual worlds to study the interaction between malware and systems, and probe for new system weaknesses).

Du, et al. [124] formulated the problem of unsupervised classification for non-uniform attack tracks in cyber domains. The authors discussed three methods from distinct fields for solving this problem. The methods are, namely, "the subsequence matching technique," "Fourier analysis" and "the social network approach." The three approaches were compared with a traditional classification algorithm, K-means clustering algorithm. Based on the preliminary results, the three approaches showed promise in the characterization and the categorization of attack tracks.

The journal Information Fusion has published a special issue on information fusion in computer security [96, 97, 152, 286, 308, 340, 436, 454]. Corona, et al. [97] gave a detailed review of issues concerning the application of information fusion techniques in computer security, with particular focus on intrusion detection in computer networks. They also discussed topics such as data organization and data reconciliation that required further research.

Morin, et al. [308] proposed a first-order logic based data model as a support tool reasoning about alerts triggered by network intrusion detection systems. They demonstrated the practicality of the proposed framework by implementing it in a hypothetical attack scenario. Maggi, et al. [286] utilized a fuzzy set theory-based technique to fuse alerts on anomalies in an intrusion detection system. The proposed method was validated in experiments using two prototypes developed earlier by the authors, namely, a host anomaly detector and a network anomaly detector. Viinikka, et al. [454] suggested an adaptive method to model and filter out intrusion detection alerts related to normal system behavior from sequences of aggregated alerts. The authors used a non-stationary autoregressive model whose parameters were estimated by a Kalman fixed-lag smoother to produce a series of differences between observations and model predictions. Anomaly alerts were signaled upon detection of residuals which exceeded pre-defined thresholds. The effectiveness of the method was demonstrated through experiments on processing huge amounts of aggregated alert sequences from an operational information network.

Sy [436] proposed a probabilistic inference-based analytical intrusion detection framework to integrate alert information obtained from sensors deployed throughout a distributive network-based intrusion detection system. The integrated information was used to assist in the generation of the most probable forensic explanation. An experimental study was conducted to evaluate the feasibility of the proposed method. The suggested method yielded favorable results, when compared to the naïve Bayes reasoning approach. Efficient detection of node replication in a wireless sensor network is required to provide authenticity of data fusion in the network.

Conti, et al. [96] developed the Information Fusion Based Clone Detection Protocol (ICD), a probabilistic, scalable distributed protocol for detection of cloned nodes. The ICD combined two different cryptographic mechanisms, namely, pseudo-random key predeployment and asymmetric cryptography. Simulation results verified the robustness of the ICD for different parameter sets considered. Panigrahi, et al. [340] presented a credit card fraud detection system which made use of a transaction history database and the integration of three approaches, namely, rule-based filtering, Dempster-Shafer (D-S) theory and Bayesian learning. For system performance analysis, stochastic models were used to generate simulated credit card transactions. The proposed fraud detection system was found to yield high accuracy in detecting fraudulent transactions.

5.3. Crisis/Disaster Management

In the event of a natural catastrophe or otherwise, there exists a large quantity of crucial data to be dealt with within a very short period of time immediately after the disaster [299]. It is essential to develop efficient data and information fusion tools for effective situation assessment and impact prediction in dynamic post-disaster scenarios, which in turn would be useful for decision making.

In view of the growing threats posed by potential use of chemical and biological agents in the military battlefield, Llinas, et al. [281] addressed issues and challenges related to the development of technologies for effective combat against these weapons of mass destruction, in both military and civilian applications. Effective execution of battle management functions depends very much on high-quality information input. The authors asserted that it was very likely that the high-quality information demands of Nuclear, Chemical, Biological and Radiological (NCBR) battle management functions could be met by many existing information fusion techniques. In addition, it was possible for transition of advanced information fusion technologies from conventional warfare settings to NCBR-specific mission applications.

Llinas [279] described the overall strategic approach (engineering methodology) to a multi-year research program which addressed issues in information fusion to support crisis centre decision makers dealing with post-event situations. Both natural and man-made disasters were considered, with emphasis placed on post-earthquake and post-chemical attack scenarios respectively. The focus was on fusion capabilities at Levels 2 and 3 (higher-level information fusion). Examples of specific research components and subsequent research plans for the program were also discussed.

Little and Rogova [274] worked on the design of a general methodology for situation assessment to support crisis management. The proposed approach utilized understanding the combination of both formal and domain-specific construction methodologies and also described a general taxonomy of relationships, one which could encapsulate many of the complexities associated with catastrophic events.

A disaster monitoring interface for an earthquake simulation was proposed by Mandiak, et al. [290]. The visualization tool was an integrated graphical user interface framework that enabled a user to easily comprehend the trend of a situation, by providing as much information (obtained via the integration of multidimensional graphic displays) as possible to him.

Rogova, et al. [369] addressed the problem of situation assessment to support casualty mitigation operations in the response phase that immediately followed an earthquake. The proposed methodology was based on the cognitive work analysis and ontological analysis of a specific emergency management domain, developed within the framework of a formal ontology.

5.4. Fault Detection and Identification/Diagnosis

The main issues of concern when applying information fusion to fault diagnosis are the acquisition of reliable information about potential faults by incorporating multiple sensors, as well as the derivation of fused decisions based on data from the multiple sensors. It is necessary to develop fusion mechanisms that minimize conflicts among the sensors, as well as imprecision and uncertainty in the sensor data.

Based on Dempster-Shafer evidence theory, a multi-sensor implementation of an engine diagnostic system was introduced by Basir and Yuan [24]. The formulation of the engine diagnostic problem in the context of the evidence theory was explained. Novel ways were introduced to enhance the effectiveness of mass functions in modeling and in evidence combination. Rational diagnosis decision making rules were proposed and the entropy of evidence was introduced to facilitate information fusion performance evaluation. Experimental results demonstrated the effectiveness of the proposed approach in resolving decision conflicts and in improving the accuracy of fault diagnosis via multi-sensor information fusion.

Fan and Zuo [133] introduced a Dempster-Shafer evidence theory-based method with the capability of increasing accuracy of decision making through multi-source information fusion. In the proposed approach, fuzzy set theory, weight of evidence and conflict resolution were introduced to address the issues of evidence sufficiency, evidence importance, and conflicting evidence in the practical application of D-S evidence theory. Test example results validated feasibility of the proposed method, as well as its improvement over the conventional D-S evidence theory in performing fault diagnosis through fusing multi-source information. In the sequel [134], successful application of the improved D-S evidence theory to machinery fault diagnosis was reported. Experimental results showed that the proposed method could enhance diagnostic accuracy and autonomy, in comparison with conventional diagnostic methods.

Bashi, et al. [22] proposed an algorithm for fault detection in large-scale systems with a large number of almost identical units operating in a shared environment. The fault detection algorithm was developed based on the estimation of a common Gaussian-mixture distribution for unit parameters via the Expectation-Maximization algorithm. The estimated common distribution incorporated and generated information from all units and was utilized for fault detection in each individual unit. The algorithm was applicable in various industrial, chemical or manufacturing processes, as well as sensor networks. In the companion paper [23], the authors described the application of their algorithm to the problem of fault detection in heating ventilation and air conditioning (HVAC) systems. Implementation details were described. Monte Carlo simulations and real data collected from three operational large HVAC systems were used in the evaluation of the performance of the proposed methodology in a realistic situation.

5.5. Biomedical Applications/Informatics

Biomedical applications/informatics generally involve voluminous data from multiple heterogeneous sources. In most circumstances, the amount of useful knowledge that can be acquired from an individual data source is limited. Information derived from multi-source data fusion is often of better quality than that obtained from the available sources separately.

Bellot, et al. [29] proposed a generic approach to fuse data in dynamical systems. A notion of qualified gain was defined to help determine the usefulness of a data fusion process developed. The method was applied to a problem of monitoring kidney disease patients who underwent dialysis at home. All the data sources and relations among them were determined. A dynamic Bayesian network-based model was used to fuse the data in order to provide daily diagnosis on the hydration state of the patients. Efficiency of the proposed approach was reflected by the experimental results obtained.

Ganta, et al. [146] described data exploration and analysis of heterogeneous biomedical informatics data sets using an online data warehouse. Experimental results obtained from applying information fusion techniques to multiple prostate cancer data sets demonstrated the feasibility of the proposed system.

Zhang, et al. [477] presented a new approach to explore the cause of human longevity based on comprehensive medical data. Expert knowledge was applied to a longevity model through artificial intelligence techniques. Firstly, fuzzy logic was used in pre-processing biomedical data. Then multiple classifier network and decision level data fusion were applied to improve the modeling accuracy. Simulation test results showed that the proposed model was able to identify individuals who belong to longevity group with high accuracy.

Muller, et al. [311] developed a modular data fusion system with Dempster-Shafer framework. An architecture of fusion was built from this system by chaining two types of elementary modules. Modules of the first type were used for symbolic interpretation of numerical reports from sensors, while those of the second type were used for the combination of these symbolic data to obtain relevant synthetical information for diagnosis. The data fused were generated by tagged Magnetic Resonance Imaging[3] and Positron Emission Tomography.[4] D-S theory was applied to model the uncertainty of the data and the rules of decision. The fusion architecture was applied to the assessment of left ventricular my-

[3]Magnetic Resonance Imaging (MRI): an imaging technique based on the principles of Nuclear Magnetic Resonance, a spectroscopic technique used by scientists to elucidate chemical structure and molecular dynamics. MRI is used primarily in medical settings to produce high quality images of the inside of the human body.

[4]Positron Emission Tomography (PET): a highly specialized imaging technique that uses short-lived radioactive substances to produce three-dimensional colored images of those substances functioning within the body. These images are called PET scans and the technique is termed PET scanning.

ocardial viability.[5] To obtain geometrical information on the potential lesions, diagnosis results obtained from the data of a patient were displayed on polar maps.

A fuzzy logic-based data fusion system for detection of life threatening patient states in cardiac care units was proposed by Kannathal, et al. [130, 226]. Heterogeneous electrophysiological and haemodynamic data were fused and analyzed. In addition, a parameter named *patient deterioration index* was proposed to evaluate the severity of the cardiac abnormality. Test results obtained showed that the proposed approach could give highly accurate clinical diagnosis in monitoring the patients.

5.6. Environment

Human activities and environmental modifications can influence the ecosystem in multiple ways. The impact can be local, regional or even global. It is necessary to develop efficient systems to monitor and control activities that produce effects on the environment.

Hubert-Moy, et al. [202] applied Dempster-Shafer's theory of evidence to support spatio-temporal monitoring and projections of land use and land cover changes. Data from spatial and temporal sources were fused to obtain spatial prediction of the location of winter bare fields for the following season on a watershed located in an intensive agricultural region. A highly accurate prediction on the presence of bare soils was achieved over the entire area of interest. The spatial distribution of misrepresented fields provided a good indicator for identification of change factors.

Heiden, et al. [188] proposed a methodology to facilitate derivation of quantitative parameters for advanced evaluation of urban biotopes,[6] an essential task in ecological urban planning. The proposed approach involved the analysis of airborne hyperspectral data and automated identification of urban surface cover types based on their material-specific spectral reflectance characteristics. The results were then integrated with vector-based urban biotope mapping, an existing database. Finally, the required quantitative parameters were derived from the resultant database. Spatial and statistical analyses showed that using quantitative parameters to complement the predominately descriptive information contained in urban biotope mapping yielded improved evaluation of urban biotopes.

Two data-driven tools, support vector machines[7] and relevance vector machines,[8] were successfully applied to perform reliable soil moisture estimation by Khalil, et al. [230]. The effectiveness and efficiency of the proposed models in soil moisture prediction were evaluated with the use of weather information. The performance and generalization capabilities of the two machines were also compared. Support vector machines and relevance vector machines could be utilized in industries such as large scale water management to attain high-level inference via information, feature and decision level fusion processes.

In order to improve management of irrigation systems, good quality of spatial and temporal data on evapotranspiration, the combination of soil evaporation and plant transpiration, was essential. However, it was not easy to attain good quality for remote sensing evapotranspiration data. Chemin and Honda [85] reported an investigation on the use of genetic algorithms in assimilating parameters of an agrohydrological[9] model. The aim of the research was to find optimized parameters that would enable the model to obtain simulated evapotranspiration output that converged to observed remote sensing evapotranspiration data. The proposed methodology involved the fusion of observed remote sensing data of high spatial resolution, as well as those of low spatial resolution.

Šeric, et al. [397] presented an advanced communication and networking environment with all applications and services being focused on users. The authors detailed environmental intelligence based on a collection (network) of observers. Observer network theory was derived from the formal theory of perception and formed the basis for the design of their forest fire monitoring system. The proposed system was implemented on a multi agent framework. The efficiency of the forest fire observer was evaluated in test examples, using numerical measures proposed by the authors.

Zervas, et al. [476] proposed a multisensor data fusion based method for fire detection. The authors described the system architecture and the application of Dempster-Shafer evidential theory for inference on the probability of fire in a geographical region monitored using a wireless network of environmental (temperature and humidity) and vision sensors. The feasibility of the proposed approach was verified by simulation test results.

[5] A *ventricle* is a heart chamber which collects blood from an *atrium* (another heart chamber that is smaller than a ventricle) and pumps it out of the heart. A *myocardium* is a muscular tissue of the heart. *Ventricular myocardial viability* is the potential for improvement of dysfunction in a ventricular myocardium after a surgical procedure for the provision of a new, additional, or augmented blood supply.

[6] Urban biotope: an area with uniform environment occupied by a unified urban community.

[7] Support vector machine [453]: a constructive machine learning procedure based on statistical learning theory. It can be used to learn a variety of representations, such as neural nets, splines, and so on.

[8] Relevance vector machine [444]: a machine learning technique based on Bayesian theory that has an identical functional form to the support vector machine.

[9] Agrohydrological: of or to do with *agrohydrology*, a research area that deals with climate, soil, and water and how these natural resources are managed in sustainable plant production.

5.7. Industrial Applications

In the recent years, many industrial applications that utilize high-level fusion techniques for problem-solving have emerged [112]. Some instances of research work from prominent areas are reviewed below.

Qiu [354] presented the development of an effective data link between manufacturing and office planning to facilitate the deployment of an integrated plant-wide information system. The information-centric data fusion framework was proposed to help integrate all levels of data, with the aim of achieving synchronization and timely delivery of necessary information, in the information system. Details on the usefulness and practicality of the proposed model in the realization of a desired plant-wide real time information system were described.

A multi-layered fusion architecture and implementation for classifiers with binary and continuous outputs were described by Goebel and Yan [154]. The fusion scheme was structured into three major components which were partitioned into layers. The classifier outputs were transformed into a single continuous domain through logical tasks performed within the layers. The modular design of the fusion architecture allowed relatively easy addition/removal of modules, as well as the re-use of the core fusion engine for other domains. The proposed fusion framework was applied to a system monitoring environment of industrial equipment. The test results obtained were compared to those achieved by a a baseline approach. An improvement in performance over the latter was shown.

Roussel, et al. [371] proposed a Bayesian inference-based fusion method to combine the outputs of various sensors. The mathematical theory concerning the Bayesian approach was discussed and the method was applied to the problem of white grapes variety classification. The classification results verified the effectiveness of the proposed method in grape variety discrimination, an important task for manufacturers in the wine industry who need to determine accurately the origins and/or varieties of the grapes used for production.

Majidi and Moshiri [289] presented a computer vision system for classification of fruits. Estimation of the volume of a fruit was carried out by training a neural network with simple features of profile images of the fruit. Inspection of fruit surface defects was based on fusion of side images of the whole area of the fruit. A set of basic color parameters of the fruit surface was then extracted and the fruit was classified via high level fusion of these visual features. Test results showed that the proposed method had acceptable performance in regard to the execution time required.

Ong and Ibañez-Guzmán [333] reviewed multi-sensor management for sensor fusion with respect to the guidance of unmanned vehicles. An information-oriented concept of perception management was introduced for multi-sensor systems. An outline of the concept of a design framework for sensor perception system was also given.

De Vin, et al. [118, 119] reported how information fusion research could benefit manufacturing applications. One particular area of interest was virtual manufacturing. An information fusion framework involving modeling and simulation was proposed for decision support in manufacturing. Relevant fused information regarding the past, present and future of the manufacturing system were extracted for future use. Interaction of the information fusion process with active databases (capable of propagating abnormal conditions or events to decision level), sensors and the simulation model was described. In [118], they also discussed some analogies between manufacturing and defense tasks, as well as aspects in which the manufacturing sector could benefit from defense research.

Razavi, et al. [361] developed a belief function-based data fusion algorithm for detecting dislocations (changes between discrete sequential locations) of materials on a construction site. The authors focused on the detection of dislocations in a noisy information environment. Each piece of material to be monitored had a Radio Frequency Identification tag attached to it. The technical feasibility and the cost-effectiveness of the proposed method were demonstrated by the implementation results in a construction field experiment.

6. SUMMARY AND FURTHER RESEARCH

In this survey paper, we review some process models that have been developed for data and information fusion. We also present an overview of research publications related to high-level information fusion, which is gaining interest in the recent years after much research focus has been placed on low-level information fusion. We also discuss relevant application areas that involve high-level data and information fusion.

Active research and development on high-level fusion is ongoing among the DIF community. There are many important topics and techniques that have not been covered in this paper. Some examples are belief networks, situation logic, network analysis, graph theory, social network analysis, scene and situation characterization and multi-resolution inferencing. Future research areas of interest include the following examples.

- Comparative assessment of different functional and process models for data/information fusion;
- Critical or comparative evaluation of high-level fusion techniques in applicability to various application problems: functionality, uncertainty, complexity, data and state diversity and dynamics, knowledge representation, knowledge extraction and discovery, context exploitation, situation characterization and prediction, and so on.

TABLE V
Topics for Further Exploration

Topic	Reference
Adversarial intent inference	[12, 25, 28, 38, 86, 136, 139, 140, 143, 147, 153, 198, 210, 247, 249, 284, 298, 317, 319–321, 344, 363, 365, 372, 388, 394, 401, 419, 432, 437, 438, 472]
Biologically-inspired and biomedical applications/informatics	[18, 75, 114, 144, 167, 179, 332, 375, 446]
Electronic and physical anomaly/intrusion detection	[56, 69, 78, 103, 115, 163, 182, 206, 207, 231, 264, 287, 310, 365, 396, 426, 441, 442, 449, 456, 472, 474]
Human cognition related research (cognitive fusion, HCI, etc.)	[30, 39, 116, 179, 181, 326, 412]
Image analysis/processing	[74, 185, 342, 458]
Information warfare	[84, 126, 246, 254, 468]
Interoperability of joint and coalition military forces	[123, 335, 336, 345, 462]
Network-centric warfare/operations and network-based defense	[150, 335, 336, 372, 435, 468]
Ontology-based approaches to high-level information fusion	[35, 44, 60, 72, 77, 82, 83, 105, 204, 237, 265, 275–277, 299, 304, 315, 316, 329, 364, 408, 410, 475]
Resource allocation/management	[3, 6, 11, 13, 15, 31, 45, 64, 73, 75, 91, 125, 138, 189–192, 196, 197, 211, 215, 255, 288, 312, 316, 358, 386, 389, 403, 431, 437, 459, 460]

The variety of application areas which apply DIF techniques has increased tremendously since they were first applied in defense research in the 1970s. The scope of applications is still expanding fast, both in the military arena and civilian sectors (including commercial and industrial applications). Table V provides some examples of high-level fusion concepts and contexts with much potential for exploration.

With rapid advancement in various technologies and accessibility to vast data and information sources, complex information fusion problems are very likely to arise in many applications that involve far more concepts and contexts than the few listed above. It is becoming increasingly necessary to explore the possibility of expanding the base of diverse disciplines (including theories and techniques) upon which existing tools have been built. A lot more research is needed and can be done to develop novel useful tools (including theories, algorithms and architectures) for solving high-level information fusion problems.[10] In addition, efficiency and effectiveness in this multidisciplinary field of research are likely to be enhanced if collaborative relationships can be established/strengthened among the various research groups [9, 278].

APPENDIX

Table VI: List of acronyms.

[10]The following paper surveys various topics and challenges in high-level information fusion: E. P. Blasch, D. A. Lambert, P. Valin, M. M. Kokar, J. Llinas, S. Das, C. Chong, and E. Shahbazian, High level information fusion (HLIF): Survey of models, issues, and grand challenges, *IEEE Aerospace and Electronic Systems Magazine*, **27**, 9 (2012), 4–20.

TABLE VI
List of Acronyms

Acronym	Definition
ANOVA	Analysis of variance
ATR	Automatic target recognition
C2	Command and control
C4I	Command, control, communications, computers and intelligence
C4ISR	Command, control, communications, computers, intelligence, surveillance and reconnaissance
COA	Course of action
DF	Data fusion
DFIG	Data Fusion Information Group
DIF	Data and information fusion
D-S	Dempster-Shafer
DOE	Design of experiments
EW	Early warning
HCI	Human-computer interaction
HRR	High range resolution
IF	Information fusion
INTEL	Intelligence
JDL	Joint Directors of Laboratories
NBD	Network-based defense
NCBR	Nuclear, chemical, biological and radiological
NCW	Network-centric warfare
OODA	Observe, orient, decide, and act
OSPA	Optimal subpattern assignment
PA	Performance assessment
PE	Performance evaluation
RADAR	Radio detecting and ranging
SA	Situation assessment
SAW	Situation awareness
SM	Sensor management
SONAR	Sound navigation and ranging
TRIP	Transformation of Requirements for the Information Process
VDF	Visual Data-Fusion

REFERENCES

[1] S. Acharya, D. Bucci, and M. Kam
Representation and fusion of conditionally refined opinions using evidence trees.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 506.

[2] S. Acharya and M. Kam
Evidence combination for hard and soft sensor data fusion.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 133.

[3] H. Aftab, N. Raj, P. Cuff, and S. Kulkarni
Mutual information scheduling for ranking.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 160.

[4] S. Ahlberg, P. Hörling, K. Johansson, K. Jöred, H. Kjellström, C. Mårtenson, G. Neider, J. Schubert, P. Svenson, P. Svensson, and J. Walter
An information fusion demonstrator for tactical intelligence processing in network-based defense.
Information Fusion, **8**, 1 (2007), 84–107.

[5] T. Aldinger and J. Kao
Data fusion and theater undersea warfare—an oceanographer's perspective.
In *Proceedings of the MTS/IEEE Oceans Conference*, 4 (2004), 2008–2012.

[6] H. Alex, M. Kumar, and B. Shirazi
MidFusion: An adaptive middleware for information fusion in sensor network applications.
Information Fusion, **9**, 3 (2008), 332–343.

[7] M. K. Allouche
Real-time use of Kohonen's self-organizing maps for threat stabilization.
Information Fusion, **6**, 2 (2005), 153–163.

[8] X. D. An and G. Q. Liu
Application of neural network in the field of target threat evaluation.
In *Proceedings of the IEEE International Joint Conference on Neural Networks*, 6 (1999), 4237–4240.

[9] C. Anken, N. Gemelli, P. LaMonica, R. Mineo, and J. Spina
Intelligent systems technology for higher level fusion.
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 1316–1323.

[10] R. Antony and J. Karakowski
First-principle approach to functionally decomposing the JDL fusion model: Emphasis on soft target data.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14.

[11] E. Aoki, A. Bagchi, P. Mandal, and Y. Boers
A theoretical look at information-driven sensor management criteria.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 156.

[12] G. Arnold, A. M. Fullenkamp, A. Bornstein, F. Morelli, T. Brown, P. Iyer, J. Lavery, S. McElhinny, E. Schmeisser, M. Strand, J. A. Karakowski, M. Scanlon, and N. Srour
Research directions in remote detection of covert tactical adversarial intent of individuals in asymmetric operations.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[13] G. Asnis and S. Blackman
Optimal allocation of multi-platform sensor resources for multiple target tracking.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 157.

[14] A. Auger and J. Roy
Expression of uncertainty in linguistic data.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14 II.

[15] J. M. Aughenbaugh and B. R. La Cour
Measurement prioritization for optimal Bayesian fusion.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1423.

[16] V. Avasarala, T. Mullen, and D. Hall
A market-based approach to sensor management.
Journal of Advances in Information Fusion, **4**, 1 (2009), 52–71.

[17] K. Baclawski, M. Kokar, J. Letkowski, C. Matheus, and M. Malczewski
Formalization of situation awareness.
In *Proceedings of the OOPSLA Workshop on Behavioral Semantics*, (2002), 1–15.

[18] D. Banks, G. Datta, A. Karr, J. Lynch, J. Niemi, and F. Vera
Bayesian CAR models for syndromic surveillance on multiple data streams: Theory and practice.
Information Fusion, **13**, 2 (2012), 105–116.

[19] Y. Bar-Shalom, X-R. Li, and T. Kirubarajan
Estimation with Applications to Tracking and Navigation: Theory, Algorithms, and Software.
John Wiley & Sons, 2001.

[20] M. Baran, D. Natale, R. Tutwiler, M. McQuillan, C. Griffin, J. Daughtry, J. Rimland, and D. Hall
Hard sensor fusion for COIN inspired situation awareness.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 135.

[21] J. Barker, R. Green, P. Thomas, G. Brown, and D. Salmond
Information fusion based decision support via hidden Markov models and time series anomaly detection.
In *Proceedings of the International Conference on Information Fusion*, (2009), 764–771.

[22] A. Bashi, V. P. Jilkov, and X. R. Li
Fault detection for systems with multiple unknown modes and similar units—Part I.
In *Proceedings of the International Conference on Information Fusion*, (2009), 732–739.

[23] A. Bashi, V. P. Jilkov, and X. R. Li
Fault detection for systems with multiple unknown modes and similar units—Part II: Application to HVAC.
In *Proceedings of the International Conference on Information Fusion*, (2009), 740–747.

[24] O. Basir and X. Yuan
Engine fault diagnosis based on multi-sensor information fusion using Dempster-Shafer evidence theory.
Information Fusion, **8**, 4 (2007), 379–386.

[25] R. Baxter, N. M. Robertson, and D. Lane
Probabilistic behaviour signatures: Feature-based behaviour recognition in data-scarce domains.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[26] J. M. Beaver, R. A. Kerekes, and J. N. Treadwell
An information fusion framework for threat assessment.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1903–1910.

[27] M. Bedworth and J. O'Brien
The Omnibus model: A new model of data fusion?
IEEE Aerospace and Electronic Systems Magazine, **15**, 4 (2000), 30–36.

[28] B. Bell, E. Santos, Jr., and S. M. Brown
Making adversary decision modeling tractable with intent inference and information fusion.
In *Proceedings of the Conference on Computer Generated Forces and Behavioral Representation*, (2002), 535–542.

[29] D. Bellot, A. Boyer, and F. Charpillet
A new definition of qualified gain in a data fusion process: Application to telemedicine.
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 865–872.

[30] N. Belov and P. Gerken
Mixed initiative soft data fusion associate.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1897–1902.

[31] A. R. Benaskeur, H. Irandoust, P. Mcguire, and R. Brennan
Holonic control-based sensor management.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1132.

[32] A. Benavoli, B. Ristic, A. Farina, M. Oxenham, and L. Chisci
An approach to threat assessment based on evidential networks.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1213.

[33] A. Benavoli, B. Ristic, A. Farina, M. Oxenham, and L. Chisci
An application of evidential networks to threat assessment.
IEEE Transactions on Aerospace and Electronic Systems, **45**, 2 (2009), 620–639.

[34] A. Berrached, M. Beheshti, A. de Korvin, and R. Alo
Applying fuzzy relation equations to threat analysis.
In *Proceedings of the Annual Hawaii International Conference on System Sciences*, (2002), 684–688.

[35] J. Besombes, V. Nimier, and L. Cholvy
Information evaluation in fusion using information correlation.
In *Proceedings of the International Conference on Information Fusion*, (2009), 264–269.

[36] J. Besombes and A. Revault d'Allonnes
An extension of STANAG2022 for information scoring.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14.

[37] S. S. Blackman and R. Popoli
Design and Analysis of Modern Tracking Systems.
Artech House, 1999.

[38] P. Bladon, P. S. Day, T. Hughes, and P. Stanley
High-level fusion using Bayesian networks: Applications in command and control.
In [328, Paper 4].

[39] E. Blasch
Assembling a distributed fused information-based human-computer cognitive decision making tool.
IEEE Aerospace and Electronic Systems Magazine, **15**, 5 (2000), 11–17.

[40] E. Blasch
Level 5 (user refinement) issues supporting information fusion management.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 130.

[41] E. Blasch
Sensor, user, mission (SUM) resource management and their interaction with Level 2/3 fusion.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 394.

[42] E. Blasch
Introduction to Level 5 fusion: The role of the user.
In [272, Chap. 19].

[43] E. Blasch, P. C. G. Costa, K. B. Laskey, D. Stampouli, G. W. Ng, J. Schubert, R. Nagi, and P. Valin
Issues of uncertainty analysis in high-level information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2012.

[44] E. Blasch, E. Dorion, P. Valin, É. Bossé, and J. Roy
Ontology alignment in geographical hard-soft information fusion systems.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[45] E. Blasch, I. Kadar, K. Hintz, J. Biermann, C. Chong, J. Salerno, and S. Das
Resource management and its interaction with Level 2/3 fusion—from the Fusion-2006 panel discussion.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1203.

[46] E. Blasch, I. Kadar, J. Salerno, M. M. Kokar, S. Das, G. M. Powell, D. D. Corkill, and E. H. Ruspini
Issues and challenges in situation assessment (Level 2 fusion).
Journal of Advances in Information Fusion, **1**, 2 (2006), 122–139.

[47] E. Blasch, J. Llinas, D. Lambert, P. Valin, S. Das, C. Chong, M. Kokar, and E. Shahbazian
High level information fusion developments, issues and grand challenges.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[48] E. Blasch and S. Plano
JDL Level 5 fusion model: User refinement issues and applications in group tracking.
Proceedings of SPIE, 4729 (2002), 270–279.

[49] E. Blasch and S. Plano
Level 5: User refinement to aid the fusion process.
Proceedings of SPIE, 5099 (2003), 288–297.

[50] E. Blasch and S. Plano
Ontological issues in higher levels of information fusion: User refinement of the fusion process.
In *Proceedings of the International Conference on Information Fusion*, 1 (2003), 634–641.

[51] E. Blasch and S. Plano
DFIG Level 5 (user refinement) issues supporting situational assessment reasoning.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), xxxv–xliii.

[52] E. Blasch and P. Valin
Track purity and current assignment ratio for target tracking and identification evaluation.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 208.

[53] E. Blasch, P. Valin, and É. Bossé
Measures of effectiveness for high-level fusion.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[54] E. Blasch, P. Valin, É. Bossé, M. Nilsson, J. van Laere, and E. Shahbazian
Implication of culture: User roles in information fusion for enhanced situational understanding.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1272–1279.

[55] Y. Boers and H. Driessen
Modified Riccati equation and its application to target tracking.
IEE Proceedings—Radar, Sonar and Navigation, **153**, 1 (2006), 7–12.

[56] N. A. Bomberger, B. J. Rhodes, M. Seibert, and A. M. Waxman
Associative learning of vessel motion patterns for maritime situation awareness.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 234.

[57] N. A. Bomberger, A. M. Waxman, B. J. Rhodes, and N. A. Sheldon
A new approach to higher-level information fusion using associative learning in semantic networks of spiking neurons.
Information Fusion, **8**, 3 (2007), 227–251.

[58] É. Bossé, J. Roy, and S. Paradis
Modeling and simulation in support of the design of a data fusion system.
Information Fusion, **1**, 2 (2000), 77–87.

[59] É. Bossé, J. Roy, and S. Wark
Concepts, Models, and Tools for Information Fusion.
Artech House, 2007.

[60] É. Bossé, P. Valin, A-C. Boury-Brisset, and D. Grenier
Exploitation of a priori knowledge for information fusion.
Information Fusion, **7**, 2 (2006), 161–175.

[61] T. C. Bradley
Dynamic resource allocation and management for Level 4 fusion.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1459–1466.

[62] C. Brax, A. Karlsson, S. F. Andler, R. Johansson, and L. Niklasson
Evaluating precise and imprecise state-based anomaly detectors for maritime surveillance.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[63] C. Brax, L. Niklasson, and R. Laxhammar
An ensemble approach for increased anomaly detection performance in video surveillance data.
In *Proceedings of the International Conference on Information Fusion*, (2009), 694–701.

[64] T. Bréhard, P-A. Coquelin, E. Duflos, and P. Vanheeghe
Optimal policies search for sensor management: Application to the ESA radar.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS04.

[65] B. Brehmer
The dynamic OODA loop: Amalgamating Boyd's OODA loop and the cybernetic approach to command and control.
In *Proceedings of the International Command and Control Research and Technology Symposium*, 2005.

[66] R. Breton and R. Rousseau
THE C-OODA: A cognitive version of the OODA loop to represent C2 activities.
In *Proceedings of the International Command and Control Research and Technology Symposium*, 2005.

[67] R. Browne
C4I defensive infrastructure for survivability against multi-mode attacks.
In *Proceedings of the 21st Century Military Communications Conference*, 1 (2000), 417–424.

[68] D. W. Burgess and C. L. Levins
Intelligent sensor resource management using evolutionary computing techniques.
In *Proceedings of the International Conference on Integration of Knowledge Intensive Multi-Agent Systems*, (2003), 325–329.

[69] S. R. Byers and S. J. Yang
Real-time fusion and projection of network intrusion activity.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session RS07.

[70] Y-C. Cai, L. He, W-M. Zhang, and Z. Liu
A situation and threat assessment model based on group analysis.
In *Proceedings of the International Conference on Machine Learning and Cybernetics*, 1 (2005), 356–361.

[71] R. Canavan, C. McCullough, and W. J. Farrell
Track-centric metrics for track fusion systems.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1147–1154.

[72] G. T. Capraro, G. B. Berdan, J. Spina, R. A. Liuzzi, and P. B. Berra
An architecture for providing information anytime, anywhere and on any device—an ontological approach.
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 1331–1339.

[73] A. Carmi
Sensor scheduling via compressed sensing.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[74] G. A. Carpenter, S. Martens, and O. J. Ogas
Self-organizing information fusion and hierarchical knowledge discovery: A new framework using ARTMAP neural networks.
Neural Networks, **18**, 3 (2005), 287–295.

[75] H. S. Carvalho, W. B. Heinzelman, A. L. Murphy, and C. J. N. Coelho
A general data fusion architecture.
In *Proceedings of the International Conference on Information Fusion*, 2 (2003), 1465–1472.

[76] R. N. Carvalho and K. C. Chang
A performance evaluation tool for multi-sensor classification systems.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1123–1130.

[77] R. N. Carvalho, P. C. G. Costa, K. B. Laskey, and K. C. Chang
PROGNOS: Predictive situational awareness with probabilistic ontologies.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[78] R. N. Carvalho, R. Haberlin, P. C. G. Costa, K. B. Laskey, and K. C. Chang
Modeling a probabilistic ontology for maritime domain awareness.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 170.

[79] M. G. Ceruti, A. Ashenfelter, R. Brooks, G. Chen, S. Das, G. Raven, M. Sudit, and E. Wright
Pedigree information for enhanced situation and threat assessment.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 43.

[80] H-M. Chai and B-S. Wang
A fuzzy logic approach for force aggregation and classification in situation assessment.
In *Proceedings of the International Conference on Machine Learning and Cybernetics*, 3 (2007), 1220–1225.

[81] K. C. Chang and J. P. Hill
Level I and Level II target valuations for sensor management.
Journal of Advances in Information Fusion, **1**, 2 (2006), 95–107.

[82] S. K. Chang, E. Jungert, and X. Li
A progressive query language and interactive reasoner for information fusion support.
Information Fusion, **8**, 1 (2007), 70–83.

[83] A. I. Chao, E. Starczewski, B. C. Krikeles, and A. E. Lusignan
An extensible, ontology-based, distributed information system architecture.
In *Proceedings of the International Conference on Information Fusion*, 1 (2003), 642–649.

[84] A. R. Chaturvedi, M. Gupta, S. R. Mehta, and W. T. Yue
Agent-based simulation approach to information warfare in the SEAS (Synthetic Environment for Analysis and Simulation) environment.
In *Proceedings of the Annual Hawaii International Conference on System Sciences*, 2000.

[85] Y. Chemin and K. Honda
Spatiotemporal fusion of rice actual evapotranspiration with genetic algorithms and an agrohydrological model.
IEEE Transactions on Geoscience and Remote Sensing, **44**, 11 Part 2 (2006), 3462–3469.

[86] G. Chen, D. Shen, C. Kwan, J. B. Cruz, Jr., and M. Kruger
Game theoretic approach to threat prediction and situation awareness.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 245.

[87] H. Chen, G. Chen, E. P. Blasch, P. Douville, and K. Pham
Information theoretic measures for performance evaluation and comparison.
In *Proceedings of the International Conference on Information Fusion*, (2009), 874–881.

[88] K. Chen, X-S. Luo, and X. Liu
Sensor resource management research based on intelligent agent in naval multi-platform cooperative engagement.
In *Proceedings of the International Conference on Machine Learning and Cybernetics*, 1 (2005), 132–136.

[89] X. Chen, W. Hu, and H. Yang
A probabilistic fuzzy method for emitter identification based on genetic algorithm.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 164.

[90] X-Z. Chen, Q-H. Zheng, X-H. Guan, C-G. Lin, and J. Sun
Multiple behavior information fusion based quantitative threat evaluation.
Computers & Security, **24**, 3 (2005), 218–231.

[91] B. Cheung, S. Davey, and D. Gray
Comparison of the PMHT path planning algorithm with the genetic algorithm for multiple platforms.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[92] S. Chin, J. Hill, and X. Shen
Opportunistic sensor resource management for extended regions.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1138–1145.

[93] J. W. Choi, J. W. Joo, and D. L. Cho
Situation/threat assessment fusion system (STAFS).
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 1374–1380.

[94] C-Y. Chong and M. Liggins, II
Fusion technologies for drug interdiction.
In *Proceedings of the IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems*, (1994), 435–441.

[95] R. Chou, Y. Boers, M. Podt, and M. Geist
Performance evaluation for particle filters.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 247.

[96] M. Conti, R. Di Pietro, L. V. Mancini, and A. Mei
Distributed data source verification in wireless sensor networks.
Information Fusion, **10**, 4 (2009), 342–353.

[97] I. Corona, G. Giacinto, C. Mazzariello, F. Roli, and C. Sansone
Information fusion for computer security: State of the art and open issues.
Information Fusion, **10**, 4 (2009), 274–284.

[98] P. C. G. Costa, E. Blasch, K. B. Laskey, S. Andler, J. Dezert, A-L. Jousselme, and G. Powell
Uncertainty evaluation: Current status and major challenges.
In *Proceedings of the International Conference on Information Fusion*, 2012.

[99] P. C. G. Costa, R. N. Carvalho, K. B. Laskey, and C. Y. Park
Evaluating uncertainty representation and reasoning in HLF systems.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 26.

[100] P. C. G. Costa, K-C. Chang, K. B. Laskey, and R. N. Carvalho
A multi-disciplinary approach to high level fusion in predictive situational awareness.
In *Proceedings of the International Conference on Information Fusion*, (2009), 248–255.

[101] P. C. G. Costa, K-C. Chang, K. B. Laskey, T. Levit, and W. Sun
High-level fusion: Issues in developing a formal theory.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[102] I. W. Dall
Threat assessment without situation assessment.
In *Proceedings of International Conference on Information, Decision and Control*, (1999), 365–370.

[103] F. Dambreville
Evaluation of a sentry system against noisy optimal intrusions.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session RS07.

[104] F. Dambreville and J-P. Le Cadre
Spatio-temporal multi-mode information management for moving target detection.
Information Fusion, **5**, 3 (2004), 169–178.

[105] R. Dapoigny and P. Barlatier
Formal foundations for situation awareness based on dependent type theory.
Information Fusion, **14**, 1 (2013), 87–107.

[106] S. Das
High-level Data Fusion.
Artech House, 2008.

[107] S. Das, R. Grey, and P. Gonsalves
Situation assessment via Bayesian belief networks.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 664–671.

[108] S. Das and D. Lawless
Truth maintenance system with probabilistic constraints for enhanced Level two fusion.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 993–1000.

[109] S. Das, J. Llinas, G. Pavlin, D. Snyder, A. Steinberg, and K. Sycara
Agent based information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2007.

[110] B. V. Dasarathy
Decision Fusion.
IEEE Computer Society Press, 1994.

[111] B. V. Dasarathy
Sensor fusion potential exploitation—innovative architectures and illustrative applications.
Proceedings of the IEEE, **85**, 1 (1997), 24–38.

[112] B. V. Dasarathy
Industrial applications of multi-sensor multi-source information fusion.
In *Proceedings of the IEEE International Conference on Industrial Technology*, 1 (2000), 5–11.

[113] B. V. Dasarathy
Information fusion—what, where, why, when, and how?
Information Fusion, **2**, 2 (2001), 75–76.

[114] B. V. Dasarathy
Information fusion in the context of biomedical applications.
Information Fusion, **3**, 2 (2002), 89.

[115] B. V. Dasarathy
Intrusion detection.
Information Fusion, **4**, 4 (2003), 243–245.

[116] B. V. Dasarathy
Information fusion in the context of human-machine interfaces.
Information Fusion, **6**, 2 (2005), 117–118.

[117] S. G. Dastidar, K. Sambhoos, J. Llinas, and C. Bowman
Performance evaluation methods for data-fusion capable tactical platforms.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1427–1435.

[118] L. J. De Vin, S. F. Andler, A. H. C. Ng, P. R. Moore, J. Pu, and B. C-B. Wong
Information fusion: What can the manufacturing sector learn from the defence industry?
In *Proceedings of the International Manufacturing Conference*, (2005), 363–371.

[119] L. J. De Vin, A. H. C. Ng, J. Oscarsson, and S. F. Andler
Information fusion for simulation based decision support in manufacturing.
In *Proceedings of the International Conference on Flexible Automation and Intelligent Manufacturing*, (2005), 136–144.

[120] J. Dezert, D. Han, Z. Liu, and J-M. Tacnet
Hierarchical DSmP transformation for decision-making under uncertainty.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 96.

[121] J. Dezert, P. Wang, and A. Tchamova
On the validity of Dempster-Shafer Theory.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 97.

[122] S. S. Dhillon, K. Chakrabarty, and S. S. Iyengar
Sensor placement for grid coverage under imprecise detections.
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 1581–1587.

[123] E. Dorion, C. J. Matheus, and M. M. Kokar
Towards a formal ontology for military coalitions operations.
In *Proceedings of the International Command and Control Research and Technology Symposium*, 2005.

[124] H. Du, C. Murphy, J. Bean, and S. J. Yang
Toward unsupervised classification of non-uniform cyber attack tracks.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1919–1925.

[125] E. Duflos, M. de Vilmorin, and P. Vanheeghe
Time allocation of a set of radars in a multitarget environment.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1260.

[126] A. J. Elbirt
Information warfare: Are you at risk?
IEEE Technology and Society Magazine, **22**, 4 (2003/2004), 13–19.

[127] M. R. Endsley
Towards a theory of situation awareness in dynamic systems.
Human Factors, **37**, 1 (1995), 32–64.

[128] M. R. Endsley
Theoretical underpinnings of situation awareness: A critical review.
In M. R. Endsley, D. J. Garland (Eds.), *Situation Awareness: Analysis and Measurement*, Lawrence Earlbaum Associates, 2000, pp. 3–32.

[129] E. B. Entin
The effects of dynamic updating of tactical information on situation awareness and performance in an attack helicopter domain.
In *Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics*, 4 (1998), 3602–3607.

[130] K. Er, U. R. Acharya, N. Kannathal, and C. M. Lim
Data fusion of multimodal cardiovascular signals.
In *Proceedings of the Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, (2005), 4689–4692.

[131] T. Erlandsson, T. Helldin, G. Falkman, and L. Niklasson
Information fusion supporting team situation awareness for future fighting aircraft.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[132] J. Esteban, A. Starr, R. Willetts, P. Hannah, and P. Bryanston-Cross
A review of data fusion models and architectures: Towards engineering guidelines.
Neural Computing and Applications, **14**, 4 (2005), 273–281.

[133] X. Fan and M. J. Zuo
Fault diagnosis of machines based on D-S evidence theory. Part 1: D-S evidence theory and its improvement.
Pattern Recognition Letters, **27**, 5 (2006), 366–376.

[134] X. Fan and M. J. Zuo
Fault diagnosis of machines based on D-S evidence theory. Part 2: Application of the improved D-S evidence theory in gearbox fault diagnosis.
Pattern Recognition Letters, **27**, 5 (2006), 377–385.

[135] B. Fassinut-Mombot and J-B. Choquel
A new probabilistic and entropy fusion approach for management of information sources.
Information Fusion, **5**, 1 (2004), 35–47.

[136] D. Fava, J. Holsopple, S. J. Yang, and B. Argauer
Terrain and behavior modeling for projecting multistage cyber attacks.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1384.

[137] G. Ferrin, L. Snidaro, S. Canazza, and G. L. Foresti
Soft data issues in fusion of video surveillance.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14 II.

[138] R. Focke, L. Wabeke, J. de Villiers, and M. Inggs
Implementing interval algebra to schedule mechanically scanned multistatic radars.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 159.

[139] P. H. Foo, G. W. Ng, K. H. Ng, and R. Yang
Application of intent inference for surveillance and conformance monitoring to aid human cognition.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1309.

[140] P. H. Foo, G. W. Ng, K. H. Ng, and R. Yang
Application of intent inference for surveillance and conformance monitoring.
Journal of Advances in Information Fusion, **4**, 1 (2009), 3–26.

[141] F. Fooladvandi, C. Brax, P. Gustavsson, and M. Fredin
Signature-based activity detection based on Bayesian networks acquired from expert knowledge.
In *Proceedings of the International Conference on Information Fusion*, (2009), 436–443.

[142] G. Fountain and S. Drager
High performance real-time fusion architecture.
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 1478–1485.

[143] J. Franke, S. M. Brown, B. Bell, and H. Mendenhall
Enhancing teamwork through team-level intent inference.
In *Proceedings of the International Conference on Artificial Intelligence*, 2000.

[144] R. D. Fricker and D. Banschbach
Optimizing biosurveillance systems that use threshold-based event detection methods.
Information Fusion, **13**, 2 (2012), 117–128.

[145] A. Gad and M. Farooq
Data fusion architecture for maritime surveillance.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 448–455.

[146] S. R. Ganta, J. Kasturi, J. Gilbertson, and R. Acharya
An online analysis and information fusion platform for heterogeneous biomedical informatics data.
In *Proceedings of the IEEE Symposium on Computer-Based Medical Systems*, (2005), 153–158.

[147] B. J. Garrick, J. E. Hall, M. Kilger, J. C. McDonald, T. O'Toole, P. S. Probst, E. R. Parker, R. Rosenthal, A. W. Trivelpiece, L. A. Van Arsdale, and E. L. Zebroski
Confronting the risks of terrorism: Making the right decisions.
Reliability Engineering and System Safety, **86**, 2 (2004), 129–176.

[148] A. E. Gelfand, C. Smith, M. Colony, and C. Bowman
Performance evaluation of decentralized estimation systems with uncertain communication.
In *Proceedings of the International Conference on Information Fusion*, (2009), 786–793.

[149] J. George, J. L. Crassidis, and T. Singh
Threat assessment using context-based tracking in a maritime environment.
In *Proceedings of the International Conference on Information Fusion*, (2009), 187–194.

[150] R. Ghanadan, P. Tufano, J. Hsu, J. Gu, and C. Connelly
Flexible access secure transfer (FAST) tactical communications waveform for airborne networking.
In *Proceedings of the IEEE Military Communications Conference*, (2005), 1–7.

[151] F. Ghassemi and V. Krishnamurthy
Upper bounds for the sensor subset selection problem.
In *Proceedings of the International Conference on Information Fusion*, (2009), 110–117.

[152] G. Giacinto, F. Roli, and C. Sansone
Information fusion in computer security.
Information Fusion, **10**, 4 (2009), 272–273.

[153] R. Glinton, J. Giampapa, and K. Sycara
A Markov random field model of context for high-level information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 375.

[154] K. F. Goebel and W. Yan
Fusing binary and continuous output of multiple classifiers.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 380–387.

[155] J. Gómez-Romero, J. García, M. Kandefer, J. Llinas, J. M. Molina, M. A. Patricio, M. Prentice, and S. C. Shapiro
Strategies and techniques for use and exploitation of contextual information in high-level fusion architectures.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[156] J. Gómez-Romero, M. A. Patricio, J. García, and José M. Molina
Ontological respresentation of context knowledge for visual data fusion.
In *Proceedings of the International Conference on Information Fusion*, (2009), 2136–2143.

[157] P. G. Gonsalves, J. E. Burge, and K. A. Harper
Architecture for genetic algorithm-based threat assessment.
In *Proceedings of the International Conference on Information Fusion*, 2 (2003), 965–971.

[158] P. G. Gonsalves, R. Cunningham, N. Ton, and D. Okon
Intelligent threat assessment processor (ITAP) using genetic algorithms and fuzzy logic.
In *Proceedings of the International Conference on Information Fusion*, 2 (2000), ThB1-18–ThB1-24.

[159] P. G. Gonsalves and G. J. Rinkus
Intelligent fusion and asset management processor.
In *Proceedings of the IEEE Information Technology Conference*, (1998), 15–18.

[160] A. A. H. E. Goossens, G. J. M. Koeners, and E. Theunissen
Development and evaluation of Level 3 situation awareness support functions for a UAV operator station.
In *Proceedings of the Digital Avionics Systems Conference*, 2 (2004), 12.D.4-1–12.D.4-8.

[161] A. Gorji, R. Tharmaras,a and T. Kirubarajan
Performance measures for multiple target tracking problems.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 206.

[162] V. Gorodetsky, O. Karsaev, and V. Samoilov
On-line update of situation assessment: A generic approach.
International Journal of Knowledge-Based and Intelligent Engineering Systems, **9**, 4 (2005), 351–365.

[163] J. H. Graham and Y. Yu
Computer system security threat evaluation based upon artificial immunity model and fuzzy logic.
In *Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics*, 2 (2005), 1297–1302.

[164] J. L. Graham, J. Rimland, and D. Hall
A COIN-inspired synthetic dataset for qualitative evaluation of hard and soft fusion systems.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 132.

[165] C. Grappiolo, S. Whiteson, G. Pavlin, and B. Bakker
Integrating distributed Bayesian inference and reinforcement learning for sensor management.
In *Proceedings of the International Conference of Information Fusion*, (2009), 93–101.

[166] S. Green and R. Watkin
Information provision for the dismounted close combat soldier.
In *Proceedings of the International Conference of Information Fusion*, 2010.

[167] J. Greensmith, U. Aickelin, and G. Tedesco
Information fusion for anomaly detection with the dendritic cell algorithm.
Information Fusion, **11**, 1 (2010), 21–34.

[168] G. Gross, R. Nagi, and K. Sambhoos
Soft information, dirty graphs and uncertainty representation/processing for situation understanding.
In *Proceedings of the International Conference of Information Fusion*, 2010.

[169] G. Gross, R. Nagi, and K. Sambhoos
Continuous preservation of situational awareness through incremental/stochastic graphical methods.
In *Proceedings of the International Conference of Information Fusion*, 2011, Paper 136.

[170] G. Gross, R. Nagi, and K. Sambhoos
Towards hard+soft data fusion: Processing architecture and implementation for the joint fusion and analysis of hard and soft intelligence data.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 450.

[171] M. Guerriero, L. Svensson, D. Svensson, and P. Willett
Shooting two birds with two bullets: How to find minimum mean OSPA estimates.
In *Proceedings of the International Conference of Information Fusion*, 2010.

[172] L. Guibas
The identity management problem—A short survey.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS13.

[173] B. Guo, M. S. Nixon, and T. Damarla
Acoustic vehicle classification by fusing with semantic annotation.
In *Proceedings of the International Conference of Information Fusion*, (2009), 232–239.

[174] C. M. Hall, D. L. Hall, S. A. H. McMullen, M. J. McMullen, and B. K. Pursel
Perspectives on visualization and virtual world technologies for mult-sensor data fusion.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14.

[175] D. L. Hall, J. Graham, L. D. More, and J. C. Rimland
Test and evaluation of soft/hard information fusion systems: A test environment, methodology and initial data sets.
In *Proceedings of the International Conference of Information Fusion*, 2010.

[176] D. L. Hall and J. Llinas
An introduction to multisensor data fusion.
Proceedings of the IEEE, **85**, 1 (1997), 6–23.

[177] D. L. Hall and J. Llinas (Eds.)
Handbook of Multisensor Data Fusion.
CRC Press, 2001.

[178] D. L. Hall, J. Llinas, M. McNeese, and T. Mullen
A framework for dynamic hard/soft fusion.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS13.

[179] D. L. Hall and S. A. H. McMullen
Mathematical Techniques in Multisensor Data Fusion (2nd ed.).
Artech House, 2004.

[180] D. L. Hall, M. D. McNeese, E. Rotthoff, T. Shaw, and J. Z. Wang
Improving the fusion process using semantic level whole-brain analysis.
In *Proceedings of the Military Sensing Series National Symposium on Sensor and Data Fusion*, 2005.

[181] M. J. M. Hall, S. A. Hall, and T. Tate
Removing the HCI bottleneck: How the human computer interface (HCI) affects the performance of data fusion systems.
In [177, Chap. 19].

[182] E. M. Hamed and T. S. Sobh
A flexible fuzzy threat evaluation computer system.
In *Proceedings of the International Conference on Electrical, Electronic and Computer Engineering*, (2004), 23–27.

[183] D. Han, J. Dezert, and C. Han
New basic belief assignment approximations based on optimization.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 71.

[184] D. Han, J. Dezert, J-M. Tacnet, and C. Han
A fuzzy-cautious OWA approach with evidential reasoning.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 70.

[185] U. Handmann, T. Kalinke, C. Tzomakas, M. Werner, and W. V. Seelen
An image processing system for driver assistance.
Image and Vision Computing, **18**, 5 (2000), 367–376.

[186] M. Hannigan, J. Llinas, and K. Sambhoos
Specificity and merging challenges in soft data association.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 65.

[187] M. Hannigan, D. McMaster, and J. Llinas
Data association and soft data streams.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[188] U. Heiden, K. Segl, S. Roessner, and H. Kaufmann
Ecological evaluation of urban biotope types using airborne hyperspectral HyMap data.
In *Proceedings of the Joint Workshop on Remote Sensing and Data Fusion over Urban Areas*, (2003), 18–22.

[189] A. O. Hero, III, D. A. Castañón, D. Cochran, and K. Kastella (Eds.)
Foundations and Applications of Sensor Management.
Springer, 2008.

[190] A. O. Hero, III and C. M. Kreucher
Network sensor management for tracking and localization.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1459.

[191] J. P. Hill and K. C. Chang
Improved representations of sensor exploitation for automatic sensor management.
In *Proceedings of the International Conference of Information Fusion*, 1 (2003), 688–694.

[192] J. P. Hill and K. C. Chang
Sensor resource management with hierarchical target valuation models.
In *Proceedings of the International Conference on Information Fusion*, 2 (2004), 1187–1194.

[193] J. P. Hill and K. C. Chang
Sensor resource management with Level 2 fusion using Markov chain models.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1444–1450.

[194] F. Hinchion, P. Mulgaonkar, D. Wilkins, and S. Galuga
Peer to peer information management for tactical situation awareness systems.
In *Proceedings of the IEEE Military Communications Conference*, 1 (2003), 179–185.

[195] M. L. Hinman
Some computational approaches for situation assessment and impact assessment.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 687–693.

[196] M. J. Hirsch, R. Nagi, and D. Sudit
A stochastic optimization framework for resource management and course of action analysis.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS04.

[197] M. Hirsch, H. Ortiz-Peña, and M. Sudit
Decentralized cooperative urban tracking of multiple ground targets by a team of autonomous UAVs.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 158.

[198] J. Högberg and L. Kaati
Weighted unranked tree automata as a framework for plan recognition.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[199] P. Hörling (Ed.), C. Mårtenson, J. Schubert, R. Suzić, and P. Svenson
A survey of some information fusion research fields applicable in operations other than war.
User Report FOI-R-2000-SE, Swedish Defence Research Agency, Sweden, 2006.

[200] C. Y. Huang
Real-time automated decision-making in advanced airborne early warning system.
In *Proceedings of the IEEE National Aerospace and Electronics Conference*, 1 (1990), 434–439.

[201] M. F. Huber, A. Kuwertz, F. Sawo. and U. D. Hanebeck
Distributed greedy sensor scheduling for model-based reconstruction of space-time continuous physical phenomena.
In *Proceedings of the International Conference on Information Fusion*, (2009), 102–109.

[202] L. Hubert-Moy, S. Corgne, G. Mercier, and B. Solaiman
Land use and land cover change prediction with the theory of evidence: A case study in an intensive agricultural region of France.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 114–121.

[203] A. Hunter
Belief modeling for maritime surveillance.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1926–1932.

[204] R. Iannella
Modeling and integration of severe weather advisories for situational awareness.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 22.

[205] J. E. Introne, I. Levit, S. Harrison, and S. Das
A data fusion approach to biosurveillance.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1359–1366.

[206] S. Jajodia, P. Liu, V. Swarup, and Cliff Wang (Eds.)
Cyber Situational Awareness—Issues and Research.
Springer, 2010.

[207] G. Jakobson
Mission cyber security situation assessment using impact dependency graphs.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 34.

[208] T. Jan
Neural network based threat assessment for automated visual surveillance.
In *Proceedings of the IEEE International Joint Conference on Neural Networks*, 2 (2004), 1309–1312.

[209] M. Jenkins and A. Bisantz
Identification of human-interaction touch points for intelligence analysis information fusion systems.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 134.

[210] F. Johansson and G. Falkman
A Bayesian network approach to threat evaluation with application to an air defense scenario.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session RS21.

[211] F. Johansson and G. Falkman
Real-time allocation of defensive resources to rockets, artillery, and mortars.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[212] F. Johansson and G. Falkman
SWARD: System for weapon allocation research and development.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[213] L. R. M. Johansson, C. Mårtenson, R. Suzić, and P. Svenson
Stochastic dynamic programming for resource allocation.
Technical report FOI-R-1666-SE, Swedish Defence Research Agency, Sweden, 2005.

[214] L. R. M. Johansson and N. Xiong
Perception management: An emerging concept for information fusion.
Information Fusion, **4**, 3 (2003), 231–234.

[215] R. Johansson and C. Mårtenson
Information acquisition strategies for Bayesian network-based decision support.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[216] W. T. Johnson and I. W. Dall
From kinematics to symbolics for situation and threat assessment.
In *Proceedings of International Conference on Information, Decision and Control*, (1999), 497–502.

[217] R. E. T. Jones, E. S. Connors, and M. R. Endsley
Incorporating the human analyst into the data fusion process by modeling situation awareness using fuzzy cognitive maps.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1265–1271.

[218] J. Josephson
Abductive inferencing for integrating information from human and robotic sources.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS13.

[219] J. R. Josephson, J. Eckroth, and T. N. Miller
Estimation of adversarial social networks by fusion of information from a wide range of sources.
In *Proceedings of the International Conference on Information Fusion*, (2009), 2144–2152.

[220] I. Kadar
Knowledge representation issues in perceptual reasoning managed situation assessment.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), xiii–xv.

[221] I. Kadar and John Salerno
Issues and challenges in higher level fusion: Threat/impact assessment.
In *Proceedings of the International Conference on Information Fusion*, 2009.

[222] I. Kadar, J. Salerno, J. Biermann, É. Bossé, S. Das, P. Gonsalves, D. Lambert, B. Rhodes, and E. Ruspini
Results from levels 2/3 fusion implementations: Issues, challenges, retrospectives and perspectives for the future.
In *Proceedings of the International Conference on Information Fusion*, 2007.

[223] I. Kadar, P. Svensson, J. Biermann, H. I. Christensen, A. Steinberg, M. M. Kokar, R. P. Mahler, and J. Llinas
Challenges in higher level fusion: Unsolved, difficult, and misunderstood problems/approaches in levels 2–4 fusion research.
In *Proceedings of the International Conference on Information Fusion*, 2004.

[224] B. Kahler and E. Blasch
Impact of HRR radar processing on moving target identification performance.
In *Proceedings of the International Conference on Information Fusion*, (2009), 858–865.

[225] M. Kandefer and S. Shapiro
Evaluating spreading activation for soft information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 66.

[226] N. Kannathal, U. R. Acharya, E. Y. K. Ng, S. M. Krishnan, C. M. Lim, and S. Laxminarayan
Cardiac health diagnosis using data fusion of cardiovascular and haemodynamic signals.
Computer Methods and Programs in Biomedicine, **82**, 2 (2006), 87–96.

[227] J. A. Karakowski
Towards visual data fusion.
In *Proceedings of the Military Sensing Series National Symposium on Sensor and Data Fusion International Open Session*, 1998.

[228] A. Karlsson
Dependable and generic high-level information fusion—Methods and algorithms for uncertainty management.
Technical report HS-IKI-TR-07-003, School of Humanities and Informatics, University of Skövde, Sweden, 2007.

[229] B. Khaleghi, A. Khamis, and F. Karray
Random finite set theoretic based soft/hard data fusion with application for target tracking.
In *Proceedings of the IEEE Conference on Multisensor Fusion and Integration for Intelligent Systems*, (2010), 50–55.

[230] A. Khalil, M. K. Gill, and M. McKee
New applications for information fusion and soil moisture forecasting.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1622–1628.

[231] S. Kim, R. W. Żbikowski, A. Tsourdos, and B. A. White
Airborne monitoring of ground traffic behaviour for hidden threat assessment.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[232] L. A. Klein
Sensor and Data Fusion: A Tool for Information Assessment and Decision Making.
SPIE Press, 2004.

[233] W. Koch, L. Snidaro, K. Rein, J. Garcia, S. F. Andler, and G. W. Ng
Multi-level fusion: Issues in bridging the gap between high and low level fusion.
In *Proceedings of the International Conference on Information Fusion*, 2012.

[234] M. M. Kokar
Situation awareness: Issues and challenges.
In *Proceedings of the International Conference on Information Fusion*, 1 (2004), 533–534.

[235] M. M. Kokar
Situation assessment: Procedural vs. logical.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), xx–xxiii.

[236] M. M. Kokar, M. D. Bedworth, and C. B. Frankel
A reference model for data fusion systems.
Proceedings of SPIE, 4051 (2000), 191–202.

[237] M. M. Kokar, C. J. Matheus, and K. Baclawski
Ontology-based situation awareness.
Information Fusion, **10**, 1 (2009), 83–98.

[238] M. M. Kokar, C. J. Matheus, K. Baclawski, J. A. Letkowski, M. Hinman, and J. Salerno
Use cases for ontologies in information fusion.
In *Proceedings of the International Conference on Information Fusion*, 1 (2004), 415–422.

[239] M. M. Kokar, C. J. Matheus, J. A. Letkowski, K. Baclawski, and P. Kogut
Association in level 2 fusion.
Proceedings of SPIE, 5434 (2004), 228–237.

[240] M. M. Kokar and G. W. Ng
High-level information fusion and situation awareness.
Information Fusion, **10**, 1 (2009), 2–5.

[241] M. M. Kokar, G. J. Toth, K. Laskey, G. M. Powell, K. Wallenius, M. Sudit, and M. Hultner
Higher-level information fusion: Challenges to the academic community.
In *Proceedings of the International Conference on Information Fusion*, 2008.

[242] M. M. Kokar and J. Wang
An example of using ontologies and symbolic information in automatic target recognition.
Proceedings of SPIE, 4731 (2002), 40–50.

[243] M. M. Kokar and J. Wang
Using ontologies for recognition: An example.
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 1324–1343.

[244] W. Komorniczak, J. Pietrasinski, and B. Solaiman
The data fusion approach to the priority assignment in the multifunction radar.
In *Proceedings of the International Conference on Microwaves, Radar and Wireless Communications*, 2 (2002), 647–650.

[245] U. Konda, T. Singh, P. Singla, and P. D. Scott
Uncertainty propagation in puff-based dispersion models using polynomial chaos.
In *Proceedings of the International Conference on Information Fusion*, (2009), 710–716.

[246] A. Kott (Ed.)
Information Warfare and Organizational Decision-making.
Artech House, 2007.

[247] A. Kott and W. M. McEneaney (Eds.)
Adversarial Reasoning: Computational Approaches to Reading the Opponent's Mind.
Taylor & Francis, 2006.

[248] A. Kott and W. Milks
Approaches to validation of information fusion systems.
In *Proceedings of the International Conference on Information Fusion*, (2009), 882–889.

[249] A. Kott, R. Singh, W. M. McEneaney, and W. Milks
Hypothesis-driven information fusion in adversarial, deceptive environments.
Information Fusion, **12**, 2 (2011), 131–144.

[250] B. Kovalerchuk
Modeling ATR processes to predict their performance by using invariance, robustness and self-refusal approach.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1139–1146.

[251] P. Krauthausen and U. D. Hanebeck
Intention recognition for partial-order plans using dynamic Bayesian networks.
In *Proceedings of the International Conference on Information Fusion*, (2009), 444–451.

[252] C. Kreucher, D. Blatt, A. Hero, and K. Kastella
Adaptive multi-modality sensor scheduling for detection and tracking of smart targets.
Digital Signal Processing, **16**, 5 (2006), 546–567.

[253] C. Kreucher, K. Kastella, and A. O. Hero, III
Sensor management using an active sensing approach.
Signal Processing, **85**, 3 (2005), 607–624.

[254] G. G. Kuperman, R. D. Whitaker, and S. M. Brown
"Cyber Warrior": Information superiority through advanced multi-sensory command and control technologies.
In *Proceedings of the IEEE National Aerospace and Electronics Conference*, (2000), 263–271.

[255] A. Kuwertz, M. F. Huber, and F. Sawo
Multi-step sensor management for localizing movable sources of spatially distributed phenomena.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[256] A. Lamallem, D. Coquin, and L. Valet
Aggregation evaluation of a fusion system devoted to image interpretation.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[257] D. A. Lambert
Grand challenges of information fusion.
In *Proceedings of the International Conference on Information Fusion*, 1 (2003), 213–220.

[258] D. A. Lambert
A unification of sensor and higher-level fusion.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 79.

[259] D. A. Lambert
STDF model based maritime situation assessments.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1269.

[260] D. A. Lambert
A blueprint for higher-level fusion systems.
Information Fusion, **10**, 1 (2009), 6–24.

[261] D. A Lambert, M. L. Hinman, É. Bossé, and C. P. Blackman
A coalition approach to higher-level fusion.
In *Proceedings of the International Conference on Information Fusion*, 2009.

[262] T. Lampinen, T. Laitinen, and J. Ropponen
Joint threat assessment with asset profiling and entity Bayes net.
In *Proceedings of the International Conference on Information Fusion*, (2009), 420–427.

[263] J. Landis, L. Bai, J. Salerno, M. Hinman, and D. Boulware
Mobile agent-based distributed fusion (MADFUSION) system.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1199–1203.

[264] R. O. Lane, D. A. Nevell, S. D. Hayward, and T. W. Beaney
Maritime anomaly detection and threat assessment.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[265] K. B. Laskey, P. C. G. Costa, and T. Janssen
Probabilistic ontologies for knowledge fusion.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session RS08.

[266] C. Laudy and J-G. Ganascia
Introducing semantic knowledge in high-level fusion.
In *Proceedings of the IEEE Workshop on Situation Management*, 2009, Paper 900659.

[267] R. Laxhammar, G. Falkman, and E. Sviestins
Anomaly detection in sea traffic—A comparison of the Gaussian mixture model and the kernel density estimator.
In *Proceedings of the International Conference on Information Fusion*, (2009), 756–763.

[268] D. Li, G. Chen, E. Blasch, and K. Pham
Sensor attack avoidance: Linear quadratic game approach.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1131–1138.

[269] X. Li, G. Chen, E. Blasch, J. Patrick, C. Yang, and I. Kadar
A geometric feature-aided game theoretic approach to sensor management.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1155–1162.

[270] M. E. Liggins, II and A. Bramson
Off-board augmented fusion for improved target detection and track.
In *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, (1993), 295–299.

[271] M. E. Liggins, II and C-Y. Chong
Distributed multi-platform fusion for enhanced radar management.
In *Proceedings of the IEEE National Radar Conference*, (1997), 115–119.

[272] M. E. Liggins, D. L. Hall, and J. Llinas (Eds.)
Handbook of Multisensor Data Fusion: Theory and Practice (2nd edition).
CRC Press, 2009.

[273] H. Ling, Y. Wu, E. Blasch, G. Chen, H. Lang, and L. Bai
Evaluation of visual tracking in extremely low frame rate wide area motion imagery.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 245.

[274] E. G. Little and G. L. Rogova
Ontology meta-model for building a situational picture of catastrophic events.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), 796–803.

[275] E. G. Little and G. L. Rogova
An ontological analysis of threat and vulnerability.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 302.

[276] E. G. Little and G. L. Rogova
Designing ontologies for higher level fusion.
Information Fusion, **10**, 1 (2009), 70–82.

[277] E. Little, K. Sambhoos, and J. Llinas
Enhancing graph matching techniques with ontologies.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14 II.

[278] J. Llinas
Developing a cost-effective approach to information fusion research and development.
In *Proceedings of International Conference on Information, Decision and Control*, (1999), 511.

[279] J. Llinas
Information fusion for natural and man-made disasters.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 570–576.

[280] J. Llinas, C. Bowman, G. L. Rogova, A. Steinberg, E. Waltz, and F. White
Revisiting the JDL data fusion model II.
In *Proceedings of the International Conference on Information Fusion*, 2 (2004), 1218–1230.

[281] J. Llinas, M. D. Moskal, and T. McMahon
Information fusion for nuclear, chemical, biological & radiological (NCBR) battle management support/disaster response management support.
Center for MultiSource Information Fusion, School of Engineering and Applied Sciences, University at Buffalo, 2002.

[282] J. Llinas, R. Nagi, D. Hall, and J. Lavery
A multi-disciplinary university research initiative in hard and soft information fusion: Overview, research strategies and initial results.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[283] J. Llinas, D. Talreja, and C. Bowman
Design of a performance evaluation methodology for data fusion-based multiple target tracking systems II: Analysis methodology.
In *Proceedings of the Military Sensing Series National Symposium on Sensor and Data Fusion*, 2004.

[284] C. Lloyd, D. Nicholson, and M. Williams
Information fusion algorithms and analysis for an exemplar detection of intent problem.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[285] C. G. Looney and L. R. Liang
Cognitive situation and threat assessments of ground battlespaces.
Information Fusion, **4**, 4 (2003), 297–308.

[286] F. Maggi, M. Matteucci, and S. Zanero
Reducing false positives in anomaly detectors through fuzzy alert aggregation.
Information Fusion, **10**, 4 (2009), 300–311.

[287] G. B. Magklaras and S. M. Furnell
Insider threat prediction tool: Evaluating the probability of IT misuse.
Computers & Security, **21**, 1 (2002), 62–73.

[288] R. Mahler
Unified sensor management using CPHD filters.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1172.

[289] B. Majidi and B. Moshiri
Industrial assessment of horticultural products' quality using image data fusion.
In *Proceedings of the International Conference on Information Fusion*, 2 (2003), 868–873.

[290] M. Mandiak, Y. Kim, P. Shah, and T. Kesavadas
Development of an integrated GUI framework for post-disaster data fusion visualization.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1131–1137.

[291] C. Mårtenson and P. Svenson
Evaluating sensor allocations using equivalence classes of multi-target paths.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), 574–581.

[292] C. J. Matheus
Using ontology-based rules for situation awareness and information fusion.
In *Proceedings of the W3C Workshop on Rule Languages for Interoperability*, 2005.

[293] C. J. Matheus, K. Baclawski, and M. M. Kokar
Derivation of ontological relations using formal methods in a situation awareness scenario.
Proceedings of SPIE, 5099 (2003), 298–309.

[294] C. J. Matheus, M. M. Kokar, and K. Baclawski
A core ontology for situation awareness.
In *Proceedings of the International Conference on Information Fusion*, 1 (2003), 545–552.

[295] C. J. Matheus, M. M. Kokar, K. Baclawski, J. J. Letkowski, C. Call, M. Hinman, J. Salerno, and D. Boulware
SAWA: An assistant for higher-level fusion and situation awareness.
Proceedings of SPIE, 5813 (2005), 75–85.

[296] C. J. Matheus, M. M. Kokar, K. Baclawski, J. J. Letkowski, C. Call, M. Hinman, J. Salerno, and D. Boulware
Lessons learned from developing SAWA: A situation awareness assistant.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 969–976.

[297] C. J. Matheus, B. Ulicny, M. M. Kokar, and G. M. Powell
Towards the formal representation of temporal aspects of enemy/threat courses of action.
In *Proceedings of the International Conference on Information Fusion*, (2009), 240–247.

[298] G. M. Mathews, D. Nicholson, A. McCabe, and Mark Williams
Plan detection under partially observable and cluttered conditions.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[299] J. Mattioli, N. Museux, M. Hemaissia, and C. Laudy
A crisis response situation model.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1217.

[300] P. Maupin, A-L. Jousselme, C. Saurel, and O. Poitou
Situation analysis and performance measurement: Application to personnel recovery.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[301] P. Maupin, A-L. Jousselme, H. Wehn, S. Mitrovic-Minic, and J. Happe
A situation analysis toolbox: Application to coastal and offshore surveillance.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[302] K. McConky, R. Nagi, M. Sudit, W. Rose, and G. Katz
Significant Information Encapsulation and Valence Exploitation (SIEVE) for discovery.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 67.

[303] D. McMaster, R. Nagi, and K. Sambhoos
Temporal alignment in soft information processing.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 68.

[304] D. McMichael, G. Jarrad, S. Williams, and M. Kennett
Force deployment analysis with generalized grammar.
Information Fusion, **10**, 1 (2009), 51–69.

[305] J. Molina, J. García, F. J. Jiménez, and J. R. Casar
Surveillance multisensor management with fuzzy evaluation of sensor task priorities.
Engineering Applications of Artificial Intelligence, **15**, 6 (2002), 511–527.

[306] L. R. Moore, III and D. Gonzales
Measuring the value of high level fusion.
Technical report ADA465987, Rand Corporation, 2004.

[307] S. Mori, C-Y. Chong, and K. C. Chang
Performance prediction of feature aided track-to-track association.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 243.

[308] B. Morin, L. Mé, H. Debar, and M. Ducassé
A logic-based model to support alert correlation in intrusion detection.
Information Fusion, **10**, 4 (2009), 285–299.

[309] N. Morizio
Context selection for linguistic data fusion.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14.

[310] R. Moskovitch, N. Nissim, R. Englert, and Y. Elovici
Active learning to improve the detection of unknown computer worms activity.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session RS07.

[311] C. Muller, M. Rombautt, and M. Janier
Dempster Shafer approach for high level data fusion applied to the assessment of myocardial viability.
Lecture Notes in Computer Science, 2230 (2001), 104–112.

[312] D. Mušicki
Target existence based resource allocation.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1258.

[313] N. Nadarajah and P. Teunissen
Attitude determination of LEO satellites using an array of GNSS sensors.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 323.

[314] S. Nagappa, D. Clark, and R. Mahler
Incorporating track uncertainty into the OSPA metric.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 207.

[315] R. Nagi, M. Sudit, and J. Llinas
An approach for Level 2/3 fusion technology development in urban/asymmetric scenarios.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 236.

[316] G. W. Ng
Intelligent Systems—Fusion, Tracking and Control.
Research Studies Press Ltd. and Institute of Physics Publishing, 2003.

[317] G. W. Ng
Brain-mind Machinery: Brain-inspired Computing and Mind Opening.
World Scientific, 2009.

[318] G. W. Ng and K. H. Ng
Sensor management—what, why and how.
Information Fusion, **1**, 2 (2000), 67–75.

[319] G. W. Ng, K. H. Ng, K. H. Tan, and C. H. K. Goh
The ultimate challenge of commander's decision aids: The cognition based dynamic reasoning machine.
In *Proceedings of the Army Science Conference*, 2006, Paper BO-05.

[320] G. W. Ng, K. H. Ng, K. H. Tan, and C. H. K. Goh
Novel methods for fusing Bayesian network knowledge fragments in D'Brain.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1305.

[321] G. W. Ng, K. H. Ng, R. Yang, and P. H. Foo
Intent inference for attack aircraft through fusion.
Proceedings of SPIE, 6242 (2006), 624206.

[322] G. W. Ng, C. H. Tan, T. P. Ng, and S. Y. Siow
Assessment of data fusion systems.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 63.

[323] X. T. Nguyen
Threat assessment in tactical airborne environments.
In *Proceedings of the International Conference on Information Fusion*, 2 (2002), 1300–1307.

[324] D. M. Nicol
Modeling and simulation in security evaluation.
IEEE Security & Privacy, **3**, 5 (2005), 71–74.

[325] M. Nilsson
Human decision making and information fusion: Extending the concept of decision support.
Technical report HS-IKI-TR-07-002, School of Humanities and Informatics, University of Skövde, Sweden, 2007.

[326] M. Nilsson and T. Ziemke
Rethinking Level 5: Distributed cognition and information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 98.

[327] North Atlantic Treaty Organization Research and Technology Organisation Information Systems Technology Panel Symposium on "Military Data and Information Fusion"
Meeting Proceedings RTO-MP-IST-040, 2004.

[328] North Atlantic Treaty Organization Research and Technology Organisation Information Systems Technology Panel Specialists' Meeting on "Information Fusion for Command Support"
Meeting Proceedings RTO-MP-IST-055, 2006.

[329] C. Nowak
On ontologies for high-level information fusion.
In *Proceedings of the International Conference on Information Fusion*, 1 (2003), 657–664.

[330] C. Nowak and D. A. Lambert
The semantic challenge for situation assessments.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), 804–811.

[331] N. Okello and G. Thorns
Threat assessment using Bayesian networks.
In *Proceedings of the International Conference on Information Fusion*, 2 (2003), 1102–1109.

[332] B. Olsson, P. Nilsson, B. Gawronska, A. Persson, T. Ziemke, and S. F. Andler
An information fusion approach to controlling complexity in bioinformatics research.
In *Proceedings of the IEEE Computational Systems Bioinformatics Conference Workshops*, (2005), 299–304.

[333] J. K. Ong and J. Ibañez-Guzmán
Perception management for the guidance of unmanned vehicles.
In *Proceedings of the IEEE Conference on Cybernetics and Intelligent Systems*, 2 (2004), 914–919.

[334] M. Oxenham
Enhancing situation awareness for air defence via automated threat analysis.
In *Proceedings of the International Conference on Information Fusion*, 2 (2003), 1086–1093.

[335] M. Oxenham, S. Challa, and M. Morelande
Decentralised fusion of disparate identity estimates for shared situation awareness.
In *Proceedings of the International Conference on Information Fusion*, 1 (2004), 167–174.

[336] M. Oxenham, S. Challa, and M. Morelande
Fusion of disparate identity estimates for shared situation awareness in a network-centric environment.
Information Fusion, **7**, 4 (2006), 395–417.

[337] M. G. Oxenham, D. J. Kewley, and M. J. Nelson
Performance assessment of data fusion systems.
In *Proceedings of the Australian Data Fusion Symposium*, (1996), 36–41.

[338] K. Palaniappan, F. Bunyak, P. Kumar, I. Ersoy, S. Jaeger, K. Ganguli, A. Haridas, J. Fraser, R. M. Rao, and G. Seetharaman
Efficient feature extraction and likelihood fusion for vehicle tracking in low frame rate airborne video.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[339] J. Palmer and S. Young
Aggregation gisting.
In *Proceedings of the International Conference on Information Fusion*, (2009), 2130–2135.

[340] S. Panigrahi, A. Kundu, S. Sural, and A. K. Majumdar
Credit card fraud detection: A fusion approach using Dempster-Shafer theory and Bayesian learning.
Information Fusion, **10**, 4 (2009), 354–363.

[341] S. Paradis, A. Benaskeur, M. Oxenham, and P. Cutler
Threat evaluation and weapons allocation in network-centric warfare.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1078–1085.

[342] O. Parsons and G. A. Carpenter
ARTMAP neural networks for information fusion and data mining: Map production and target recognition methodologies.
Neural Networks, **16**, 7 (2003), 1075–1089.

[343] G. Petterson, L. Axelsson, T. Jensen, M. Karlsson, and A. Malmberg
Multi-source integration and temporal situation assessment in air combat.
In *Proceedings of International Conference on Information, Decision and Control*, (1999), 371–375.

[344] A. Pfeffer, S. Das, D. Lawless, and B. Ng
Factored reasoning for monitoring dynamic team and goal formation.
Information Fusion, **10**, 1 (2009), 99–106.

[345] T. Pham, G. H. Cirincione, D. Verma, and G. Pearson
Intelligence, surveillance, and reconnaissance fusion for coalition operations.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS19.

[346] M. T. Plehn
Control Warfare: Inside the OODA Loop.
Master's thesis, School of Advanced Airpower Studies, Air University, Maxwell Air Force Base, 2000.

[347] A. Polychronopoulos, A. Amditis, U. Scheunert, and T. Tatschke
Revisiting JDL model for automotive safety applications: The PF2 functional model.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 259.

[348] R. L. Popp, A. W. Bailey, and J. N. Tsitsiklis
Dynamic airborne sensor resource management for ground moving target tracking and classification.
In *Proceedings of the IEEE Aerospace Conference*, 3 (2000), 405–415.

[349] G. M. Powell
Tactical situation assessment challenges and implications for computational support.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), xxiv–xxvi.

[350] M. A. Pravia, O. Babko-Malaya, M. K. Schneider, J. V. White, C-Y. Chong, and A. S. Willsky
Lessons learned in the creation of a data set for hard/soft information fusion.
In *Proceedings of the International Conference on Information Fusion*, (2009), 2114–2121.

[351] M. A. Pravia, R. K. Prasanth, P. O. Arambel, C. Sidner, and C-Y. Chong
Generation of a fundamental data set for hard/soft information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS13.

[352] K. Premaratne, M. N. Murthi, J. Zhang, M. Scheutz, and P. H. Bauer
A Dempster-Shafer theoretic conditional approach to evidence updating for fusion of hard and soft data.
In *Proceedings of the International Conference on Information Fusion*, (2009), 2122–2129.

[353] M. Prentice and S. Shapiro
Using propositional graphs for soft information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 69.

[354] R. G. Qiu
A data fusion framework for an integrated plant-wide information system.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 101–107.

[355] C. Qu and Y. He
A method of threat assessment using multiple attribute decision making.
In *Proceedings of the International Conference on Signal Processing*, 2 (2002), 1091–1095.

[356] Z. Qureshi and P. Urlings
Situation awareness and automation: Issues and design approaches.
In *Proceedings of International Conference on Information, Decision and Control*, (1999), 605–610.

[357] U. D. Ramdaras and F. Absil
Networks of maritime radar systems: Sensor selection algorithm for $P_D < 1$ based on the modified Riccati equation.
In *Proceedings of the IEEE Nonlinear Statistical Signal Processing Workshop: Classical, Unscented and Particle Filtering Methods*, (2006), 152–155.

[358] U. D. Ramdaras and F. Absil
Sensor selection: The modified Riccati equation approach compared with other selection schemes.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1184.

[359] U. D. Ramdaras and F. Bolderheij
Performance-based sensor selection for optimal target tracking.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1687–1694.

[360] S. Rawat, J. Llinas and C. Bowman
Design of a performance evaluation methodology for data fusion-based multiple target tracking systems.
Proceedings of SPIE, 5099 (2003), 139–151.

[361] S. N. Razavi, C. T. Haas, P. Vanheeghe, and E. Duflos
Real world implementation of belief function theory to detect dislocation of materials in construction.
In *Proceedings of the International Conference on Information Fusion*, (2009), 748–755.

[362] S. Reece, S. Roberts, C. Claxton, and D. Nicholson
Multi-sensor fault recovery in the presence of known and unknown fault types.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1695–1703.

[363] K. Rein, M. Frey, U. Schade, and S. Kawaletz
ALARM for early warning: A lightweight analysis for recognition of menace.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[364] B. J. Rhodes
Taxonomic knowledge structure discovery from imagery-based data using the neural associative incremental learning (NAIL) algorithm.
Information Fusion, **8**, 3 (2007), 295–315.

[365] J. Rickard
Level 2/3 fusion in conceptual spaces.
In *Proceedings of the International Conference on Information Fusion*, 2006, Paper 16.

[366] J. Rimland and J. Llinas
Network and infrastructure considerations for hard and soft information fusion processes.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 296.

[367] B. Ristic, B-N. Vo, and D. Clark
Performance evaluation of multi-target tracking using the OSPA metric.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[368] G. Rogova and É. Bossé
Information quality in information fusion.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[369] G. L. Rogova, P. D. Scott, and C. Lollett
Higher level fusion for post-disaster casualty mitigation operations.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 938–945.

[370] R. Rousseau and R. Breton
The M-OODA: a model incorporating control functions and teamwork in the OODA loop.
In *Proceedings of the Command and Control Research and Technology Symposium*, 2004.

[371] S. Roussel, V. Bellon-Maurel, J-M. Roger, and P. Grenier
Fusion of aroma, FT-IR and UV sensor data based on the Bayesian inference. Application to the discrimination of white grape varieties.
Chemometrics and Intelligent Laboratory Systems, **65**, 2 (2003), 209–219.

[372] J. N. Roux and J. H. van Vuuren
Threat evaluation and weapon assignment decision support: A review of the state of the art.
ORiON, **23**, 2 (2007), 151–187.

[373] J. Roy and A. B. Guyard
Multiple hypothesis situation analysis support system prototype.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[374] V. Royer and J-F. Challine
A platform for interoperable fusion models.
In *Proceedings of the International Conference on Information Fusion*, 2 (2000), ThB1-3–ThB1-10.

[375] V. Sakkalis, M. Zervakis, and S. Micheloyannis
Biopattern initiative: Towards the development and integration of next-generation information fusion approaches.
In *Proceedings of the Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, 1 (2004), 399–402.

[376] J. Salerno
Information fusion: A high-level architecture overview.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 680–686.

[377] J. Salerno, M. Hinman, D. Boulware, and P. Bello
Information fusion for situational awareness.
In *Proceedings of the International Conference on Information Fusion*, 1 (2003), 507–513.

[378] J. Salerno, J. Llinas, and Gabriel Jakobson
Social, Cultural, and Cognitive Aspects of Situation Management: Issues and Challenges.
In *Proceedings of the International Conference on Information Fusion*, 2011.

[379] J. J. Salerno, M. Sudit, S. J. Yang, G. P. Tadda, I. Kadar, and J. Holsopple
Issues and challenges in higher level fusion: Threat/impact assessment and intent modeling (a panel summary).
In *Proceedings of the International Conference on Information Fusion*, 2010.

[380] P. Salmon, N. Stanton, G. Walker, and D. Green
Situation awareness measurement: A review of applicability for C4i environments.
Applied Ergonomics, **37**, 2 (2006), 225–238.

[381] K. Sambhoos, C. Bowman, and J. Llinas
A case study with design of experiments: Performance evaluation methodology for Level 1 distributed data fusion processes.
Information Fusion, **12**, 2 (2011), 93–104.

[382] K. Sambhoos and J. Llinas
Evaluation methods for distributed multi-platform systems in electronic warfare and information-warfare related missions.
In *Proceedings of the International Conference on Information Fusion*, (2009), 163–170.

[383] K. Sambhoos, J. Llinas, and E. Little
Graphical methods for real-time fusion and estimation with soft message data.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14.

[384] K. Sambhoos, R. Nagi, M. Sudit, and T. Rickard
Hierarchical higher level data fusion using fuzzy hamming and hypercube clustering.
Journal of Advances in Information Fusion, **3**, 2 (2008), 90–106.

[385] K. Sambhoos, R. Nagi, M. Sudit, and A. Stotz
Enhancements to high level data fusion using graph matching and state space search.
Information Fusion, **11**, 4 (2010), 351–364.

[386] K. Sambhoos, M. Temel, F. Pan, and M. Sudit
Level 4 fusion for biosensor location and setting.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS04.

[387] K. Sambhoos, M. Temel, F. Pan, and M. Sudit
Process refinement using biosensor location problem.
In *Proceedings of the International Conference on Information Fusion*, (2009), 256–263.

[388] E. Santos, Jr.
A cognitive architecture for adversary intent inferencing: Knowledge structure and computation.
Proceedings of SPIE, 5091 (2003), 182–193.

[389] C. O. Savage and B. F. La Scala
Sensor management for tracking smart targets.
Digital Signal Processing, **19**, 6 (2009), 968–977.

[390] M. K. Schneider
Computing maximal track clusters for sensor resource management.
In *Proceedings of the International Conference on Information Fusion*, (2009), 78–84.

[391] K. A. Scholte and W. L. van Norden
Applying the PCR6 rule of combination in real time classification systems.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1665–1672.

[392] J. Schubert, P. Svenson, and C. Mårtenson
System prediction combining state estimation with an evidential influence diagram.
In *Proceedings of the International Conference on Information Fusion*, (2009), 428–435.

[393] R. Schubert, H. Klöden, G. Wanielik, and S. Kälberer
Performance evaluation of multiple target tracking in the absence of reference data.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[394] T. Schuck and E. Blasch
Description of the Choquet integral for tactical knowledge representation.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[395] D. Schuhmacher, B-T. Vo, and B-N. Vo
A consistent metric for performance evaluation of multi-object filters.
IEEE Transactions on Signal Processing, **56**, 8 (2008), 3447–3457.

[396] S. Schwoegler, J. Holsopple, S. Blackman, and M. Hirsch
On the application of multiple hypothesis tracking to the cyber domain.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 33.

[397] L. Šerić, D. Stipaničev, and M. Štula
Observer network and forest fire detection.
Information Fusion, **12**, 3 (2011), 160–175.

[398] G. P. Settembre, D. Nardi, R. Pigliacampo, A. Farinelli, and M. Rossi
Solving disagreements in a multi-agent system performing situation assessment.
In *Proceedings of the International Conference on Information Fusion*, (2009), 717–724.

[399] E. Shahbazian, D. E. Blodgett, and P. Labbé
The extended OODA model for data fusion systems.
In *Proceedings of the International Conference on Information Fusion*, (2001), FrB1-19–FrB1-25.

[400] B. Shapo and C. Kreucher
Track-before-fuse error bounds for tracking passive targets.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 210.

[401] D. Shen, G. Chen, L. Haynes, and E. Blasch
Strategies comparison for game theoretic cyber situational awareness and impact assessment.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1317.

[402] Y. Shu and F. Kazuo
Team situation awareness as a basis of team intention inference: Definition and framework.
In *Proceedings of the SICE Annual Conference*, 3 (2002), 1641–1646.

[403] C. Simonin, J-P. Le Cadre, and F. Dambreville
The cross-entropy method for solving a variety of hierarchical search problems.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1483.

[404] M. A. Skoglund, F. Gustafsson, and K. Jönsson
Modeling and sensor fusion of a remotely operated underwater vehicle.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 372.

[405] P. R. Smart, N. R. Shadbolt, L. A. Carr, and M. C. Schraefel
Knowledge-based information fusion for improved situational awareness.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1017–1024.

[406] F. Smarandache and J. Dezert
Extended PCR rules for dynamic frames.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 39.

[407] F. Smarandache, D. Han, and A. Martin
Comparative study of contradiction measures in the theory of belief functions.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 49.

[408] A. Smirnov, M. Pashkin, N. Shilov, T. Levashova, and A. Kashevnik
Context-aware operational decision support.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1464.

[409] D. Smith and S. Singh
Approaches to multisensor data fusion in target tracking: A survey.
IEEE Transactions on Knowledge and Data Engineering, **18**, 12 (2006), 1696–1710.

[410] L. Snidaro, M. Belluz, and G. L. Foresti
Domain knowledge for surveillance applications.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1367.

[411] J. Snyder, S. Marcus, and R. Williams
Fusion challenges in non-traditional warfare—the road to an asymmetric fusion model.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14 II.

[412] M. A. Solano, S. Ekwaro-Osire, and M. M. Tanik
High-Level fusion for intelligence applications using Recombinant Cognition Synthesis.
Information Fusion, **13**, 1 (2012), 79–98.

[413] X. Song, P. Willett, and S. Zhou
Posterior Cramér-Rao bounds for Doppler biased multistatic range-only tracking.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 209.

[414] D. Stampouli, D. Vincen, and G. Powell
Situation assessment for a centralised intelligence fusion framework for emergency services.
In *Proceedings of the International Conference on Information Fusion*, (2009), 179–186.

[415] F. Stein, J. Garska, and P. L. McIndoo
Network-centric warfare: Impact on army operations.
In *Proceedings of the IEEE/AFCEA Conference on Information Systems for Enhanced Public Safety and Security*, (2000), 288–295.

[416] A. N. Steinberg
Unification across JDL data fusion levels 1 and 2.
In *Proceedings of the International Conference on Information Fusion*, 1 (2004), 530–532.

[417] A. N. Steinberg
Threat assessment technology development.
In *Proceedings of the International and Interdisciplinary Conference on Modeling and Using Context*, (2005), 490–500.

[418] A. N. Steinberg
An approach to threat assessment.
In *Proceedings of the International Conference on Information Fusion*, 2 (2005), 1256–1263.

[419] A. N. Steinberg
Predictive modeling of interacting agents.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1136.

[420] A. N. Steinberg
Context-sensitive data fusion using structural equation modeling.
In *Proceedings of the International Conference on Information Fusion*, (2009), 725–731.

[421] A. N. Steinberg
Foundations of situation and threat assessment.
In [272, Chap. 18].

[422] A. N. Steinberg and C. L. Bowman
Revisions to the JDL data fusion model.
In [272, Chap. 3].

[423] A. N. Steinberg, C. L. Bowman, and F. E. White
Revisions to the JDL data fusion model.
Proceedings of SPIE, 3719 (1999), 430–441.

[424] A. N. Steinberg and G. Rogova
Situation and context in data fusion and natural language understanding.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS14 II.

[425] A. Stotz, R. Nagi, and M. Sudit
Incremental graph matching for situation awareness.
In *Proceedings of the International Conference on Information Fusion*, (2009), 452–459.

[426] A. Stotz and M. Sudit
Information Fusion Engine for Real-time Decision-making (INFERD): A perceptual system for cyber attack tracking.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1359.

[427] O. Straka, J. Duník, and M. Šimandl
Performance evaluation of local state estimation methods in bearings-only tracking problems.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 246.

[428] S. Stubberud, P. J. Shea, and D. Klamer
Metrics for Level 2 fusion association.
In *Proceedings of the International Conference on Information Fusion*, 2 (2003), 1180–1186.

[429] S. C. Stubberud and K. A. Kramer
Level 2 fusion: Situational assessment composition fusion with uncertain classification.
In *Proceedings of the International Conference on Systems Engineering*, (2005), 348–354.

[430] S. C. Stubberud and K. A. Kramer
Incorporation of uncertainty into Level 2 fusion association metrics.
In *Proceedings of the International Conference on Intelligent Sensors, Sensor Networks and Information Processing*, (2005), 205–210.

[431] P. Svenson
Equivalence classes of future paths for sensor allocation and threat analysis.
Proceedings of SPIE, 5913 (2005), 363–369.

[432] P. Svenson, T. Berg, P. Hörling, M. Malm, and C. Mårtenson
Using the impact matrix for predictive situational awareness.
In *Proceedings of the International Conference on Information Fusion*, 2007, Paper 1298.

[433] P. Svenson, R. Forsgren, B. Kylesten, P. Berggren, R. F. Wong, M. S. Choo, and J. Y. H. Kho
Swedish-Singapore studies of Bayesian modelling techniques for tactical intelligence analysis.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[434] P. Svenson and C. Mårtenson
SB-Plan: Simulation-based support for resource allocation and mission planning.
In *Proceedings of the Conference on Civil and Military Readiness*, 2006, Paper S3.6.

[435] P. Svensson
Technical survey and forecast for information fusion.
In [327, Paper 4].

[436] B. K. Sy
Integrating intrusion alert information to aid forensic explanation: An analytical intrusion detection framework for distributive IDS.
Information Fusion, **10**, 4 (2009), 325–341.

[437] K. Sycara, R. Glinton, B. Yu, J. Giampapa, S. Owens, M. Lewis, and LTC C. Grindle
An integrated approach to high-level information fusion.
Information Fusion, **10**, 1 (2009), 25–50.

[438] K. Sycara and M. Lewis
From data to actionable knowledge and decision.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 577–584.

[439] C. H. Tan, H. Chen, and G. W. Ng
Assessment of data fusion systems Part 2: Sensor network architectures.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[440] G. Terejanu, P. Singla, T. Singh, and P. D. Scott
Decision based uncertainty propagation using adaptive Gaussian mixtures.
In *Proceedings of the International Conference on Information Fusion*, (2009), 702–709.

[441] C. Thomas and N. Balakrishnan
Performance enhancement of intrusion detection systems using advances in sensor fusion.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS01.

[442] C. Thomas and N. Balakrishnan
Modified evidence theory for performance enhancement of intrusion detection systems.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session RS07.

[443] P. Z. Thunemann, R. Mattikalli, S. Arroyo, and P. Frank
Characterizing the tradeoffs between different sensor allocation and management algorithms.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1473–1480.

[444] M. Tipping
The relevance vector machine.
Advances in Neural Information Processing Systems, 12 (2000), 652–658.

[445] G. Toth, M. Kokar, and M. Liggins
Directions for higher-level fusion research: Needs and capabilities.
In *Proceedings of the International Conference on Information Fusion*, 2009.

[446] J. Twycross and U. Aickelin
Information fusion in the immune system.
Information Fusion, **11**, 1 (2010), 35–44.

[447] B. Ulicny, C. J. Matheus, G. Powell, and M. M. Kokar
Current approaches to automated information evaluation and their applicability to priority intelligence requirement answering.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[448] P. Valin, É. Bossé, A. Guitouni, H. Wehn, and J. Happe
Testbed for distributed high-level information fusion and dynamic resource management.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[449] A. van den Broek, R. Neef, P. Hanckmann, S. van Gosliga, and D. van Halsema
Improving maritime situational awareness by fusing sensor information and intelligence.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 171.

[450] J. van Laere
Challenges for IF performance evaluation in practice.
In *Proceedings of the International Conference on Information Fusion*, (2009), 866–873.

[451] A. V. van Leijen, W. L. van Norden, and F. Bolderheij
Unification of radar and sonar coverage modeling.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1673–1678.

[452] T. van Valkenburg-van Haarst, A. V. van Leijen, and F. C. A. Groen
Colour as an attribute for automated detection in maritime environments.
In *Proceedings of the International Conference on Information Fusion*, (2009), 1679–1686.

[453] V. Vapnik
The Nature of Statistical Learning Theory.
Springer, 1995.

[454] J. Viinikka, H. Debar, L. Mé, A. Lehikoinen, and M. Tarvainen
Processing intrusion detection alert aggregates with time series modeling.
Information Fusion, **10**, 4 (2009), 312–324.

[455] E. Waltz and J. Llinas
Multisensor Data Fusion.
Artech House, 1990.

[456] Y. Wang, H. Yang, X. Wang, and R. Zhang
Distributed intrusion detection system based on data fusion method.
In *Proceedings of the World Congress on Intelligent Control and Automation*, 5 (2004), 4331–4334.

[457] R. B. Washburn, M. K. Schneider, and J. J. Fox
Stochastic dynamic programming based approaches to sensor resource management.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 608–615.

[458] A. M. Waxman, D. A. Fay, B. J. Rhodes, T. S. McKenna, R. T. Ivey, N. A. Bomberger, V. K. Bykoski, and G. A. Carpenter
Information fusion for image analysis: Geospatial foundations for higher-level fusion.
In *Proceedings of the International Conference on Information Fusion*, 1 (2002), 562–569.

[459] M. Wei, G. Chen, E. Blasch, H. Chen, and J. B. Cruz, Jr.
Game theoretic multiple mobile sensor management under adversarial environments.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS04.

[460] K. White, J. Williams, and P. Hoffensetz
Radar sensor management for detection and tracking.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS04.

[461] M. Wielgus, M. Bartys, A. Antoniewicz, and B. Putz
Fast and adaptive bidimensional empirical mode decomposition for the real-time video fusion.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 223.

[462] M. Witkowski, G. White, P. Louvieris, G. Görbil, E. Gelenbe, and L. Dodd
High-level information fusion and mission planning in highly anisotropic threat spaces.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session SS13.

[463] L. Xie, Y. Zhu, B. Huang, and Y. Zheng
Kalman filtering approach to multirate information fusion for soft sensor development.
In *Proceedings of the International Conference on Information Fusion*, 2012, Paper 196.

[464] N. Xiong and P. Svensson
Multi-sensor management for information fusion: Issues and approaches.
Information Fusion, **3**, 2 (2002), 163–186.

[465] Y. Xun, M. M. Kokar, and K. Baclawski
Control based sensor management for a multiple radar monitoring scenario.
Information Fusion, **5**, 1 (2004), 49–63.

[466] R. R. Yager
A framework for reasoning with soft information.
Information Sciences, **180**, 8 (2010), 1390–1406.

[467] R. R. Yager
Hard and soft information fusion using measures.
In *Proceedings of the International Conference on Intelligent Systems and Knowledge Engineering*, (2010), 13–16.

[468] T. Yan and B. Wang
Grid architecture model of network centric warfare.
Journal of Systems Engineering and Electronics, **17**, 1 (2006), 121–125.

[469] C. Yang, E. Blasch, and I. Kadar
Geometric factors in target positioning and tracking.
In *Proceedings of the International Conference on Information Fusion*, (2009), 85–92.

[470] C. Yang, I. Kadar, and E. Blasch
Performance-driven resource management in layered sensing.
In *Proceedings of the International Conference on Information Fusion*, (2009), 850–857.

[471] C. Yang, L. Kaplan, E. Blasch, and M. Bakich
Optimal placement of heterogeneous sensors in target tracking.
In *Proceedings of the International Conference on Information Fusion*, 2011, Paper 244.

[472] S. J. Yang, A. Stotz, J. Holsopple, M. Sudit, and M. Kuhl
High level information fusion for tracking and projection of multistage cyber attacks.
Information Fusion, **10**, 1 (2009), 107–121.

[473] X. Yuan, C. Han, and F. Lian
Fusion performance analysis with the correlation.
In *Proceedings of the International Conference on Information Fusion*, 2010.

[474] J. Yuill, F. Wu, J. Settle, F. Gong, R. Forno, M. Huang, and J. Asbery
Intrusion-detection for incident-response, using a military battlefield-intelligence process.
Computer Networks, **34**, 4 (2000), 671–697.

[475] M. Zandipour, B. J. Rhodes, and N. A. Bomberger
COALESCE: A probabilistic ontology-based scene understanding approach.
In *Proceedings of the International Conference on Information Fusion*, 2008, Session RS08.

[476] E. Zervas, A. Mpimpoudis, C. Anagnostopoulos, O. Sekkas, and S. Hadjiefthymiades
Multisensor data fusion for fire detection.
Information Fusion, **12**, 3 (2011), 150–159.

[477] X. Zhang, M. Zhang, and S. Sun
An artificial intelligent approach on longevity modeling.
In *Proceedings of the IEEE International Conference on Tools with Artificial Intelligence*, (2005), 462–465.

[478] Y. Zhang, R. Nagi, and M. Sudit
Analytic network process for model elicitation in nation-building simulations.
In *Proceedings of the International Conference on Information Fusion*, (2009), 155–162.

[479] Q. Zhu, S. L. Aldridge, and T. N. Resha
Hierarchical Collective Agent Network (HCAN) for efficient fusion and management of multiple networked sensors.
Information Fusion, **8**, 3 (2007), 266–280.

[480] J. H. Zwaga and H. Driessen
Tracking performance constrained MFR parameter control: Applying constraints on prediction accuracy.
In *Proceedings of the International Conference on Information Fusion*, 1 (2005), 546–551.

Pek Hui Foo received her B.Sc. (Hons.), M.Sc., and Ph.D. degrees from the National University of Singapore in 1999, 2001, and 2009 respectively.

She is currently a Senior Member of Technical Staff at DSO National Laboratories, Singapore. Her research is on applications of data and information fusion.

Gee Wah Ng received his Ph.D. from University of Manchester Institute of Science and Technology (UMIST), UK, in 1996.

In 2006 and 2007, he was a research associate in Massachusetts Institute of Technology (MIT) and visiting scholar at Boston University. He is currently a programme director in DSO National Laboratories for Information Exploitation Programme. Concurrently he is an adjunct associate professor in National University of Singapore, Engineering Science Programme. He authored three technical books and published many technical papers in international conferences and journals. The three technical books are namely *Application of Neural Networks to Adaptive Control of Nonlinear Systems* (John Wiley and Sons Inc., ISBN 0471972630), *Intelligent Systems—Fusion, Tracking and Control* (Research Studies Press and Institute of Physics Publishing, ISBN 086380277X) and *Brain-Mind Machinery* (World Scientific Publishing Co., ISBN 9789812790255). His current R&D interests include data and information fusion, computational cognitive system (brain-inspired technology), target tracking, large scale data mining and analytics, and computational intelligence techniques for information processing.

Efficient 2D Sensor Location Estimation using Targets of Opportunity

DAVID F. CROUSE
RICHARD W. OSBORNE, III
KRISHNA PATTIPATI
PETER WILLETT
YAAKOV BAR-SHALOM

This paper discusses the Maximum Likelihood (ML) algorithm for the self-localization of passive (angular) or active (angle and range) sensors using targets of opportunity. The approach, which is considered in two dimensions, is appropriate when traditional alternatives, such as the use of known-location targets or satellite navigation systems, are unavailable. It is not assumed that the sensors can "see" each other, though they are assumed to take measurements with respect to a common (biased) axis. Unlike previous ML algorithms, we take into account the circular nature of the angular measurements, allowing for more accurate estimates to be obtained. A simple least-squares method is additionally provided for initialization. Simulations demonstrate that the accuracy of the ML estimator approaches the Cramér-Rao Lower Bound (CRLB), something that similar algorithms have been unable to achieve.

Manuscript received August 18, 2010; revised December 14, 2011; released for publication September 15, 2012.

Refereeing of this contribution was handled by Benjamin Slocumb.

This work was partially supported by the Office of Naval Research under contracts N00014-09-10613 and N00014-10-10412 and by the Army Research Office under contract W911NF-06-1-0467.

Authors' address: The Department of Electrical and Computer Engineering, University of Connecticut, 371 Fairfield Way, U-2157, Storrs, Connecticut 06269, E-mail: ({crouse, rosborne, krishna, willett, ybs}@engr.uconn.edu).

1557-6418/13/$17.00 © 2013 JAIF

1. INTRODUCTION

Due to their low cost and ease of deployment, the use of passive acoustic sensors for target tracking has seen increasing popularity. Such systems might consist of individual microphones or hydrophones that self-assemble into arrays [24], or, perhaps, sensors consisting of clusters of microphones or hydrophones, each producing measurements consisting of arrival angles and features/attributes for use in data association [19].[1] The clusters of microphones or hydrophones form individual sensors, which can also be referred to as "nodes" in the system. This paper focusses on the latter scenario, localizing sensors with measurements taken with respect to a common, unknown coordinate axis. Determining which detection on one sensor corresponds to the same target on another sensor (measurement association) might be accomplished, for example, by utilizing acoustic patterns for classification, as has previously been done to aid acoustic tracking [19]. Target tracking is not considered here. The scenarios considered focus on angular noise levels up to 2° (root-mean squared error), which is the accuracy of the sensors in [19], though acoustic sensors can often have significantly worse angular accuracies.

When considering the construction of land-based sensor networks, it cannot be assumed that satellite-based localization systems, such as GPS (USA) or GLONASS (Russia), will be available, and such signals cannot penetrate far underwater. However, many non-satellite-based location estimation algorithms, which have been primarily designed for use in underwater and wireless networks may be used. A number of methods applied to sonar channels are described in [4]. These approaches typically utilize the communication characteristics between sensors and are divided into two categories: range-based and range-free. Range-based methods utilize range (distance) measurements. Range-free schemes do not utilize range information. Both techniques might take advantage of moving anchor nodes that broadcast their position [6, 9, 13, 22].

Our focus is on algorithms for node localization based on the angle-only observations of the nodes, though we do consider the case where range measurements are also available. Estimates based on angle-only measurements are particularly useful when the sensors have a limited broadcast range. Underwater, this might be the case when the sensor network is built using data MULEs (Mobile Ubiquitous LAN[2] Extensions) [21]. A data MULE is a mobile device that approaches the sensors to collect data. In such a network, traditional methods of sensor localization, which rely on communication channels between sensors, are not applicable.

[1] Given multiple targets in a scene, features, such as the range-Doppler profile of different targets, can be used in target tracking algorithms to determine which measurements originated from which targets.

[2] Local Area Network

The node localization algorithm considered in this paper can also be used with networks of wireless land-based acoustic sensors. Though many localization techniques using aspects of the wireless communication channel between sensors exist, estimates of the sensor locations obtained using acoustic data can reasonably be expected to be uncorrelated with those obtained using more traditional means. Multiple uncorrelated estimates can be easily fused, improving the overall accuracy. Under the typical assumption that the noise on the estimates is Gaussian distributed, estimate fusion via the least squares algorithm [1, Ch. 3] requires that the covariance matrix of the individual estimates be known. Thus, in Section 7, the Cramér Rao Lower Bound (CRLB), a lower bound on the error of an unbiased estimator, is derived for the estimation problem at hand. Since the accuracy of the ML estimation method derived in this paper approaches the CRLB, as demonstrated in Section 7, the CRLB should be an accurate approximation for the covariance matrix of the estimate at low noise levels. However, being a lower-bound, in more difficult (nonlinear) estimation problems, or when the signal to noise ratio is low, the CRLB tends to be overly optimistic. The validity of the CRLB for estimate fusion is not considered in this paper.

A maximum likelihood (ML) solution to localizing both passive and active nodes is outlined in [16]. Though sensors often cannot take measurements spanning a full 360°. Additionally, the distribution of the noise corrupting angular measurements often depends upon the geometry of the target with respect to the hydrophone or microphone array taking the measurement. Nonetheless, it is common practice for the noise corrupting angular measurements to be modeled as Gaussian, which is not bounded to a range of 0 to 2π radians. Since the sensors can face any direction, the angular measurements taken by the sensors in a global coordinate system can span the range of 0 to 2π or $-\pi$ to π, depending upon where the boundary is placed. The Gaussian noise approximation is often good *except near the discountinity* (0–2π or $-\pi$–π). Figure 1 illustrates the boundary problem. This paper rederives the ML algorithm accounting for the idiosyncrasies of circular data. Section 2 discusses the signal model and the ML solution is provided in Section 4. Since poor performance is generally observed when using a random initialization, Section 5 discusses the generation of initial estimates without prior information. Though this work focusses on angle-only networks, the case where 2D range measurements are also available is additionally considered. Section 7 demonstrates the performance of the algorithms through simulation and Section 8 summarizes the results.

With the exception of [16], few algorithms have been developed to jointly localize and determine the orientation of angle-only sensors. A significant amount of work has been done regarding localizing users within cellular networks [23], with very little attention paid to the angle-only measurement case. In what has been done, a single user must always be in range of at least two base stations (anchor nodes). Other work has considered similar issues for cellular networks [17], whereby all users are in range of a number of anchor nodes. In our solution, no target ever needs to be simultaneously observed by two nodes of a known location provided that conditions for observability, which are discussed in Section 3, are met.

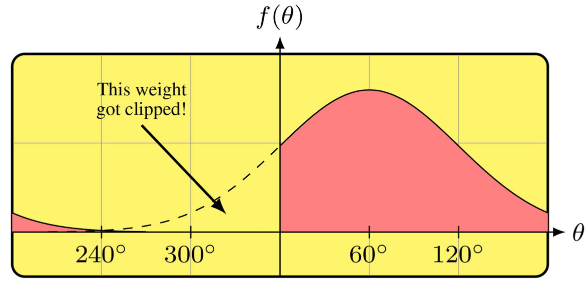


Fig. 1. The traditional linear measurement model as applied to circular data with a mean of 60° does not accurately represent the uncertainty in the likelihood of observations near the 0–2π boundary, as illustrated for the Normal distribution. In this case, a significant portion of the density is clipped. A circular measurement model (illustrated by the dashed line) more accurately reflects the underlying uncertainty.

In [18] an algorithm for localizing sensors that can see their neighbors was developed. The algorithm is deterministic and errors compound with propagation distances. An attempt to mitigate this problem was given in [11], where a linear programming method was used to improve the consistency of the angular measurements between sensors before the deterministic localization algorithm was executed. In both instances, it is generally assumed that if one sensor can see another, then the reverse is true. Thus, these algorithms are not applicable to the case where targets of opportunity are observed. Another method involving semidefinite programming was given in [3], but it requires the use of a heuristic parameter that depends upon the size and geometry of the network.

Many papers dealing with sensor registration only correct for residual bias after an initial estimate has been obtained. Most require full range and angle estimates (see [5] for an extensive bibliography), though some are adaptable to the range-only case [20]. The majority of algorithms only estimate the sensor orientations, though some can also estimate the sensor positions [14]. Most approaches utilize some type of linearization and none of them are applicable to the aforementioned estimation scenarios when no initial estimate is available.

2. DEFINITIONS AND MODELS

We assume that all angular and, if available, range measurements are taken in two dimensions with respect to a common axis, which need not be known. The sensors and the targets are assumed to be individual points in space. The measurements between sensors are assumed to be synchronized. That is, in each scan, measurements from disparate sensors represent the angle from the sensor to the targets at the same time (at the same target position).[3] It should be noted that individual observations may occur simultaneously or at different times if a target is stationary. A measurement of the same target at a different time shall simply be considered as another target in the context of this problem, since tracking is not performed (admittedly such information would help, but here we ignore it). If multiple targets are present at the same time, then classification information may be used to associate measurements between the sensors. We will not address the problem where measurements cannot be associated between sensors, in which case there may be multiple possible solutions for the target location based upon a particular set of observations.

When dealing with angles, it will become necessary to utilize a four-quadrant inverse tangent function with range $(-\pi,\pi]$, which is defined as follows

$$\mathrm{atan}_2[y,x] \triangleq \begin{cases} \arctan\left[\dfrac{y}{x}\right] & x>0 \\ \arctan\left[\dfrac{y}{x}\right]+\pi & x<0, \quad y\geq 0 \\ \arctan\left[\dfrac{y}{x}\right]-\pi & y<0, \quad x<0 \\ \dfrac{\pi}{2} & x=0, \quad y>0 \\ -\dfrac{\pi}{2} & x=0, \quad y<0 \\ 0 & x=0, \quad y=0 \end{cases} \tag{1}$$

where arctan represents the standard inverse tangent function with range $(-\pi/2,\pi/2)$.

Let $\theta_{s,t}$ and $r_{s,t}$ be the angular and (if available) range measurements from sensor s observing target t. Both shall be assumed corrupted with zero-mean additive noise:

$$\theta_{s,t} = \theta_{s,t}^{\text{true}} + w_{s,t}^{\theta} \tag{2}$$

$$r_{s,t} = r_{s,t}^{\text{true}} + w_{s,t}^{r}. \tag{3}$$

All of the additive noises are assumed independent. The range noise, $w_{s,t}^{r}$, is assumed to be distributed Gaussian $\mathcal{N}\{0,\sigma_r^2\}$. The Gaussian noise assumption is commonly used despite the fact that one will never measure a negative range.[4] As the targets can be assumed to be far from the sensors compared to the standard deviation of the noise ($>30\sigma$), this approximation is accurate. However, the use of a Gaussian approximation for noise corrupting angular measurements is more problematic.

When dealing with angular measurements, many traditional statistical concepts need to be redefined due to the "wrapping" of the distribution about the circle and due to problems at the 0–2π boundary. For example, the traditional notions of mean and variance no longer provide useful quantities; a sample mean of $-\pi$ and π would yield zero, which is the worst possible estimate, since $-\pi$ and π represent the same point on the circle. For this reason, a lot of research has been done with regard to statistical methods relating to directional data in multiple dimensions [7, 8, 15]. Our definition of the mean direction and circular standard deviation are based on the following trigonometric moments

$$\alpha \triangleq \mathrm{E}[\cos\theta], \qquad \beta \triangleq \mathrm{E}[\sin\theta]. \tag{4}$$

The mean resultant length, ρ, and mean direction, μ_θ, are defined in terms of these moments through the polar relation

$$\alpha + j\beta = \rho e^{j\mu_\theta}. \tag{5}$$

Thus, the magnitude of $\alpha + j\beta$ is the mean resultant length, and its phase is the circular mean. The circular standard deviation is defined as

$$\sigma_\theta \triangleq \sqrt{-2\ln\rho}. \tag{6}$$

The maximum likelihood sensor localization algorithm derived in [16] using common targets of opportunity assumed that angular measurements of the target locations were corrupted with additive Gaussian noise. That is, the distribution of the angular measurement was

$$p(\theta) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\theta-\mu)^2}{2\sigma^2}\right). \tag{7}$$

As will be demonstrated in Section 7, maximization based upon the algorithm in [16] yields results that are often very useful but, depending upon the geometry of the sensors and the observed targets, can also be quite far from the CRLB. In other words, the estimator cannot be statistically efficient without accounting for the circularity of the measurements. This paper accounts for the circularity of the noise.

In the following subsections, we consider three noise distributions appropriate for circular data: the wrapped normal distribution, the clipped mod normal distribution and the von Mises distribution. At low circular standard deviations, less than around 20° or $\pi/9$ radians, all of the distributions are essentially the same, as shown in Fig. 2. However, as the standard deviations increase, the differences become more profound, with the von Mises distribution distinguishing itself the most from the other two.

If an angular measurement is truly corrupted with additive Gaussian noise, which is often a good assumption due to the Central Limit Theorem, then the noise effectively gets wrapped on the region from 0 to 2π or $-\pi$ to π, depending upon where one wishes to make the cut. For this reason, the wrapped normal distribution,

[3]This is equivalent to saying that the sensors are assumed to be synchronized and the propagation delay between a target and a sensor is assumed to be negligible.

[4]The normal distribution is unbounded, implying that there exists a probability, however small, of measuring a negative range.

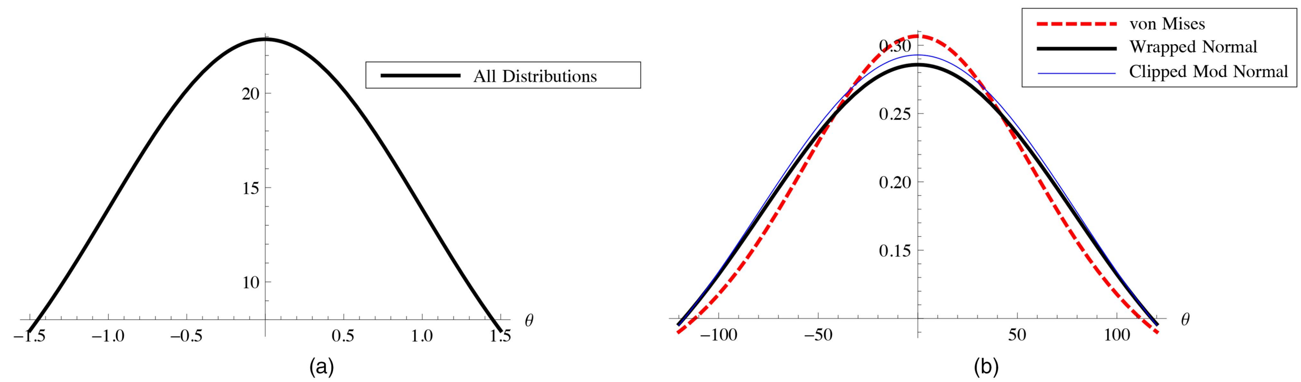


Fig. 2. When the circular standard deviation is low, as in (a), then the distributions shown are essentially the same. Increasing the circular standard deviation, the von Mises differentiates itself first, and only when the circular standard deviation is very high are the two normal distributions significantly different. The plot ranges are $\pm 1.5\sigma_\theta$. (a) $\sigma_\theta = 1°$. (b) $\sigma_\theta = 80°$.

discussed in Section 2.1, is the most natural distribution to use. Moreover, the circular mean and standard deviation in the wrapped normal distribution have an easy-to-understand meaning: they are equal to the mean and standard deviation of the linear normal distribution that got wrapped to the circle [15]. Indeed, if the standard deviation is small and the mean is far from the boundary, then both distributions are nearly the same.

On the other hand, array processing generally does not directly yield angular measurements, but rather unit vectors pointing toward the targets. Processing the measurements in their original (array) coordinates can possibly avoid the boundary issues illustrated in Fig. 1. However, many sensors only provide angular measurements. If one converts such unit vectors having components corrupted with Gaussian noise into angles, then the angular measurements are von Mises distributed [15, pg. 42]. The von Mises distribution is discussed in Section 2.3.

2.1. The Wrapped Normal Distribution

The *wrapped normal distribution* is obtained when Gaussian noise is added to a circular datum. As discussed, for example in [7], [15], if the additive noise is distributed as $\mathcal{N}\{\mu,\sigma^2\}$, then the wrapped normal distribution has the following PDF

$$p(\theta) = \frac{1}{2\pi}\left(1 + 2\sum_{k=1}^{\infty}\rho^{k^2}\cos[k(\theta-\mu)]\right) \tag{8}$$

where

$$\rho = e^{-\sigma^2/2} \tag{9}$$

and $-\pi \le \theta < \pi$. The aforementioned definitions of the mean direction and circular standard deviation are such that $\mu_\theta = \mu$ and $\sigma_\theta = \sigma$.

2.2. The Clipped Mod Normal Distribution

We developed the clipped mod normal distribution as a simple approximation that avoids the infinite sum present in the wrapped normal distribution. It comes from the assumption that almost all of the mass of the Gaussian noise added to the measurement is within $\pm\pi$ of the mean. In this case, shifting the cutting region as far as possible from the mean and discarding the mass that would have been wrapped (in this case, almost nothing) and renormalizing the distribution is a good approximation of the distribution on the circle. Thus, the wrapped normal distribution may be approximated by the following shifted and clipped distribution

$$p(\theta) = \frac{1}{c}\exp\left(-\frac{[m(\theta-\mu)]^2}{2\sigma^2}\right) \tag{10}$$

where

$$c = \Phi\left[\frac{\pi}{\sigma}\right] - \Phi\left[-\frac{\pi}{\sigma}\right] \tag{11}$$

$$m(\theta) = \begin{cases} \theta - 2\pi & \text{if} \quad \theta > \pi \\ \theta + 2\pi & \text{if} \quad \theta < -\pi \\ \theta & \text{otherwise} \end{cases} \tag{12}$$

and $-\pi \le \{\mu,\theta\} < \pi$, Φ is the cumulative distribution of the standard normal distribution, and c is the normalizing constant. Because the function $m(\theta-\mu)$ is squared, other forms for m are also valid. Note, however, that one cannot replace m with the modulo over π or over 2π, because that would assign large penalties to small negative offsets.

2.3. The von Mises Distribution

The von Mises distribution was derived in 1918 by Richard von Mises in an attempt to statistically determine whether the atomic weights of elements were integer multiples of a common base unit of weight, whereby non-integer measurements would be attributed to noise.[5] The von Mises distribution (on the circle) has been widely studied, in part to due to its similarity to the wrapped Cauchy distribution and the wrapped normal

[5]He concluded that the likelihood at the zero point (that the atomic weights are integers) was nine times greater than the average likelihood across the rest of the circle, which did not tell him very much.

distribution. More information on the von Mises distribution and its multidimensional generalizations (von Mises-Fisher distributions) can be found in [7], [8], [15]. The von Mises distribution is given by

$$p(\theta) = \frac{1}{2\pi I_0(\kappa)} \exp(\kappa \cos[\theta - \mu_\theta]) \tag{13}$$

where μ_θ is the mean direction, $-\pi \leq \{\theta, \mu_\theta\} < \pi$, and $0 \leq \kappa < \infty$. The value κ is a measure of the concentration of the distribution and is inversely proportional to the circular standard deviation. The function I_ν is a modified Bessel function of the first kind. For an integer ν, I_ν is given by

$$I_\nu(\kappa) = \frac{1}{\pi} \int_0^\pi \exp[\kappa \cos[\theta]] \cos[\nu\theta] d\theta. \tag{14}$$

The mean resultant length of the von Mises distribution is

$$\rho = \frac{I_1(\kappa)}{I_0(\kappa)}. \tag{15}$$

The von Mises distribution approximates a wrapped Normal distribution having the same mean and circular standard deviation. Given κ, the circular standard deviation can be calculated from (15) and (6). Finding a value of κ corresponding to a particular circular standard deviation can be performed using a simple numerical search. However, the mean resultant length in (15) can be hard to evaluate for accurate measurements. For example, if $\sigma_\theta = \pi/180$, that is a 1° standard deviation, a reasonable value for an accurate sensor, then $\kappa \approx 3283$. Though ρ is just under one, $I_1(\kappa) \approx 5.81 \times 10^{1423}$, a number that cannot be stored in a computer's double-precision floating point register. However, the ratio in (15) may be expressed as an infinite sum. Methods for computing the ratio are compared in [10] and the most efficient method for this problem is summarized in Appendix B. The calculation of this ratio is important for evaluating the CRLB, as discussed in Section 6.

3. THE OBSERVABILITY OF THE SENSOR LOCATIONS

The requirements for observability in the angle-only case shall be considered. Figure 3 shows a system with three sensors, s_1, s_2, and s_3. Suppose that the location of s_1 is known. In this case, all angles in the system may be preserved by scaling everything around s_1. That is, the locations of s_2 and s_3 are not observable if only s_1 is known. Now suppose that both s_1 and s_3 have known locations. In this case, the locations of the targets t_1 and t_2 can be uniquely determined (we know this from the angle-side-angle theorem of planar geometry).

Given that the locations of s_1 and s_3 are known, if only target t_1 were observed, then, having angle-only measurements, the location of s_2 could not be uniquely determined, because the observed angle at s_2 simply defines a line passing through t_1. If both t_1 and t_2 were observed, then, as shown in Fig. 3(b) angles θ_1, θ_2 and θ_3

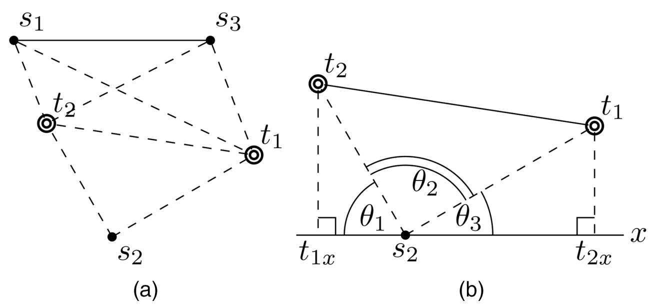


Fig. 3. A system with three sensors and two targets in (a). If the location of two sensors is known, then the locations of the targets may be uniquely determined as well as the third sensor location.

are all known. Similarly, because s_1 and s_3 have known locations, the locations of t_1 and t_2 are known exactly. Thus, the location of s_2 may be solved by considering the intersection of the line passing through t_1 at an angle of θ_3 with respect to the x-axis with the line passing through t_2 at an angle of $\theta_1 + \theta_3$ with respect to the x-axis. Note that this will not work if t_1 and t_2 are collinear with respect to s_2.

All together, in order for the locations of sensors in a network consisting of angle-only observations to be observable, the locations of at least two sensors must be known a priori (anchor nodes). Additionally, at least two targets must be observed by the sensors. However, the anchor nodes need not observe common targets if connected subsets of sensors between them can observe the targets seen by the anchor nodes. Section 5 presents an algorithm for generating initial estimates of the sensor locations by solving a set of linear equations. In that case, this observability criterion manifests itself as a requirement that the matrix in the linear equation be invertible. The fact that the anchor nodes need not be simultaneously seen by all sensors is made clearer through simulation in Section 7 where the anchor nodes never see the same targets simultaneously.

When range measurements are available, the situation becomes simpler, because a single sensor can uniquely identify the location of a target. Thus, the location of any additional sensor seeing the target may also be determined. This means that the location of only one sensor needs to be known. However, if measurements are taken with respect to a common, unknown axis, then two sensor locations must be known in order to resolve the angular ambiguity.

4. MAXIMUM LIKELIHOOD ESTIMATION

This section presents a general formulation of the maximum likelihood estimator using range measurements and angular measurements taken with respect to a common, unknown axis. If range measurements are not available or if the measurement axis is known, then the appropriate terms in the objective function and gradient should be omitted. Defining $w_{i,j}^r$ and $w_{i,j}^\theta$ to be uncorrelated additive noise corrupting the range and angular components of the measurement from sensor i to target location j, $\flat$ to be the "bias" on the angular measurements (representing the fact that they are taken with respect to an unknown axis), then, using the measurement model given in Section 2, a measurement consisting of a range, $r_{i,j}$ and angle, $\theta_{i,j}$ is given by

$$r_{i,j} = \sqrt{(y_t^j - y_s^i)^2 + (x_t^j - x_s^i)^2} + w_{i,j}^r \tag{16}$$

$$\theta_{i,j} = \mathrm{atan}_2[y_t^j - y_s^i, x_t^j - x_s^i] + \flat + w_{i,j}^\theta \tag{17}$$

where (x_t^j, y_t^j) are the Cartesian coordinates of target j, and (x_s^i, y_s^i) are the coordinates of sensor i. The assumption that $w_{i,j}^r$ and $w_{i,j}^\theta$ be uncorrelated does not always hold. However, there do not appear to be readily available probability distributions that can jointly represent the linear nature of the range measurement and the circular nature of the angular measurement.

Defining the vectors $\mathbf{s}$ and $\mathbf{t}$ to be the sets of all unknown sensor and target locations, the likelihood function is the product of the likelihoods for the ranges and angles

$$\Lambda(\mathbf{s}, \mathbf{t}, \theta_b) = \prod_{i,j} p_r(r_{i,j}|\mathbf{s}, \mathbf{t}, \flat) p_\theta(\theta_{i,j}|\mathbf{s}, \mathbf{t}, \flat) \tag{18}$$

where the product in (18) is over all pairs (i,j) where sensor i observes target j. It is assumed that the range measurements are corrupted with normally distributed noise as

$$p(r_{i,j}|\mathbf{s}, \mathbf{t}, \flat) \sim \mathcal{N}\left[\sqrt{(y_t^j - y_s^i)^2 + (x_t^j - x_s^i)^2}, (\sigma_{i,j}^r)^2\right]. \tag{19}$$

The distribution of $\theta_{i,j}$ depends upon which model from Section 2 we are using.

Determining the maximum of (18) is equivalent to finding the minimum of the negative log-likelihood of (18), designated as $\lambda(\mathbf{s}, \mathbf{t})$, which (discarding constant terms) has the following form:

$$\lambda(\mathbf{s}, \mathbf{t}, \flat)$$

$$= \overbrace{\sum_{i,j} \frac{-1}{2(\sigma_{i,j}^r)^2}\left(r_{i,j} - \sqrt{(y_t^j - y_s^i)^2 + (x_t^j - x_s^i)^2}\right)^2}^{\lambda_r}$$

$$+ \underbrace{\sum_{i,j} K_{s,t} f\left(\theta_{i,j} - \mathrm{atan}_2\left[\frac{y_t^j - y_s^i}{x_t^j - x_s^i}\right] - \flat\right)}_{\lambda_\theta}. \tag{20}$$

The value $K_{i,j}$ and function $f(\cdot)$ are given depending upon the distribution of $\theta_{i,j}$ according to Table I.

In the simulations, the minimization of (20) was carried out using the Quasi-Newton optimization algorithm [2]. For this the gradient of $\lambda(\mathbf{s}, \mathbf{t}, \flat)$ is needed. This gradient is the sum of the gradients of λ_r and λ_θ, which are defined in (20). The gradient elements of λ_r are given by

TABLE I
Values and Functions Dependent upon the Distribution of $\theta_{i,j}$ that are used in Expressions for the Likelihood
(the function m is defined in (12))

	Clipped Mod Normal	von Mises	Wrapped Normal
$K_{i,j}$	$1/(2(\sigma_{i,j}^\theta)^2)$	$\kappa_{i,j}$	1
$f(\cdot)$	$m(\cdot)^2$	$\cos[\cdot]$	$\log[1 + 2\Sigma_{k=1}^\infty \rho^{k^2} \cos[k(\cdot)]$
$\mathfrak{F}(\cdot)$	$2m(\cdot)$	$\sin[\cdot]$	$(2\Sigma_{k=1}^\infty \rho^{k^2} k \sin[k(\cdot)])/$ $(1 + 2\Sigma_{k=1}^\infty \rho^{k^2} \cos[k(\cdot))$
$\mathfrak{s}$	-1	1	1

$$\frac{\partial \lambda_r}{\partial a^i} = -\sum_j \frac{r_{i,j} - \sqrt{d_{i,j}}}{(\sigma_{i,j}^r)^2 \sqrt{d_{i,j}}} c_a^r(i,j) \tag{21}$$

$$\frac{\partial \lambda_r}{\partial b^j} = -\sum_i \frac{r_{i,j} - \sqrt{d_{i,j}}}{(\sigma_{i,j}^r)^2 \sqrt{d_{i,j}}} c_b^r(i,j) \tag{22}$$

$$\frac{\partial \lambda_r}{\partial \flat} = 0 \tag{23}$$

with $a \in \{x_s, y_s\}$ and $b \in \{x_t, y_t\}$. The constants are

$$d_{i,j} = (x_t^j - x_s^i)^2 + (y_t^j - y_s^i)^2 \tag{24}$$

$$c_{x_s}^r(i,j) = (x_t^j - x_s^i) \tag{25}$$

$$c_{y_s}^r(i,j) = (y_t^j - y_s^i) \tag{26}$$

$$c_{x_t}^r(i,j) = -(x_t^j - x_s^i) \tag{27}$$

$$c_{y_t}^r(i,j) = -(y_t^j - y_s^i). \tag{28}$$

With the quantities $\mathfrak{F}$ and $\mathfrak{s}$ given by Table I, the gradient elements of $\lambda_\theta(\mathbf{s}, \mathbf{t})$ are

$$\frac{\partial \lambda_\theta}{\partial a^i} = \sum_j K_{i,j} \mathfrak{F}(\theta_{i,j} - \mathrm{atan}_2[y_t^j - y_s^i, x_t^j - x_s^i] - \flat) c_a^\theta(i,j) \tag{29}$$

$$\frac{\partial \lambda_\theta}{\partial b^j} = \sum_i K_{i,j} \mathfrak{F}(\theta_{i,j} - \mathrm{atan}_2[y_t^j - y_s^i, x_t^j - x_s^i] - \flat) c_b^\theta(i,j) \tag{30}$$

$$\frac{\partial \lambda_\theta}{\partial \flat} = \mathfrak{s} \sum_{i,j} K_{i,j} \mathfrak{F}(\theta_{i,j} - \mathrm{atan}_2[y_t^j - y_s^i, x_t^j - x_s^i] - \flat) \tag{31}$$

where the c^θ terms are

$$c_{x_s}^\theta(i,j) = (y_t^j - y_s^i)/d_{i,j} \tag{32}$$

$$c_{y_s}^\theta(i,j) = -(x_t^j - x_s^i)/d_{i,j} \tag{33}$$

$$c_{x_t}^\theta(i,j) = -(y_t^j - y_s^i)/d_{i,j} \tag{34}$$

$$c_{y_t}^\theta(i,j) = (x_t^j - x_s^i)/d_{i,j} \tag{35}$$

and $d_{i,j}$ is as defined in (24).

In order to be able to perform likelihood maximization, initial estimates of the quantities being estimated are needed. The following section discusses how these may be obtained.

5. ALGORITHMS FOR GENERATING INITIAL ESTIMATES

5.1. Initial Estimates Without Range Measurements

1) *Joint Estimation of Sensor and Target Locations*: The angular measurement of sensor i observing target j, $\theta_{i,j}$, taken with respect to a known, common axis may be expressed as follows

$$\tan[\theta_{i,j}] = \frac{y_t^j - y_s^i}{x_t^j - x_s^i} \tag{36a}$$

$$\cot[\theta_{i,j}] = \frac{x_t^j - x_s^i}{y_t^j - y_s^i}. \tag{36b}$$

These equations may be rearranged to get

$$x_t^j \tan[\theta_{i,j}] - x_s^i \tan[\theta_{i,j}] - y_t^j + y_s^i = 0 \tag{37a}$$

$$y_t^j \cot[\theta_{i,j}] - y_s^i \cot[\theta_{i,j}] - x_t^j + x_s^i = 0. \tag{37b}$$

Thus, using (37a) and (37b), one can generate a linear system of equations that, in the absence of measurement noise and assuming that a sufficient number of equations are linearly independent, can be solved exactly for the sensor and target locations, given that enough sensors have a priori known locations. Sensors with known locations are necessary for the uniqueness of a nontrivial solution and are simply put on the right-hand side of the equation.

For example, consider the presence of three sensors observing two targets. Assuming that the location of sensors one and three are known a priori, the location of the two targets and the third sensor is given by the linear set of equations in (38), which is derived using (37a).

$$\underbrace{\begin{bmatrix} 0 & 0 & \tan[\theta_{1,1}] & -1 & 0 & 0 \\ -\tan[\theta_{2,1}] & 1 & \tan[\theta_{2,1}] & -1 & 0 & 0 \\ 0 & 0 & \tan[\theta_{3,1}] & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tan[\theta_{1,2}] & -1 \\ -\tan[\theta_{2,2}] & 1 & 0 & 0 & \tan[\theta_{2,2}] & -1 \\ 0 & 0 & 0 & 0 & \tan[\theta_{2,2}] & -1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_s^2 \\ y_s^2 \\ x_t^1 \\ y_t^1 \\ x_t^2 \\ y_t^2 \end{bmatrix}}_{\mathbf{s}}$$

$$= \underbrace{\begin{bmatrix} \tan[\theta_{1,1}]x_s^1 - y_s^1 \\ 0 \\ \tan[\theta_{3,1}]x_s^3 - y_s^3 \\ \tan[\theta_{1,2}]x_s^1 - y_s^1 \\ 0 \\ \tan[\theta_{3,2}]x_s^3 - y_s^3 \end{bmatrix}}_{\mathbf{b}}. \tag{38}$$

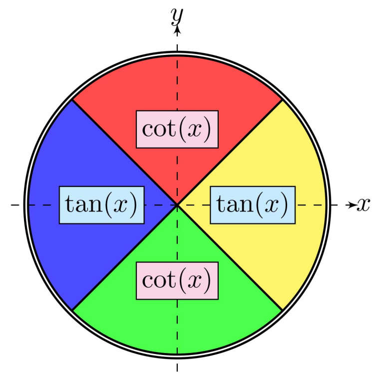


Fig. 4. The quadrants in which one should use the tangent or cotangent so as to minimize the effects of measurement error.

Thus, in this case, the location of the second sensor and the target at both times is the solution to

$$\mathbf{As} = \mathbf{b}. \tag{39}$$

A necessary condition for the observability of the system is that the locations of at least two sensors are known. However, the two sensors with known locations do not necessarily have to observe the same target at the same time for the matrix $\mathbf{A}$ to have full rank.

Due to the use of the tangent in (37a), serious estimation inaccuracies will occur if the targets used for estimation are close to $\pm 90^\circ$ with respect to any of the observing sensors. This is because the measurement error causes the measured angle to be above or below $\pm 90^\circ$, changing a very large positive entry in the $\mathbf{A}$ matrix to a very large negative value or vice versa. This problem can be minimized by using Equation (37b), which uses the cotangent, when the observation is between 45° and 135° or between -45° and -135°, as shown in Fig. 4.

2) *Estimation of the Sensor Locations Alone*: If the target locations are not needed, they can be eliminated from the estimation. We shall once again assume that all angles are taken with respect to a reference direction common for all sensors. In this subsection, we shall also assume that each target is observed simultaneously by at least three sensors with an appropriate (non-collinear) geometry. We define the measurements as being taken with respect to the x-axis in our 2-D coordinate system. For simplicity of notation, let us define the following functions

$$\Delta_{a,b}^T(j) \triangleq \tan[\theta_{a,j}] - \tan[\theta_{b,j}] \tag{40}$$

$$\Delta_{a,b}^C(j) \triangleq \cot[\theta_{a,j}] - \cot[\theta_{b,j}] \tag{41}$$

$$\Psi_{a,b}(j) \triangleq 1 - \cot[\theta_{a,j}]\tan[\theta_{b,j}] \tag{42}$$

where a and b are sensor indices and j is a target index.

As proven in Appendix A, given any three sensors simultaneously observing the target, one can combine (36a) and (40) using (37a) for each sensor to get an expression relating the sensor locations independent of the Cartesian location of the target. For sensors 1 through 3, this gives us equations (43a)–(43d).

$$0 = y_s^1 \Delta_{2,3}^T(j) + x_s^1 \tan[\theta_{1,j}]\Delta_{3,2}^T(j) + y_s^2 \Delta_{3,1}^T(j) + x_s^2 \tan[\theta_{2,t}]\Delta_{1,3}^T(j) + y_s^3 \Delta_{1,2}^T(j) + x_s^3 \tan[\theta_{3,j}]\Delta_{2,1}^T(j) \tag{43a}$$

$$0 = y_s^1 \cot[\theta_{1,j}]\Delta_{2,3}^T(j) + x_s^1 \Delta_{3,2}^T(j) - y_s^2 \Psi_{1,3}(j) + x_s^2 \tan[\theta_{2,j}]\Psi_{1,3}(j) + y_s^3 \Psi_{1,2}(j) - x_s^3 \tan[\theta_{3,j}]\Psi_{1,2}(j) \tag{43b}$$

$$0 = y_s^1 \cot[\theta_{1,j}]\Psi_{2,3}(j) - x_s^1 \Psi_{2,3}(j) - y_s^2 \cot[\theta_{2,j}]\Psi_{1,3}(j) + x_s^2 \Psi_{1,3}(j) + y_s^3 \Delta_{2,1}^C(j) + x_s^3 \tan[\theta_{3,j}]\Delta_{1,2}^C(j) \tag{43c}$$

$$0 = y_s^1 \cot[\theta_{1,j}]\Delta_{3,2}^C(j) + x_s^1 \Delta_{2,3}^C(j) + y_s^2 \cot[\theta_{2,j}]\Delta_{1,3}^C(j) + x_s^2 \Delta_{3,1}^C(j) + y_s^3 \cot[\theta_{3,j}]\Delta_{2,1}^C(j) + x_s^3 \Delta_{1,2}^C(j). \tag{43d}$$

As was the case in the previous section, the equations derived in this section can be used with multiple observations of the targets over time to reduce the solution of sensor locations to that of solving $\mathbf{As} = \mathbf{b}$, where in this case $\mathbf{s}$ consists of only the sensor locations.

Note that as the number of sensors increases, the number of possible equations that can be written increases rapidly. However, the equations are not all independent. For example, for N sensors observing a common target, there are $\binom{N}{3}$ possible variants of (43a) that can be written depending upon which three targets are put into the equation. However, for $N > 3$ only N of these equations are linearly independent and the rest do not provide any new information, because they are not based on new observations. Linearly dependent equations may be removed by using the Modified Gram-Schmidt Orthonormalization Algorithm [12] or other, similar methods, though, as demonstrated in Section 7, this can hurt the performance of the algorithm.

5.2. Initial Estimates With Range Measurements

1) *Jointly Estimating Sensor and Target Locations*: When range measurements are available, the estimation problem becomes much simpler. Letting the range measurement of sensor i observing target j be $r_{i,j}$, we can write

$$r_{i,j}\cos[\theta_{i,j}] = x_t^j - x_s^i \tag{44}$$

$$r_{i,j}\sin[\theta_{i,j}] = y_t^j - y_s^i. \tag{45}$$

As was true in the angular case we can collect these linear equations and solve them for the sensor and target locations. In this instance, the $\mathbf{A}$ matrix is particularly simple, being composed only of ± 1 and 0 elements.

2) *Estimating the Sensor Locations Alone*: When two sensors simultaneously observe the same target, we can eliminate the target location from the estimation problem by manipulating (45) and (44) to get

$$r_{2,j}\cos[\theta_{2,j}] - r_{1,j}\cos[\theta_{1,j}] = x_s^1 - x_s^2 \tag{46}$$

$$r_{2,j}\sin[\theta_{2,j}] - r_{1,j}\sin[\theta_{1,j}] = y_s^1 - y_s^2. \tag{47}$$

As was the case in Section 5.1.2, we can again use the equations to find initial estimates based on a linear least squares solution.

5.3. Measurements with Respect to an Unknown, Common Axis

We consider the case where all sensors have the same unknown bias in their measurements. This might occur, for example, if all measurements are taken with respect to magnetic north, but the anchor node locations are given in terms of geographic north.

Figure 5 illustrates the scaling uncertainty that arises when only one anchor node is used. The dark circles represent sensors and the open circles anchor nodes. The dotted lines are only present to show that the transforms considered are affine (they do not distort the relative angles). In the noiseless case, if we were to remove the second anchor node and not compensate for the bias, then Figs. 5(b), (c), and (d) are three possible solutions for the system described by $\mathbf{As} = \mathbf{b}$ using the equations for angle-only observations from Section 5.1 (when range measurements are available, then only one solution exists). All of the biased solutions are rotated by the bias angle. As shown in (b) and (c), the figure can be scaled about the single anchor node without changing any of the measured angles (in the case where angles are measured to targets, the apparent locations of the targets are scaled as well). Figure 5(d) comes about due to our use of the tangent and cotangent in Section 5.1, whereby the equations do not change if all angles are flipped 180°.

In the case of only two anchor nodes and angle-only measurements, a method of estimating the sensor locations while correcting for the unknown global rotation is as follows:

1) Find an observation from the first anchor node. Assume that it is at a known, fixed distance from the first anchor node (such as 10 m). Find its location using the bias measurement under this assumption. This shall be a pseudo-anchor node.

2) Perform the sensor location estimation as described in Section 5.1 using the biased measurements, the first anchor node and the previously determined pseudo-anchor node as an anchor node assuming that the location of the second anchor node is unknown.

3) Find the vector between the first anchor node and the true location of the second anchor node (for example, for the scenario in Fig. 5(a), it has been

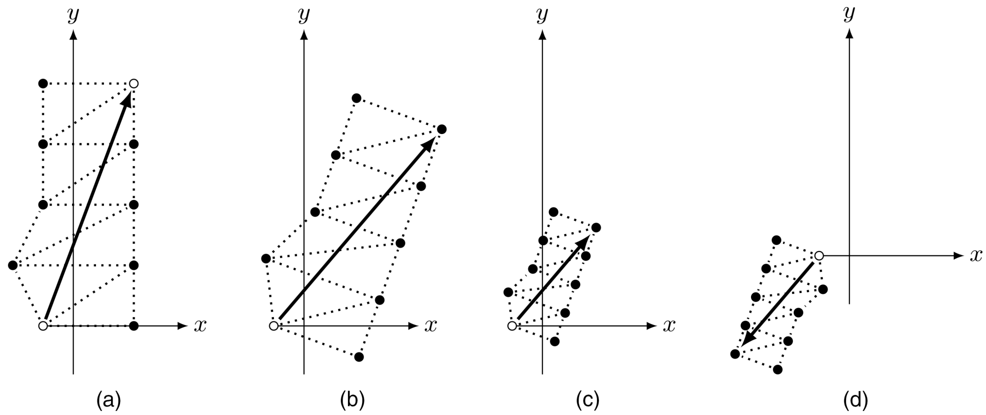


Fig. 5. The open circles represent the anchor sensors, whose locations are known a priori. The dotted lines show that the transformation being considered is affine (does not distort the relative angles between the sensors). The array points from the first (fixed) anchor node to the one that is removed. The array is the vector between the first and last nodes. Subfigure (a) shows the true setup of the problem. Subfigures (b), (c) and (d) show possible solutions when the second anchor node is removed and a common bias is left uncompensated. All scaled solutions are angularly consistent—even, as illustrated in (d), when the scaling factor is negative. (a) True sensor locations. (b) One biased solution. (c) Second biased solution. (d) Third biased solution.

drawn). We shall call this $\mathbf{v}_1$. Also find the vector between the first anchor node and the apparent position of the second anchor node as given by the previous estimation (such as the vectors in Fig. 5(b) or 5(c)). The choice of the pseudo-anchor node rules out the geometry of 5(d)); we shall call this $\mathbf{v}_2$.

4) Evaluate $\theta = \angle\mathbf{v}_2 - \angle\mathbf{v}_1$.

5) Perform the sensor location estimation again using the adjusted angles and both of the true anchor nodes to get a final estimate of the sensor locations.

The algorithm finds a solution for the biased system and then compares how that solution is rotated with respect to the true system. The first step of the aforementioned method creates a pseudo anchor point to set a reference for the scaling of the solution. This is important to make sure that we do not get a solution that is inverted by 180°, as in Fig. 5(d). Moreover, it is necessary for setting the scale of the figure. We would like to find a solution, but one possible solution places the nodes infinitesimally close to the first anchor point. The use of such a solution would be subject to precision problems on any computer.

A similar procedure can be performed if range measurements are available. In this case, there is no need to designate any node as a pseudo anchor node. When more sensors are present, one may break the observations into subsets according to the connectivity between anchor nodes, and calculate separate biases before averaging them. We shall not consider that case here.

6. THE CRAMÉR-RAO LOWER BOUND

In order to evaluate the efficiency of the estimator (how well the estimator is performing compared to a lower bound on the unbiased estimator), the Cramér Rao Lower Bound (CRLB) for the particular scenarios must be calculated [1]. The CRLB provides a lower bound on the covariance matrix of an unbiased estimator as

$$\mathrm{E}\{[\hat{\mathbf{x}} - \mathbf{x}_0][\hat{\mathbf{x}} - \mathbf{x}_0]^T\} \geq J^{-1} \tag{48}$$

where $\mathbf{x}$ is a vector parameter, $\hat{\mathbf{x}}$ is the parameter estimate, $\mathbf{x}_0$ is the true parameter value, and J is the Fisher Information Matrix (FIM).

The FIM is defined as

$$J \triangleq \mathrm{E}\{[\nabla_{\mathbf{x}}\lambda(\mathbf{x})][\nabla_{\mathbf{x}}\lambda(\mathbf{x})]^T\}|_{\mathbf{x}=\mathbf{x}_0}. \tag{49}$$

In the context of the problem at hand, $\mathbf{s}$ and $\mathbf{t}$ correspond to the variable $\mathbf{x}$. The appropriate diagonal entries of J^{-1} provide a lower bound for the mean squared error (MSE) of each estimated parameter, assuming that the estimator is unbiased. The FIM may be estimated by averaging values of $[\nabla_{\mathbf{x}}\lambda(\mathbf{x})][\nabla_{\mathbf{x}}\lambda(\mathbf{x})]^T|_{\mathbf{x}=\mathbf{x}_0}$ across Monte Carlo runs. For the case where the angular measurements have a von Mises distribution, an exact closed-form solution (in terms of modified Bessel functions) for the elements of the FIM will be presented. In the simulations when using the wrapped normal distribution, the CRLB was estimated by averaging the squared gradient across Monte Carlo runs.

Consider the elements of the FIM for normally distributed range measurements and von Mises distributed angular measurements taken with respect to a common, unknown axis (if range or angular measurements are not available or the measurement axis is known, then the appropriate terms may be omitted). Each element of the FIM is the expectation of a product of sums. Note that the expectation of cross terms[6] in the product sums involving angular measurements is zero. This is because the variables in question are independent and, thus the expectation of the product is equal to the product of the

[6]A cross term is a product such that elements involving (i_1, j_1) and (i_2, j_2) are multiplied where $i_1 \neq i_2$ or $j_1 \neq j_2$.

expectations. Based on (29) and (30), the expectations have the following form,

$$\int_{-\pi}^{\pi} \sin[\theta-\mu] e^{\kappa\cos[\theta-\mu]} d\theta = 0. \tag{50}$$

Note that the product of cross terms involving λ_r are zero, because again the variables in question are independent and the expectation can be factorized. Based on (21) and (22), the expectations shall have the following form

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \frac{r-\sqrt{d}}{\sigma^2\sqrt{d}} e^{-(r-\sqrt{d})^2/2\sigma^2} dr = 0. \tag{51}$$

Due to the assumed independence of the noise corrupting the range measurement and that corrupting the angular measurement, all cross terms between derivatives of λ_r and λ_θ are zero. Thus, we can write

$$J = \overbrace{\mathrm{E}\{[\nabla_{\mathbf{x}}\lambda_r(\mathbf{x})][\nabla_{\mathbf{x}}\lambda_r(\mathbf{x})]^T\}}^{J_r} + \overbrace{\mathrm{E}\{[\nabla_{\mathbf{x}}\lambda_\theta(\mathbf{x})][\nabla_{\mathbf{x}}\lambda_\theta(\mathbf{x})]^T\}}^{J_\theta}. \tag{52}$$

Let us compute J_θ. Because all of the cross terms are zero, we only need to concern ourselves with the expectation of the product of the gradient elements with the same (i,j) values. To simplify things, we shall note that

$$I_1(\kappa) = \frac{\kappa}{2\pi} \int_{-\pi}^{\pi} \sin^2[\theta-\mu] e^{\kappa\cos[\theta-\mu]} d\theta \tag{53}$$

which does not depend on μ. Thus, taking the expected value over the elements in the FIM, we get

$$\mathrm{E}\left[\frac{\partial\lambda_\theta}{\partial\alpha_s^{i_1}}\frac{\partial\lambda_\theta}{\partial\beta_s^{i_2}}\right] = \begin{cases} \sum_j \kappa_{i,j}\rho_{i,j} c^\theta_{\alpha_s}(i,j) c^\theta_{\beta_s}(i,j) & \text{if} \quad i_1 = i_2 = i \\ 0 & \text{otherwise} \end{cases} \tag{54}$$

$$\mathrm{E}\left[\frac{\partial\lambda_\theta}{\partial\alpha_t^{j_1}}\frac{\partial\lambda_\theta}{\partial\beta_t^{j_2}}\right] = \begin{cases} \sum_i \kappa_{i,j}\rho_{i,j} c^\theta_{\alpha_t}(i,j) c^\theta_{\beta_t}(i,j) & \text{if} \quad j_1 = j_2 = j \\ 0 & \text{otherwise} \end{cases} \tag{55}$$

$$\mathrm{E}\left[\frac{\partial\lambda_\theta}{\partial\alpha_s^{i}}\frac{\partial\lambda_\theta}{\partial\beta_t^{j}}\right] = \kappa_{i,j}\rho_{i,j} c^\theta_{\alpha_s}(i,j) c^\theta_{\beta_t}(i,j) \tag{56}$$

$$\mathrm{E}\left[\left(\frac{\partial\lambda_\theta}{\partial\flat}\right)^2\right] = \sum_{i,j} \kappa_{i,j}\rho_{i,j} \tag{57}$$

$$\mathrm{E}\left[\frac{\partial\lambda_\theta}{\partial\alpha_s^{i}}\frac{\partial\lambda_\theta}{\partial\flat}\right] = \sum_j \kappa_{i,j}\rho_{i,j} c^\theta_{\alpha_s}(i,j) \tag{58}$$

$$\mathrm{E}\left[\frac{\partial\lambda_\theta}{\partial\alpha_t^{j}}\frac{\partial\lambda_\theta}{\partial\flat}\right] = \sum_i \kappa_{i,j}\rho_{i,j} c^\theta_{\alpha_t}(i,j) \tag{59}$$

where $(\alpha,\beta) \in \{x,y\}$ and the mean resultant lengths are

$$\rho_{i,j} = \frac{I_1(\kappa_{i,j})}{I_0(\kappa_{i,j})}. \tag{60}$$

The calculation of the ratio of modified Bessel functions in the CRLB can be problematic, as mentioned in Section 2.3. An algorithm for calculating the ratio is discussed in Appendix B.

Now let us consider the calculation of J_r.

$$\mathrm{E}\left[\frac{\partial\lambda_r}{\partial\alpha_s^{i_1}}\frac{\partial\lambda_r}{\partial\beta_s^{i_2}}\right] = \begin{cases} \sum_j \frac{1}{d_{i,j}(\sigma^r_{i,j})^2} c^r_{\alpha_s}(i,j) c^r_{\beta_s}(i,j) & \text{if} \quad i_1 = i_2 = i \\ 0 & \text{otherwise} \end{cases} \tag{61}$$

$$\mathrm{E}\left[\frac{\partial\lambda_r}{\partial\alpha_t^{j_1}}\frac{\partial\lambda_r}{\partial\beta_t^{j_2}}\right] = \begin{cases} \sum_i \frac{1}{d_{i,j}(\sigma^r_{i,j})^2} c^r_{\alpha_t}(i,j) c^r_{\beta_t}(i,j) & \text{if} \quad j_1 = j_2 = j \\ 0 & \text{otherwise} \end{cases} \tag{62}$$

$$\mathrm{E}\left[\frac{\partial\lambda_r}{\partial\alpha_s^{i}}\frac{\partial\lambda_r}{\partial\beta_t^{j}}\right] = \frac{1}{d_{i,j}(\sigma^r_{i,j})^2} c^r_{\alpha_s}(i,j) c^r_{\beta_t}(i,j) \tag{63}$$

$$\mathrm{E}\left[\left(\frac{\partial\lambda_r}{\partial\flat}\right)^2\right] = \mathrm{E}\left[\frac{\partial\lambda_r}{\partial\alpha_s^{i}}\frac{\partial\lambda_r}{\partial\flat}\right] = \mathrm{E}\left[\frac{\partial\lambda_r}{\partial\alpha_t^{j}}\frac{\partial\lambda_r}{\partial\flat}\right] = 0. \tag{64}$$

7. SIMULATIONS

7.1. The Scenario

We used a scenario involving ten sensors and four targets over 20 time-steps (in the equations, observations of the same target at a different times are treated as separate "targets"). The sensors were placed in x locations in the set of $\{-50,100\}$ meters and y locations in the set of $\{0,100,200,300,400\}$ meters, with the exception of the one that would have been at $(-50,100)$, which was instead set to $(-100,100)$ in order to break the symmetry of the arrangement so that it would be clear if poor estimates flipped anything. This configuration is shown in Fig. 6. The locations of the sensors at $(-50,0)$ and $(100,400)$ were assumed to be known a priori, and they were used as anchor arrays.

The first target was located at an x location of -250 meters and traveled at a constant speed from 20 to 380 meters in y. The second target was placed at an x location of 350 meters and traveled at a constant speed from 0 to 400 meters in y. The third target started at 400 meters in x, traveled at a constant speed to 800 meters by step 10 and came back to 400 meters in the x direction by step 20. In the y direction, it traveled at a constant speed from 20 to 380. The fourth target was placed at a y location of 500 meters and traveled at a constant speed from -600 to 1000 meters in x.

To demonstrate that unlike other algorithms, no target needs to be simultaneously visible to both anchor nodes, and the targets were only visible to a subset of the sensors at each time. From steps 1 to 4, only the sensors at y locations of 0 and 100 meters could see the targets. From steps 5 through 8, the sensors between 0 and 200 meters could see the targets. From steps 9 through 12

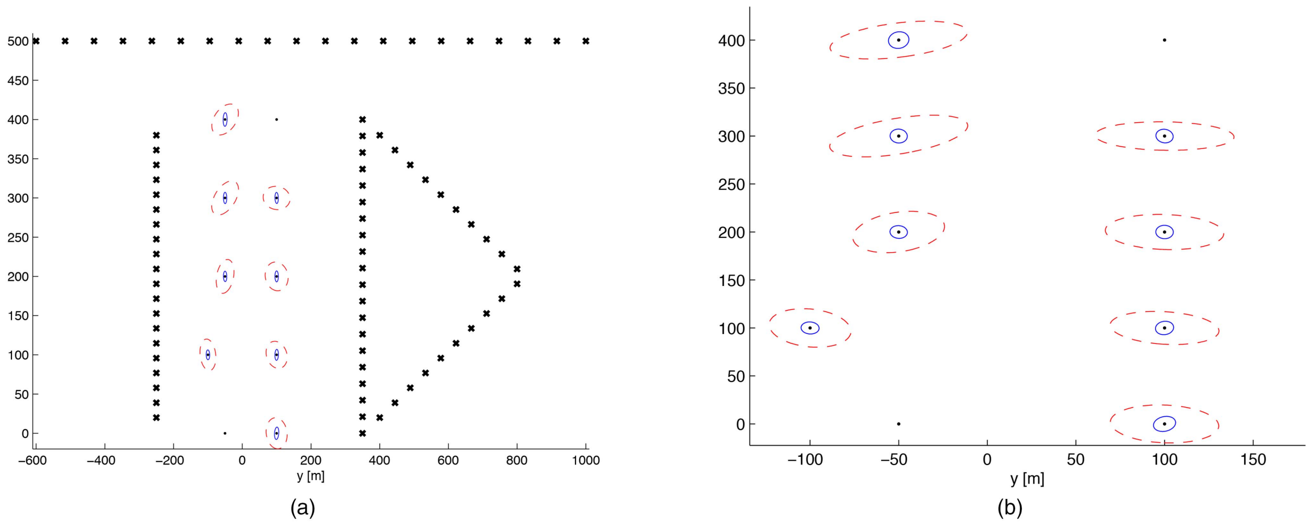


Fig. 6. The scenario showing the true sensor (the dots) and target locations (the $\times$s) used in the simulations. The anchor nodes are in the upper-right and lower-left corners. The ellipses represent the 99% confidence regions based on the CRLB for the sensor locations when the angular measurement axis is unknown. The dashed (outer) ellipses are using angle-only measurements using a von Mises angular noise distribution with $\sigma_\theta = 2^\circ = \pi/90$; the smaller, solid line ellipses are for the case with both angular and range measurements with $\sigma_r = 7.5$ meters. (a) Overall layout. (b) Magnified ellipses.

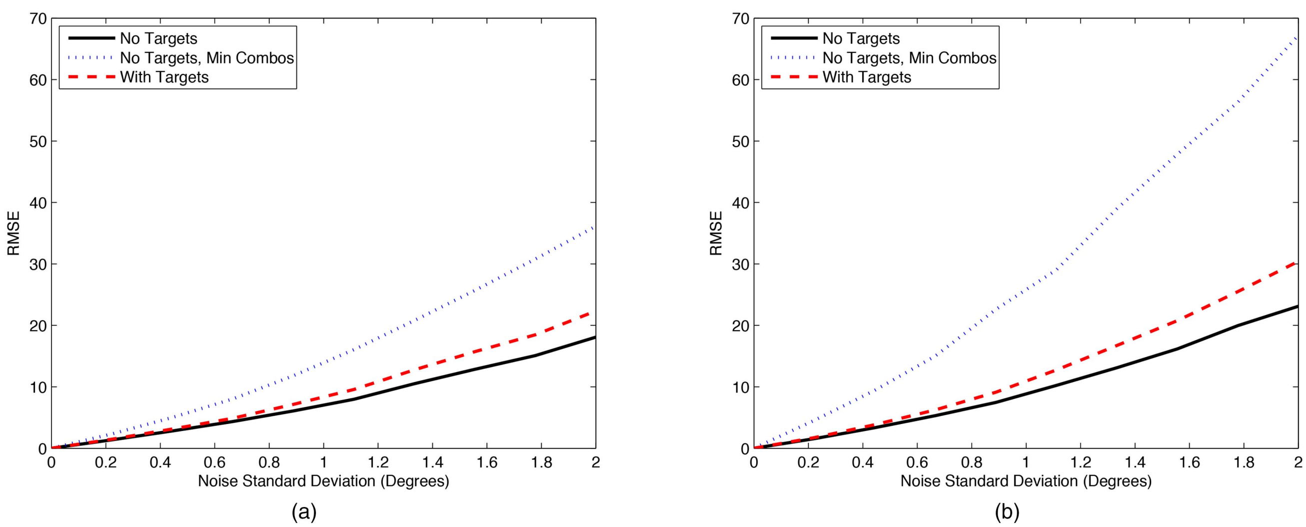


Fig. 7. The RMSE of the estimated sensor locations of the initialization algorithms. In (a) the measurements are taken with respect to a common, known axis. In (b) they are taken with respect to a common, unknown axis. 1000 Monte Carlo runs were performed. (a) Known measurement axis. (b) Unknown measurement axis.

the sensors between 100 and 300 meters could see the targets. From steps 13 through 16 the sensors between 200 and 400 meters could see the targets and from steps 17 through 20 the sensors from 300 to 400 meters in y could see the target. This means that the two sensors with known locations *never both simultaneously saw any target*.

7.2. The Initialization Algorithms

We compared the performance of the angle-only initialization algorithms under both known and unknown measurement axes, as shown in Fig. 7, where the RMSE of the sensor location estimates is shown, averaged over all sensors having unknown locations. The line labeled "No Targets" is the algorithm from Section 5.1.2 where the sensor locations are estimated without explicitly estimating the target locations. The line labeled "No Targets, Min Combos" is the same, except redundant equations of the $\binom{N}{3}$ that could be generated for each set of N sensors observing a common target were eliminated using the Gram-Schmidt algorithm [12]. The line labeled "With Targets" is the RMSE of the sensors when the target locations are jointly estimated, as given in Section 5.1.1. In all cases the solution to $\mathbf{As} = \mathbf{b}$ was found using least squares. The angular measurements were generated using the wrapped normal distribution with independence between sensors. One thousand Monte Carlo runs were performed.

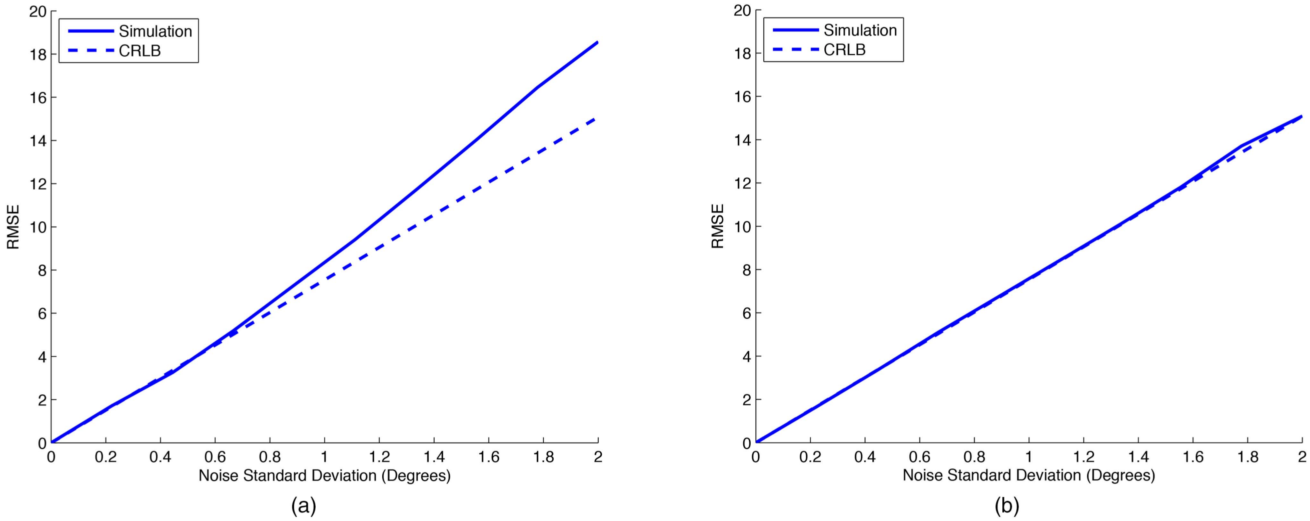


Fig. 8. The RMSE of the estimated sensor locations of the three scenarios compared to the CRLB under wrapped normal noise. In (a) we use the method of [16] utilizing a linear normal PDF that does not account for the circular nature of the measurements. In (b) we use the new ML method utilizing the clipped mod normal distribution assumption. 1,000 Monte Carlo runs were performed. (a) ML not corrected. (b) ML corrected.

The maximum noise standard deviation was set at 2°, corresponding to the accuracy of the acoustic sensors used in [19]. Higher noise standard deviations were found to produce occasionally very bad estimates (outliers). The likelihood of encountering such outliers varies depending upon the geometry. In many practical scenarios, this may not be a problem, since often, coarse estimates of the sensor locations can be obtained when they are placed and the initialization algorithm can be bypassed. The maximum noise standard deviation in the simulations in this paper was chosen sufficiently low such that extremely bad estimates did not occur, explaining the smoothness of the curves in the simulations.

7.3. ML Maximization

We compared the performance of the ML algorithm of [16] that does not take into account the circular nature of the measurements (assuming a linear normal distribution), with our ML algorithm (assuming a clipped normal distribution with the same standard deviation). The least-squares algorithm of Section 5.1.1 estimating both sensor and target locations was used to provide initial estimates. Measurements were generated using a wrapped normal distribution. All measurements were taken with respect to a common, known axis. 1000 Monte Carlo runs were performed. The results are shown in Fig. 8. Since the wrapping of the distributions depends upon where the π, $-\pi$ boundary is placed, rotating the global coordinate system changes the performance. However, if the ensemble of sensors make angular observations over the entire 360° range, then no rotation will exist where the basic linear model is nearly identical to the circular model.

In Table II, we numerically compare the effects of having different amounts of data regarding the scenario, in this case when using the von Mises noise distribution. The results are comparable when using the clipped mod normal distribution. The noise parameters for the sensors were $\sigma_\theta = 1°$ and, when range measurements were available, $\sigma_r = 7.5$ m.

TABLE II
The Average RMSE of the ML Estimates of the Sensor Locations Depending upon the Measurements Available for $\sigma_\theta = 1°$ and $\sigma_r = 7.5$ m as Obtained using the Initialization and Likelihood Maximization Algorithms Compared to the CRLB using von Mises Distributed Noise
(the results when using wrapped normal noise and performing ML maximization assuming the clipped mod normal density are similar; 1,000 Monte Carlo runs were performed)

Measurements		von Mises Simulated	von Mises CRLB
angle	known axis	5.466	5.437
	unknown axis	6.773	6.306
angle+range	known axis	2.037	2.049
	unknown axis	2.171	2.146

8. CONCLUSIONS

The importance of accounting for the circular nature of the data when performing sensor localization was highlighted. If the measurement noise is truly Gaussian, then the resulting noisy measurement will be wrapped on the unit circle, leading to the wrapped normal distribution. We introduced an approximation for the wrapped normal distribution that is very accurate for small to moderate circular standard deviation values. We derived simple linear least squares solutions for the target locations that we then used as initial estimates for performing ML estimation, as well as a method for handling a common, unknown measurement axis. When using wrapped normally-distributed noise, our estimation method accounting for the circular nature of the data and using the clipped mod normal distribution for estimation proved efficient, whereas a previously introduced (and certainly decent) ML algorithm [16] that uses a linear noise model can diverge from the CRLB, dependent upon the geometry of the sensors and where the $\pi/-\pi$ boundary for the global coordinates is placed. We also quantified the effects of having different information available when estimating the sensor locations, including the availability of range measurements and knowledge of the common measurement axis of the sensors.

APPENDIX A. DERIVATION OF (43A)

Here we derive (43a), which underlies much of the algorithm. The derivations of (43b), (43c) and (43d) are performed similarly. Equation (36a) applied to the first sensor gives us

$$\tan[\theta_{1,j}] = \frac{y_t^j - y_s^1}{x_t^j - x_s^1} \tag{65}$$

$$y_t^j = y_s^1 + (x_t^j - x_s^1)\tan[\theta_{1,j}]. \tag{66}$$

Substituting (66) into (36a) applied to the second and third sensors gives us

$$\tan[\theta_{2,j}] = \frac{y_s^1 - y_s^2 + (x_t^j - x_s^1)\tan[\theta_{1,j}]}{x_t^j - x_s^2} \tag{67}$$

$$\tan[\theta_{3,j}] = \frac{y_s^1 - y_s^3 + (x_t^j - x_s^1)\tan[\theta_{1,j}]}{x_t^j - x_s^3}. \tag{68}$$

Solving (67) for the x location of the targets gives us

$$x_t^j = \frac{y_s^2 - y_s^1 + x_s^1\tan[\theta_{1,j}] - x_s^2\tan[\theta_{2,j}]}{\tan[\theta_{1,j}] - \tan[\theta_{2,j}]}. \tag{69}$$

Substituting (69) back into (68) and simplifying gives us the form of (43a).

APPENDIX B. CALCULATING BESSEL FUNCTION RATIOS

In [10] two methods were considered for converting a continued fraction representation of the ratio of two modified Bessel functions of the first kind into sums, allowing the computation of the ratio of Bessel functions without the overflow problems associated with calculating each function alone. Though the method attributed to Gauss had better asymptotic performance, it was demonstrated that the method attributed to Perron allowed for faster calculation of the ratio of two Bessel functions to an accuracy typically desired on a computer. The method based upon work by Perron is summarized as follows:

$$\frac{I_\nu(x)}{I_{\nu-1}(x)} = \sum_{k=0}^{\infty} c_k \tag{70}$$

where $\{x,\nu\} > 0$ and

$$c_k = \prod_{n=1}^{k} p_k \tag{71}$$

$$c_0 = 1 \tag{72}$$

$$p_1 = \frac{\frac{1}{2}x\left(\nu + \frac{1}{2}\right)}{\left(\nu + \frac{x}{2}\right)\left(\nu + x + \frac{1}{2}\right) - \frac{1}{2}x\left(\nu + \frac{1}{2}\right)} \tag{73}$$

$$p_k = \frac{\frac{1}{2}x\left(\nu + k - \frac{1}{2}\right)(1 + p_{k-1})}{\left(\nu + x + \frac{k-1}{2}\right)\left(\nu + x + \frac{k}{2}\right) - \frac{1}{2}x\left(\nu + k - \frac{1}{2}\right)(1 + p_{k-1})}. \tag{74}$$

Computation of the ratio of Bessel functions may thus be approximated by summing a suitable number of terms from (70). A suitable termination criterion for $\nu = 1$ is to stop when adding the next increment no longer changes the result. For example using double-precision arithmetic, for $\kappa = 3000$ terms $k \geq 5$ no longer change the result; for $\kappa = 11$ terms $k \geq 44$ no longer change the result.

REFERENCES

[1] Y. Bar-Shalom, X. Li, and T. Kirubarajan
Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software.
J. Wiley and Sons, 2001.

[2] D. P. Bertsekas
Nonlinear Programming.
Athena Scientific, 1999.

[3] P. Biswas and Y. Ye
Semidefinite programming for ad hoc wireless network localization.
In *Proceedings of the 3rd International Symposium on Information Processing in Sensor Networks*, Barkeley, CA, 2004, pp. 46–54.

[4] V. Chandrasekhar, W. Seah, Y. S. Choo, and H. V. Ee
Localization in underwater sensor networks—survey and challenges.
In *Proceedings of the First ACM International Workshop on Underwater Networks*, 2006, pp. 33–40. [Online], available: http://wuwnet.engr.uconn.edu/papers/p033-chandrasekhar.pdf.

[5] D. F. Crouse, Y. Bar-Shalom, and P. Willett
Sensor bias estimation in the presence of data association uncertainty.
In *Proceedings of SPIE, Signal and Data Processing of Small Targets Conference*, vol. 7445-22, Aug. 2009.

[6] M. Erol, L. Vieira, and M. Gerla
AUV-aided localization for underwater sensor networks.
In *Proceedings of the International Conference on Wireless Algorithms, Systems and Applications*, Aug. 2007, pp. 44–54.

[7] N. Fisher
Statistical Analysis of Circular Data.
Cambridge, UK: Cambridge University Press, 1993.

[8] N. Fisher, T. Lewis, and B. Embleton
Statistical Analysis of Spherical Data.
Cambridge: Cambridge University Press, 1987.

[9] A. Galstyan, B. Krishnamachari, K. Lerman, and S. Pattem
Distributed online localization in sensor networks using a moving target.
In *Proceedings of the 3rd International Symposium on Information Processing in Sensor Networks*, 2004, pp. 61–70.

[10] W. Gautschi and J. Slavik
On the computation of modified Bessel function ratios.
Mathematics of Computation, **32**, 143 (July 1978), 865–875.

[11] C. Gentile and J. Shiu
Sensor location through linear programming with arrival angle constraints.
In *Proceedings of the IEEE Information Theory Society Conference on Information Sciences and Systems*, Mar. 2007.

[12] G. H. Golub and C. F. van Loan
Matrix Computations (3rd ed.).
Johns Hopkins University Press, 1996.

[13] W-L. Huang, S-J. Li, and L. Duo
Designing reduced beacon trajectory for sensor localization.
Journal of Zheijang University, **8**, 12 (2007), 1971–1982.

[14] X. Lin, Y. Bar-Shalom, and T. Kirubarajan
Multisensor-multitarget bias estimation for general asynchronous sensors.
IEEE Transactions on Aerospace and Electronic Systems, **41**, 3 (2005), 899–921.

[15] K. V. Mardia and P. E. Jupp
Directional Statistics.
Chichester, England: John Wiley & Sons, 2000.

[16] R. L. Moses, D. Krishnamurthy, and R. Patterson
An auto-calibration method for unattended ground sensors.
In *Proceedings of the IEEE International Conference on Acoustics Speech and Signal Processing*, vol. 3, 2002, pp. 2941–2944.

[17] A. Nasipuri and K. Li
A directionality based location discovery scheme for wireless sensor networks.
In *Proceedings of the 1st ACM International Workshop on Wireless Sensor Networks and Applications*, 2002, pp. 105–111.

[18] D. Niculescu and B. Nath
Ad-hoc positioning system (APS) using AOA.
In *Proceedings of the 22nd Annual Joint Conference of the IEEE Computer and Communications Societies*, San Francisco, CA, Mar. 2003.

[19] V. C. Ravindra, Y. Bar-Shalom, and T. Damarla
Feature-aided localization of ground vehicles using passive acoustic sensor arrays.
In *Proceedings of the 12th International Conference on Information Fusion*, Seattle, July 2009, pp. 70–77.

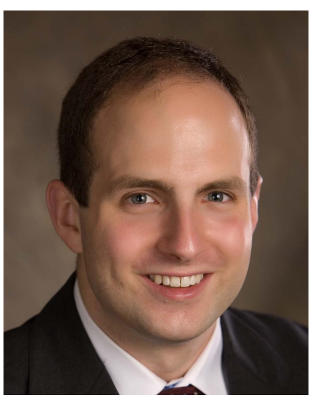
[20] B. Ristic and N. Okello
Sensor registration in ECEF coordinates using the MLR algorithm.
In *Proceedings of the Sixth International Conference of Information Fusion*, Cairns, Queensland, Australia, 2003, pp. 135–142.

[21] R. C. Shah, S. Roy, S. Jain, and W. Brunette
Data MULEs modeling a three-tier architecture for sparse sensor networks.
In *Proceedings of the First IEEE International Workshop on Sensor Network Protocols and Applications*, Seattle, May 2003, pp. 30–41.

[22] K-F. Ssu, C-H. Ou, and H. C. Jiau
Localization with mobile anchor points in wireless sensor networks.
IEEE Transactions on Vehicular Technology, **54**, 3 (May 2005), 1187–1197.

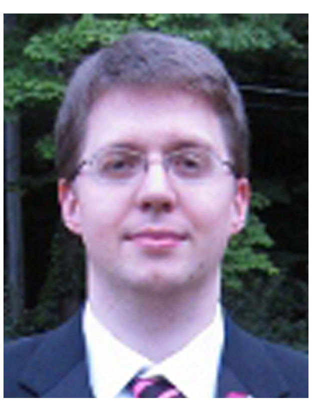
[23] G. Sun, J. Chen, W. Guo, and K. Liu
Signal processing techniques in network-aided positioning: A survey of state-of-the-art positioning designs.
IEEE Signal Processing Magazine, **22**, 4 (July 2005), 12–23.

[24] J. Zhang, M. Walpola, D. Roelant, H. Zhu, and K. Yen
Self-organization of unattended wireless acoustic sensor networks for ground target tracking.
Pervasive and Mobile Computing, **5**, 2 (Apr. 2009), 148–164.



David Frederic Crouse received his B.S., M.S., and Ph.D. degrees in electrical engineering, in 2005, 2008, and 2011, respectively, from the University of Connecticut (UCONN), Storrs, and his B.A. degree in German from the University of Connecticut in 2006.

In 2006 he spent a year with the Ruprecht-Karls-Universität, Heidelberg, Germany. He currently works at the Naval Research Laboratory in Washington, D.C. and is an associate editor for the *IEEE Aerospace and Electronic Systems Magazine*. His research interests lie in the areas of statistical signal processing and tracking.



Richard W. Osborne, III received his B.S., M.S., and Ph.D. degrees from the University of Connecticut in 2004, 2007, and 2012, respectively.

He is currently an assistant research professor in the Electrical and Computer Engineering Department at the University of Connecticut. In addition to his work at the University of Connecticut, he has consulted with a number of outside companies, and helped to develop and teach a short course on tracking and data fusion. His current research interests lie in the fields of estimation theory and information fusion.

Krishna R. Pattipati received his B.Tech. degree in electrical engineering with highest honors from the Indian Institute of Technology, Kharagpur, in 1975, and his M.S. and Ph.D. degrees in systems engineering from UConn, Storrs, in 1977 and 1980, respectively.

He was with ALPHATECH, Inc., Burlington, MA from 1980 to 1986. He has been with the Department of Electrical and Computer Engineering at UConn, where he is currently the UTC Professor of Systems Engineering. His current research activities are in the areas of agile planning, diagnosis and prognosis techniques for cyber-physical systems, multi-object tracking, and combinatorial optimization. A common theme among these applications is that they are characterized by a great deal of uncertainty, complexity, and computational intractability. He is a cofounder of Qualtech Systems, Inc., a firm specializing in advanced integrated diagnostics software tools (TEAMS, TEAMS-RT, TEAMS-RDS, TEAMATE), and serves on the board of Aptima, Inc.

Dr. Pattipati was selected by the IEEE Systems, Man, and Cybernetics (SMC) Society as the Outstanding Young Engineer of 1984, and received the Centennial Key to the Future award. He has served as the Editor-in-Chief of the *IEEE Transactions on Systems, Man, and Cybernetics—Part B* from 1998 to 2001, vice-president for technical activities of the IEEE SMC Society from 1998 to 1999, and as vice-president for conferences and meetings of the IEEE SMC Society from 2000 to 2001. He was corecipient of the Andrew P. Sage Award for the Best SMC Transactions Paper for 1999, the Barry Carlton Award for the Best AES Transactions Paper for 2000, the 2002 and 2008 NASA Space Act Awards for "A Comprehensive Toolset for Model-based Health Monitoring and Diagnosis," and "Real-time Update of Fault-Test Dependencies of Dynamic Systems: A Comprehensive Toolset for Model-Based Health Monitoring and Diagnostics," and the 2003 AAUP Research Excellence Award at UCONN. He also won the best technical paper awards at the 1985, 1990, 1994, 2002, 2004, 2005, and 2011 IEEE AUTOTEST Conferences, and at the 1997, 2004 Command and Control Conference. He is an elected Fellow of IEEE and of the Connecticut Academy of Science and Engineering.

Peter Willett is an IEEE Fellow and has been a faculty member in the Electrical and Computer Engineering Department at the University of Connecticut since 1986. His interests are in statistical signal processing, detection, machine learning, communications, data fusion, and tracking. He was Editor-in-Chief of *IEEE Transactions on AES* from 2006–2011. He has been Associate Editor of the *IEEE AES, SMC-A and SMC-B Transactions*. He remains Associate Editor of the *IEEE AES Magazine*, and ISIF's *Journal of Advances in Information Fusion*, and is also a member of the editorial board of IEEE's *Special Topics in Signal Processing* journal and a Senior Editor of *IEEE Signal Processing Letters*. He is a member of the IEEE AESS Board of Governors and is Vice-Chair of the IEEE Signal Processing Society's Sensor-Array and Multichannel Technical Committee.

Yaakov Bar Shalom was born on May 11, 1941. He received his B.S. and M.S. degrees from the Technion, Israel Institute of Technology, in 1963 and 1967 and the Ph.D. degree from Princeton University in 1970, all in electrical engineering.

From 1970 to 1976 he was with Systems Control, Inc., Palo Alto, CA. Currently he is Board of Trustees Distinguished Professor in the Department of Electrical and Computer Engineering and Marianne E. Klewin Professor in Engineering at the University of Connecticut. He is also Director of the ESP (Estimation and Signal Processing) Lab.

His current research interests are in estimation theory, target tracking and data fusion. He has published over 400 papers and book chapters in these areas and in stochastic adaptive control. He coauthored the monograph *Tracking and Data Association* (Academic Press, 1988), the graduate texts *Estimation and Tracking: Principles, Techniques* and *Software* (Artech House, 1993), *Estimation with Applications to Tracking and Navigation: Algorithms and Software for Information Extraction* (Wiley, 2001), the advanced graduate texts *Multitarget-Multisensor Tracking: Principles and Techniques* (YBS Publishing, 1995), *Tracking and Data Fusion* (YBS Publishing, 2011), and edited the books *Multitarget-Multisensor Tracking: Applications and Advances* (Artech House, Vol. I, 1990; Vol. II, 1992; Vol. III, 2000).

He has been elected Fellow of IEEE for "contributions to the theory of stochastic systems and of multi-target tracking." He has been consulting to numerous companies and government agencies, and originated the series of Multitarget-Multisensor Tracking short courses offered via UCLA Extension, at Government Laboratories, private companies and overseas.

During 1976 and 1977 he served as Associate Editor of the *IEEE Transactions on Automatic Control* and from 1978 to 1981 as Associate Editor of *Automatica*. He was Program Chairman of the 1982 American Control Conference, General Chairman of the 1985 ACC, and Cochairman of the 1989 IEEE International Conference on Control and Applications. During 1983–87 he served as Chairman of the Conference Activities Board of the IEEE Control Systems Society and during 1987–89 was a member of the Board of Governors of the IEEE CSS. He was a member of the Board of Directors of the International Society of Information Fusion (1999–2004) and served as General Chairman of FUSION 2000, President of ISIF in 2000 and 2002 and Vice President of Publications in 2004–11.

In 1987 he received the IEEE CSS Distinguished Member Award. Since 1995 he is a Distinguished Lecturer of the IEEE AESS and has given numerous keynote addresses at major national and international conferences. He is corecipient of the M. Barry Carlton Award for the best paper in the *IEEE Transactions on Aerospace and Electronic Systems* in 1995 and 2000 and recipient of the 1998 University of Connecticut AAUP Excellence Award for Research. In 2002 he received the J. Mignona Data Fusion Award from the DoD JDL Data Fusion Group. He is a member of the Connecticut Academy of Science and Engineering. In 2008 he was awarded the IEEE Dennis J. Picard Medal for Radar Technologies and Applications, and in 2012 the Connecticut Medal of Technology.

He has been listed by academic.research.microsoft (top authors in engineering) as #1 among the researchers in Aerospace Engineering based on the citations of his work.

INTERNATIONAL SOCIETY OF INFORMATION FUSION

ISIF Website: http://www.isif.org

SOCIETY VISION

The International Society of Information Fusion (ISIF) is the premier professional society and global information resource for multidisciplinary approaches for theoretical and applied information fusion technologies.

SOCIETY MISSION

Advocate

To advance the profession of fusion technologies, propose approaches for solving real-world problems, recognize emerging technologies, and foster the transfer of information.

Serve

To serve its members and engineering, business, and scientific communities by providing high-quality information, educational products, and services.

Communicate

To create international communication forums and hold international conferences in countries that provide for interaction of members of fusion communities with each other, with those in other disciplines, and with those in industry and academia.

Educate

To promote undergraduate and graduate education related to information fusion technologies at universities around the world. Sponsor educational courses and tutorials at conferences.

Integrate

Integrate ideas from various approaches for information fusion, and look for common threads and themes– look for overall principles, rather than a multitude of point solutions. Serve as the central focus for coordinating the activities of world-wide information fusion related societies or organizations. Serve as a professional liaison to industry, academia, and government.

Disseminate

To propagate the ideas for integrated approaches to information fusion so that others can build on them in both industry and academia.

Call for Papers

The Journal of Advances in Information Fusion (JAIF) seeks original contributions in the technical areas of research related to information fusion. Authors of papers in one of the technical areas listed on the inside cover of JAIF are encouraged to submit their papers for peer review at http://jaif.msubmit.net.

Call for Reviewers

The success of JAIF and its value to the research community is strongly dependent on the quality of its peer review process. Researchers in the technical areas related to information fusion are encouraged to register as a reviewer for JAIF at http://jaif.msubmit.net. Potential reviewers should notify via email the appropriate editors of their offer to serve as a reviewer.

Journal of Advances in Information Fusion Volume 8 Number 1 June 2013 ISSN 1557-6418