Fusion of Asynchronous Passive Measurements

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The use of angular information in the form of line-of-sight (LOS) measurements from passive sensors for the purposes of target localization and tracking has been extensively studied. Previous work has shown that the formation of fused *composite* measurements from a minimum number of LOS measurements (two) is statistically efficient, and therefore, the Cramér-Rao Lower Bound (CRLB) provides a valid measurement noise covariance for the resulting composite measurement. If the LOS measurements are not synchronized, however, the formation of composite measurements is not possible from two LOS observations. In this paper, two methods are presented for forming composite measurements when LOS observations are obtained asynchronously. It is demonstrated that the minimum number of LOS measurements required from two asynchronous sensors is four, and that both methods provide a statistically efficient estimate for track initialization.

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1. INTRODUCTION

Target localization and tracking algorithms often make use of passive sensors, these being for stealthy surveillance of a region. The main disadvantage of using such sensors is that they generally provide only line-of-sight (LOS) measurements, and without providing range measurements, a single passive sensor cannot fully localize a target. The angular measurements could be directly used in nonlinear tracking filters, or composite measurements can be obtained by fusing multiple angular measurements, with the resulting composite measurements passed to the tracking filter.

The fused composite measurements can provide full Cartesian position (and possibly velocity) estimates to take advantage of the ensuing linear measurement equation. If the state equation is linear, then one can use linear filters. The use of S-D assignment algorithms for association of angular measurements from passive sensors can be found in [4], [8]. In the present paper we will assume that the angular measurements have been correctly associated and will focus on the formation of composite measurements and their use in track initialization. The composite measurements could continue to be used in a linear tracking filter, or the angular measurements could be used directly in a nonlinear (EKF) filter, such as in [9], where tracking boost phase missiles with LOS measurements was examined.

Prior work on target localization through angular measurements includes [3], [5], [6], [10]–[12]. Application of Taylor-series estimation to the problem of target localization is presented in [5] and extended in [11]. In both papers, though the statistics of the estimation errors are examined, neither the CRLB nor the statistical efficiency of the procedure is investigated.

In [10], equations are derived for the covariance-based uncertainty ellipsoids, circular error probability regions, and geometric dilution of precision, along with their relation to the particular localization scheme and received signal characteristics. However, the CRLB and the statistical efficiency of the estimation scheme are not considered.

LOS measurements to "beacons" with known location have been used to determine the position and attitude of a sensor (camera) in [3]. In this formulation, the LOS angle measurements to the beacons are taken by the sensor at an unknown location and the angles are with respect to the unknown attitude of the sensor. Thus, the estimation of the sensor location and attitude has to be done simultaneously. Observability conditions and the CRLB were derived for this problem.

An investigation of the CRLB of the initial state estimate of a boost phase object using LOS measurements from geosynchronous satellites is considered in [12]. That paper, however, focused only on the behavior of the CRLB, and not on whether any estimation scheme meets the CRLB.

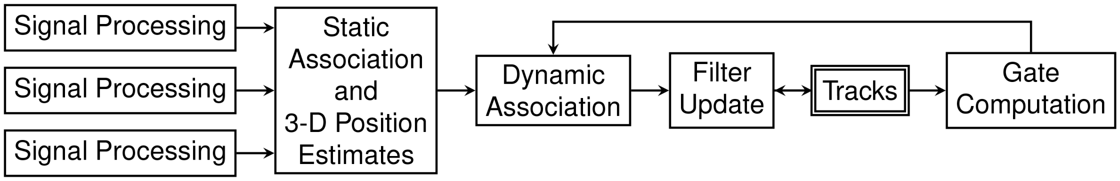

Fig. 1. Type III multisensor information processing configuration.

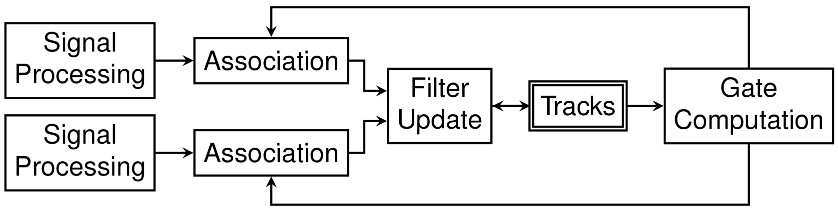

Fig. 2. Type IV multisensor information processing configuration.

The ML estimation for bearing-only target localization (triangulation) was considered in [6]. In that paper, examples in two dimensions with bearing-only measurements show that the ML estimator is unbiased and efficient only when a significant number of measurements are utilized.

None of the previously mentioned papers provided a comparison of linear and nonlinear tracking filters using angular and composite measurements. Furthermore, these papers did not examine the particular difficulties of utilizing asynchronous measurements.

In [7] the statistical efficiency of composite measurements was examined for passive sensors which provide LOS measurements. In that work, it was shown via statistical tests that the minimum number of measurements (two) provide a composite measurement with a resulting estimation error that was consistent with the Cramér-Rao Lower Bound (CRLB) of the resulting parameter estimation problem. Demonstration that the estimator in question was statistically efficient (i.e., the estimator met the CRLB) was of particular interest since the CRLB can be easily calculated and can then be used as a measurement noise covariance for linear tracking filters which utilize the resulting composite measurement.

One particular limitation of the method outlined in [7] is the need for the sensors to be perfectly synchronized. The present work expands the method of [7] for use with asynchronous passive sensors and again demonstrates the statistical efficiency of the approach. Additionally, an alternative track initialization scheme using interpolated LOS measurements is examined, and it is demonstrated that both methods are statistically efficient and the performance difference between them is statistically insignificant.

Section 2 provides an overview of the relevant data fusion configurations (Type III and Type IV—see [2]). Section 3 formulates the problem by illustrating the requirements for observability and outlining the method of forming composite measurements. Section 4 provides simulation results and Section 5 summarizes the paper and presents conclusions.

2. MULTISENSOR TRACKING CONFIGURATIONS

As defined in [2], there are four general configurations of information processing for multisensor tracking. The Type I configuration refers to single sensor tracking and provides a baseline for comparison of multisensor tracking configurations. The Type II configuration refers to single sensor tracking followed by track-to-track association and fusion. There exist a number of subsets of this configuration depending on possible levels of feedback and memory. Of particular interest to this paper, however, are the Type III and Type IV configurations.

2.1. Type III Multisensor Configuration

The Type III multisensor configuration is illustrated in Figure 1. In this configuration, each (passive) sensor performs individual signal processing to generate (LOS) measurements. The measurements are then passed on to a fusion center where the measurements are associated and combined into full three-dimensional (3D) position measurements, referred to as *supermeasurements* or *composite measurements*. The composite measurements can then undergo "dynamic association," i.e., the association of measurements to existing tracks (or, alternatively, to form new tracks). The use of composite measurements allows the tracking filter to behave as a single sensor tracker would.

2.2. Type IV Multisensor Configuration

The Type IV multisensor configuration is the fully centralized multisensor tracking configuration and illustrated in Figure 2. In this configuration, each sensor performs individual signal processing to generate measurements, and each measurement is passed to a fusion center which will then perform the association of measurements to tracks followed by track update (as well as formation/termination of new/old tracks).

3. PROBLEM FORMULATION

Assume we have N_s sensors with known position $\mathbf{s}_i = [x_i, y_i, z_i]'$, $i = 1,2,...,N_s$, in Cartesian coordinates. Each sensor provides line-of-sight (LOS) measurements, where the LOS measurement at time t_n (not necessarily the same across sensors) is to a target at the unknown position $\mathbf{x}_p(t_n) = [x(t_n), y(t_n), z(t_n)]'$, in the same Cartesian coordinates. The measurement from sensor i and at time t_n is

$$\mathbf{z}_i(t_n) = h(\mathbf{x}_p(t_n), \mathbf{s}_i) + w_i(t_n) \tag{1}$$

where $w_i(t_n)$ is zero-mean white Gaussian measurement noise with covariance matrix R_i and

$$h(\mathbf{x}_p(t_n), \mathbf{s}_i) = \begin{bmatrix} \alpha_i(t_n) \\ \epsilon_i(t_n) \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\dfrac{y(t_n) - y_i}{x(t_n) - x_i}\right) \\ \tan^{-1}\left(\dfrac{z(t_n) - z_i}{\sqrt{(x(t_n) - x_i)^2 + (y(t_n) - y_i)^2}}\right) \end{bmatrix} \tag{2}$$

Furthermore, it will be assumed that, in the asynchronous case, the measurements are provided to the fusion center with a time stamp at which the measurement was taken. This time stamp will be assumed to be known perfectly.

For a more detailed overview of the LOS measurement fusion in the synchronous case, see [7]. The procedure for the synchronous case is to utilize Iterated Least Squares (closely related to the Gauss-Newton method) with two LOS measurements to obtain a maximum likelihood (ML) estimate of the full 3D position of the target. We assume that there is no data association uncertainty between the two measurements (i.e., it is known perfectly that they belong to the same target).

For the asynchronous case, modifications are needed to account for each measurement being taken at a different time. Assuming that the measurements are taken a short time interval apart (so that the target does not have time to maneuver), the target will be well-modeled by a constant velocity motion model. In order to fit a constant velocity motion model to the target, a six dimensional state vector must be estimated, consisting of the target's position and velocity at a particular point in time. There exists, however, a subtle unobservability for this problem that will necessitate the use of more measurements than at first seems necessary.

3.1. Incomplete Observability of the Target State with Three LOS Measurements

Since each LOS measurement (1) is a two dimensional vector, three such measurements should be the minimum required to solve for a constant velocity target's state, i.e., we have six equations (observations from (2)) and six unknowns (target position and velocity in 3D space). The estimation of the constant velocity target's state at a particular point in time, however, is basically equivalent to finding three positions along the LOS vectors (one position along each vector), such that the three positions are appropriately spaced to match the constant velocity model and the three time stamps. Given three sensors with one LOS from each, the target parameter vector is fully observable (provided the sensors are not positioned on a straight line). If multiple LOS measurements are provided by the same sensor, however, there is a lack of full observability when three LOS measurements are provided if the trajectory is coplanar with the line connecting the two sensors. This incomplete observability will be demonstrated by illustrating some of the multiple solutions obtained when given two LOS measurements from one sensor and one from a second sensor.

Figure 3 depicts three possible trajectories which are found to fit the same three LOS observations provided by two sensors. In addition to the true target which was simulated to generate the observations, there are two other ghost targets (only two are shown here; there are many possible), traveling in different directions with different constant speeds, that could have produced the same observations. Since the target's state is thus unobservable with three LOS measurements, the *minimum number of observations* which can form composite measurements from two asynchronous sensors is *four*.

3.2. Formation of Composite Measurements from Asynchronous LOS

Due to the issues with observability of the six dimensional target state, a single composite measurement will be formed from a batch of four asynchronous LOS measurements. Similar to [7], the formation of the composite measurement will be done via Iterated Least Squares (ILS) [1] using the ML criterion.

We will assume that the batch of measurements provided to the fusion center is

$$\mathbf{z} = [\mathbf{z}_1(t_1)', \mathbf{z}_2(t_2)', \mathbf{z}_3(t_3)', \mathbf{z}_4(t_4)']' \quad t_1 < t_2 < t_3 < t_4 \tag{3}$$

where $\mathbf{z}_i(t_n)$ is given by (1).[1]

The composite measurement will consist of the target's state

$$\mathbf{x}(t_f) = [x(t_f), y(t_f), z(t_f), \dot{x}(t_f), \dot{y}(t_f), \dot{z}(t_f)]' \tag{4}$$

[1]The notation of (3) would seem to suggest that four sensors are used, however, any order of measurements from two to four sensors would be valid. In fact, in later simulations, the measurements will be assumed to come from two sensors at alternating times, i.e., $[z_1(t_1), z_2(t_2), z_1(t_3), z_2(t_4)]$.

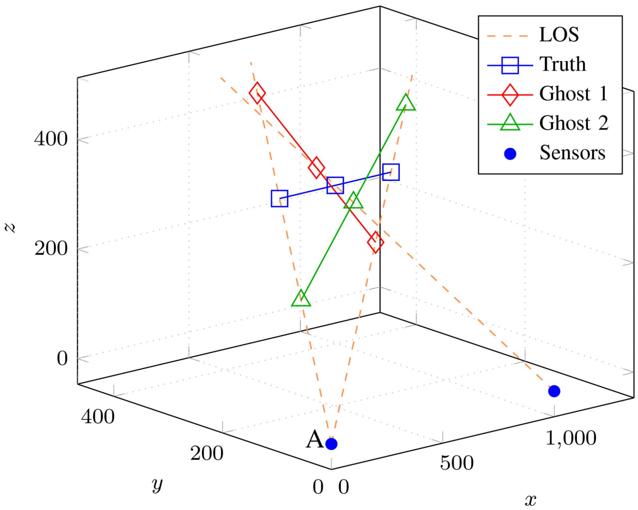

Fig. 3. Three possible target trajectories that fit three identical LOS observations. The speeds of these targets are 250 m/s (Truth), 146.6 m/s (Ghost 1), and 498.5 m/s (Ghost 2). The two LOS from A and one LOS from B allow additional possible (ghost) trajectories.

at a particular "fusion time" t_f. The ILS estimate (after the jth iteration) of the target state (4) is

$$\hat{\mathbf{x}}_{\mathrm{ILS}}^{j+1} = \hat{\mathbf{x}}_{\mathrm{ILS}}^{j} + [(H^j)'R^{-1}H^j]^{-1}(H^j)'R^{-1} \cdot [\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}_{\mathrm{ILS}}^{j})] \tag{5}$$

where

$$\mathbf{h}(\hat{\mathbf{x}}_{\mathrm{ILS}}^{j}) \triangleq \begin{bmatrix} h(F(t_1,t_f)\hat{\mathbf{x}}_{\mathrm{ILS}}^{j},\mathbf{s}_1) \\ h(F(t_2,t_f)\hat{\mathbf{x}}_{\mathrm{ILS}}^{j},\mathbf{s}_2) \\ h(F(t_3,t_f)\hat{\mathbf{x}}_{\mathrm{ILS}}^{j},\mathbf{s}_3) \\ h(F(t_4,t_f)\hat{\mathbf{x}}_{\mathrm{ILS}}^{j},\mathbf{s}_4) \end{bmatrix} \tag{6}$$

$$F(t_n,t_f) \triangleq \begin{bmatrix} 1 & 0 & 0 & t_n - t_f & 0 & 0 \\ 0 & 1 & 0 & 0 & t_n - t_f & 0 \\ 0 & 0 & 1 & 0 & 0 & t_n - t_f \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$R = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & R_4 \end{bmatrix} \tag{8}$$

and H^j is the Jacobian matrix of the measurements (given below) evaluated at the jth ILS estimate.

By using the transition matrix (7), the target state (4) is predicted to the time of each measurement in (3), and (6) provides the predicted LOS observation for use in forming the necessary residuals for the ILS iteration (5).

The Jacobian matrix is

$$H = [H_1' \quad H_2' \quad H_3' \quad H_4']' \tag{9}$$

where[2]

$$H_i = \begin{bmatrix} \frac{\partial \alpha_i}{\partial x} & \frac{\partial \alpha_i}{\partial y} & \frac{\partial \alpha_i}{\partial z} & \frac{\partial \alpha_i}{\partial \dot{x}} & \frac{\partial \alpha_i}{\partial \dot{y}} & \frac{\partial \alpha_i}{\partial \dot{z}} \\ \frac{\partial \epsilon_i}{\partial x} & \frac{\partial \epsilon_i}{\partial y} & \frac{\partial \epsilon_i}{\partial z} & \frac{\partial \epsilon_i}{\partial \dot{x}} & \frac{\partial \epsilon_i}{\partial \dot{y}} & \frac{\partial \epsilon_i}{\partial \dot{z}} \end{bmatrix} \tag{10}$$

The necessary partial derivatives with respect to the position terms of (4) are

$$\frac{\partial \alpha_i}{\partial x} = -\frac{\Delta y_i}{(\Delta x_i)^2 + (\Delta y_i)^2} \tag{11}$$

$$\frac{\partial \alpha_i}{\partial y} = \frac{\Delta x_i}{(\Delta x_i)^2 + (\Delta y_i)^2} \tag{12}$$

$$\frac{\partial \alpha_i}{\partial z} = 0 \tag{13}$$

$$\frac{\partial \epsilon_i}{\partial x} = -\frac{(\Delta x_i)(\Delta z_i)}{\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}\|\mathbf{x} - \mathbf{s}_i\|^2} \tag{14}$$

$$\frac{\partial \epsilon_i}{\partial y} = -\frac{(\Delta y_i)(\Delta z_i)}{\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}\|\mathbf{x} - \mathbf{s}_i\|^2} \tag{15}$$

$$\frac{\partial \epsilon_i}{\partial z} = \frac{\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}}{\|\mathbf{x} - \mathbf{s}_i\|^2} \tag{16}$$

where $\|\cdot\|$ denotes the Euclidean norm,

$$\begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} \triangleq \begin{bmatrix} 1 & 0 & 0 & \Delta t_n & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t_n & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t_n \end{bmatrix} \mathbf{x} - \mathbf{s}_i \tag{17}$$

and

$$\Delta t_n \triangleq t_n - t_f \tag{18}$$

The partial derivatives with respect to the velocity terms of (4) are

$$\frac{\partial \alpha_i}{\partial \dot{x}} = \Delta t_n \frac{\partial \alpha_i}{\partial x} \tag{19}$$

$$\frac{\partial \alpha_i}{\partial \dot{y}} = \Delta t_n \frac{\partial \alpha_i}{\partial y} \tag{20}$$

$$\frac{\partial \alpha_i}{\partial \dot{z}} = \Delta t_n \frac{\partial \alpha_i}{\partial z} \tag{21}$$

$$\frac{\partial \epsilon_i}{\partial \dot{x}} = \Delta t_n \frac{\partial \epsilon_i}{\partial x} \tag{22}$$

$$\frac{\partial \epsilon_i}{\partial \dot{y}} = \Delta t_n \frac{\partial \epsilon_i}{\partial y} \tag{23}$$

$$\frac{\partial \epsilon_i}{\partial \dot{z}} = \Delta t_n \frac{\partial \epsilon_i}{\partial z} \tag{24}$$

3.3. Initial Solution

In order to perform the numerical search via ILS, an initial estimate $\hat{\mathbf{x}}_{\mathrm{ILS}}^{0}$ is required. Since four LOS measurements are needed to form the composite measurement, the initialization will be done by forming two Cartesian measurements from pairs of LOS, as if they

[2] The time argument t_f has been omitted for simplicity, but note that the partial derivatives are taken with respect to the elements of (4).

were taken synchronously. Since the initialization needs only to be approximate, the error introduced by (incorrectly) assuming pairs of LOS measurements are taken synchronously will be corrected by refinement in subsequent updates.

In this case, each pair of LOS measurements will form a Cartesian position as

$$\begin{bmatrix} x^0 \\ y^0 \\ z^0 \end{bmatrix} = \begin{bmatrix} \dfrac{y_2 - y_1 + x_1 \tan\alpha_1 - x_2 \tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} \\ \dfrac{\tan\alpha_1 (y_2 + \tan\alpha_2 (x_1 - x_2)) - y_1 \tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} \\ z_1 + \tan\epsilon_1 \left| \dfrac{(y_1 - y_2)\cos\alpha_2 + (x_2 - x_1)\sin\alpha_2}{\sin(\alpha_1 - \alpha_2)} \right| \end{bmatrix} \tag{25}$$

The two Cartesian positions formed from (25) can then be differenced to provide an approximate initial velocity estimate. This procedure is analogous to two-point differencing [1] and will provide a full six-dimensional state estimate to initialize the ILS algorithm.

3.4. Formation of Composite Measurements from Interpolated Asynchronous LOS

As an alternative to the above method of forming full composite measurements by explicitly fitting to a constant velocity model, one could form the composite measurement by utilizing interpolated asynchronous LOS measurements and two-point differencing.

In the interpolation method, the successive LOS measurements from one sensor are interpolated to the time of a measurement from the second sensor, i.e., the interpolated measurement is

$$\hat{\mathbf{z}}_i(t_n) = \mathbf{z}_i(t_{n-1}) + \frac{t_n - t_{n-1}}{t_{n+1} - t_{n-1}}[\mathbf{z}_i(t_{n+1}) - \mathbf{z}_i(t_{n-1})]] \tag{26}$$

where t_n in this case would be the time of the measurement from the second sensor, and t_{n-1} and t_{n+1} are the times of the two measurements from the first sensor. This interpolated LOS and the second sensor's LOS measurement can then provide a composite position measurement using the method of [7]. This can be repeated using a different set of LOS measurements to obtain a composite position measurement at another time. Two-point differencing is then performed on the two composite position measurements, and the resulting state estimate and covariance are predicted to the fusion time t_f.

In later sections, comparisons are made between this interpolation method and the full asynchronous LOS composite measurement method of Subsection 3.2. In order to use the same number of asynchronous LOS measurements (four) in both the interpolation method and the composite measurement method, the first use of (26) will involve $[\mathbf{z}_1(t_1), \mathbf{z}_2(t_2), \mathbf{z}_1(t_3)]$, and the second will involve $[\mathbf{z}_2(t_2), \mathbf{z}_1(t_3), \mathbf{z}_2(t_4)]$. The use of the middle two LOS measurements in both composite measurements will result in correlated errors, but the two-point differencing will be carried out assuming uncorrelated errors.

3.5. Cramér-Rao Lower Bound

The Cramér-Rao Lower Bound (CRLB) provides a lower bound on the estimation error obtainable from an unbiased estimator, where

$$E\{(x - \hat{x})(x - \hat{x})'\} \geq J^{-1} \tag{27}$$

where J is the Fisher Information Matrix (FIM), x is the true value to be estimated, and $\hat{x}$ is the estimate.

The FIM is

$$J = E\{[\nabla_{\mathbf{x}} \ln \Lambda(\mathbf{x})][\nabla_{\mathbf{x}} \ln \Lambda(\mathbf{x})]'\}|_{\mathbf{x}=\mathbf{x}_{true}} \tag{28}$$

where $\Lambda(\mathbf{x})$ is the likelihood function of the parameter vector to be estimated, and the FIM is evaluated at the true parameter vector.[3]

The gradient of the log-likelihood function is

$$\nabla_{\mathbf{x}} \lambda(\mathbf{x}) = \sum_{i=1}^{N_s} H_i' R_i^{-1} (\mathbf{z}_i(t_i) - h(F(t_f, t_i)\mathbf{x}, \mathbf{s}_i)) \tag{29}$$

which, when plugged into (28) gives

$$J = \sum_{i=1}^{N_s} H_i' (R_i^{-1})' H_i |_{\mathbf{x}=\mathbf{x}_{true}} \tag{30}$$

$$= H'(R^{-1})' H |_{\mathbf{x}=\mathbf{x}_{true}} \tag{31}$$

The resulting CRLB, J^{-1}, evaluated at the final estimate $\hat{\mathbf{x}}_{\mathrm{ILS}}$, can be used as an (estimated) measurement noise covariance matrix for the resulting composite measurement. This allows $\hat{\mathbf{x}}_{\mathrm{ILS}}$ to be used as a linear measurement, avoiding the need to use LOS measurements directly in a nonlinear tracking filter.

4. SIMULATION RESULTS

In order to examine the fusion of asynchronous LOS measurements in Type III multisensor tracking configurations, a nearly constant velocity target was simulated. The motion model used was a constant velocity (CV) motion model [1].

The target's initial state was

$$\mathbf{x}(t_0) = [-4000 \quad 4000 \quad 500 \quad 100 \quad 0 \quad 0]' \tag{32}$$

Two sensors were assumed to be positioned at

$$\mathbf{s}_1 = [0 \quad 0 \quad 0]' \tag{33}$$

$$\mathbf{s}_2 = [2000 \quad y_2 \quad 0]' \tag{34}$$

[3] The strict definition of the FIM requires it to be evaluated at the true parameter, however, evaluation at the estimate (referred to as the *observed* Fisher information) generally yields a very good approximation.

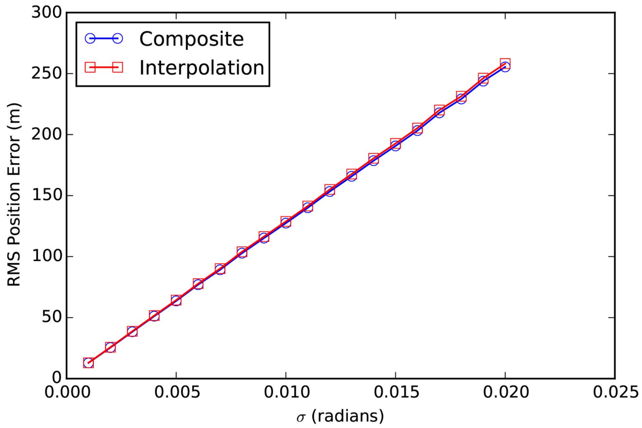

Fig. 4. RMS position error (over 10,000 Monte Carlo runs) of initial track state for various measurement noise standard deviations and y_2 = 8,000.

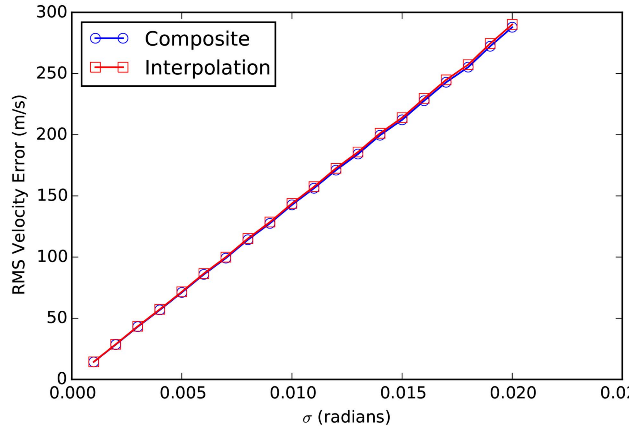

Fig. 5. RMS velocity error (over 10,000 Monte Carlo runs) of initial track state for various measurement noise standard deviations and y_2 = 8,000.

where two sensor-target geometries are tested by using $y_2 = 0$ or $y_2 = 8000$. When $y_2 = 0$, the target trajectory is coplanar with the line connecting the sensors, demonstrating in a practical fashion that the fourth LOS measurement provides observability for the problem described in Subsection 3.1.

The two sensors are assumed to provide measurements at a sampling interval of $T = 1$ s, however, Sensor 2 provides measurements offset $T/2$ after Sensor 1; meaning there is one LOS measurement provided every $T/2$, as opposed to two LOS measurements provided every T.

With synchronous measurements, the Type III configuration could provide composite measurements of the target position in Cartesian space once every second, formed in the same manner as in [7]. With asynchronous measurements, the Type III configuration will provide composite measurements of the target position *and velocity* at the time of the final LOS observation in the batch; but only one composite measurement will be generated at intervals of $2T$ (since four LOS measurements are needed).

Two methods of initializing target tracks using a batch of four LOS measurements will be compared. The formation of composite position and velocity estimates described in Section 3.2 can be used, with the CRLB covariance used as the initial track covariance. Alternatively, the method of Section 3.4—a combination of interpolation of the LOS measurements and two-point differencing [1]—will be used.

Figure 4 shows the RMS position error of the initial track state for both initialization methods. The measurement noise standard deviation is varied from 1 mrad to 20 mrad, $y_2 = 8000$, and 10,000 Monte Carlo runs are performed. The two methods perform nearly identically.

Figure 5 shows the RMS velocity error of the initial track state for both initialization methods. Once again, the velocity error is nearly identical for both methods.

Figure 6 shows the normalized estimation error squared (NEES) for the two initialization methods. The

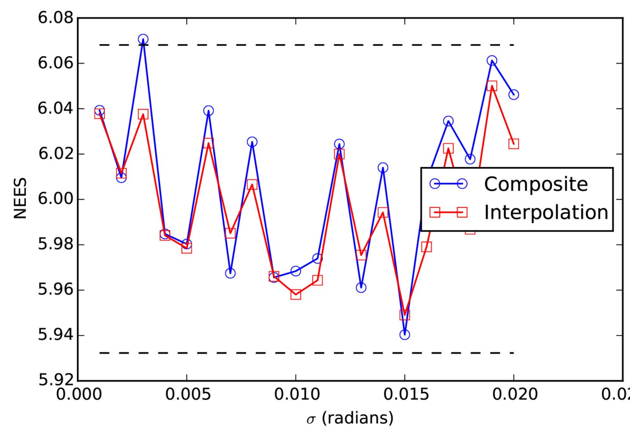

Fig. 6. Normalized estimation error squared (NEES) over 10,000 Monte Carlo runs, with y_2 = 8,000.

NEES provides a way of evaluating the consistency of the estimation errors with the covariances provided by each estimation method. The dashed line of the figure shows the 95% probability region for the NEES, demonstrating that the estimate errors are commensurate with their corresponding calculated covariances. In the case of the composite measurement, the covariance is provided by the CRLB. In the case of the interpolation method, the covariance is given by the two-point differencing procedure [1], where the measurement noise covariance for each of the two composite position measurements is given by the CRLB as outlined in [7]. Note that, since an interpolated measurement is used, the measurement noise covariance of the interpolated measurement is not equal to the single LOS measurement noise covariance. Due to the interpolation (and the fact that, in this case, the interpolation is performed at the midpoint between two measurements), the measurement noise covariance of the interpolated LOS measurements is half that of an individual measurement. The fact that the measurement noises in the interpolations are correlated, however, will be neglected.

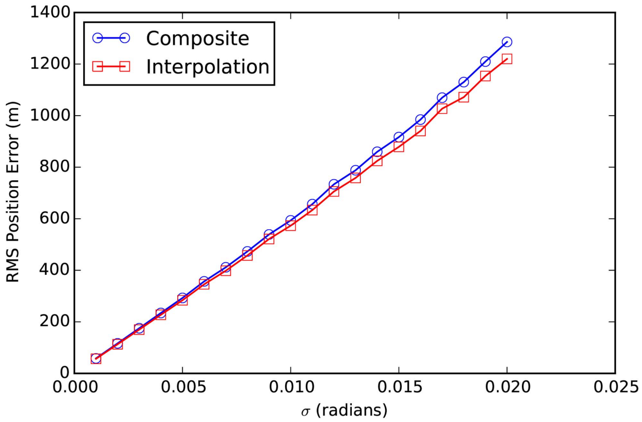

Fig. 7. RMS position error (over 10,000 Monte Carlo runs) of initial track state for various measurement noise standard deviations and $y_2 = 0$.

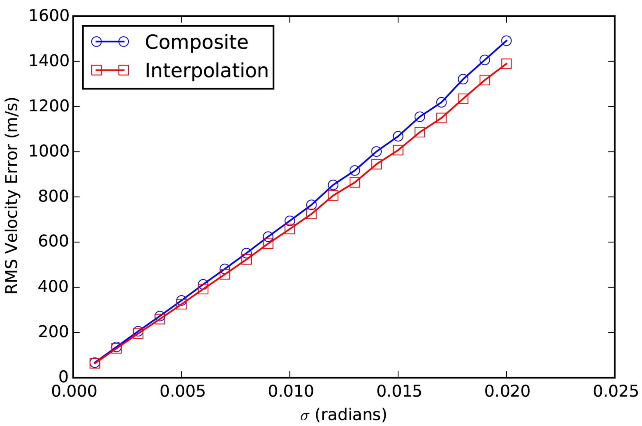

Fig. 8. RMS velocity error (over 10,000 Monte Carlo runs) of initial track state for various measurement noise standard deviations and $y_2 = 0$.

The method which fuses all four LOS measurements into a composite position and velocity estimate provides a consistent covariance using the CRLB, demonstrating that the estimator is statistically efficient with a batch of four LOS measurements. The interpolation method, however, also provides consistency, in spite of the fact that the interpolation measurement noises are assumed uncorrelated.

Figures 7–8 show the RMSE position and velocity error when $y_2 = 0$. In this case, there appears to be a slight improvement in performance when using interpolated measurements. In order to test this, the statistical significance of the error difference was examined. The squared error of each element of the state (position and velocity in x, y, and z) was normalized by its respective error covariance and averaged over the Monte Carlo runs. This provides a statistical test involving a chi-square random variable (similar to the NEES), where a non-zero mean in the difference of the errors (i.e., a significant improvement in one method over the other) will manifest as a value outside of the $1-\alpha$ probability region. For the average over 10,000 Monte Carlo runs and

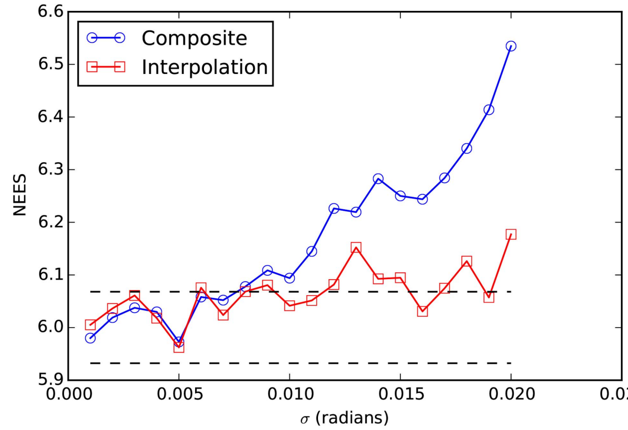

Fig. 9. Normalized estimation error squared (NEES) over 10,000 Monte Carlo runs, with $y_2 = 0$.

$\alpha = 0.05$, this value is 1.02. The maximum normalized squared difference of the errors over all measurement noise values and dimensions was 0.021, well below the threshold required to reject the hypothesis that there is no statistically significant difference in accuracy.

Figure 9 shows the NEES for the case where $y_2 = 0$. For this more difficult geometry, where the target trajectory is coplanar with the line connecting the sensors, the statistical efficiency breaks down for the composite measurement method when the measurement noise standard deviation increases. The interpolation method, however, is more resistant to the difficulties imposed by the marginally observable geometry. The correlations introduced by the interpolation method work in our favor in the marginally observable case by reinforcing a solution that is skewed towards the middle two measurements. This in turn reinforces a more "straight line" solution over the ML solution's fit to the four noisy data points.

Following track initialization, the track maintenance phase can be carried out either in the Type III configuration (where batches of measurements are fused into full composite measurements of position and velocity) or the Type IV configuration where the track is updated one measurement at a time (in a nonlinear tracking filter). For the examples considered here, the choice of fusion configuration for the track maintenance phase made no significant difference in tracking performance over the course of the target's trajectory. It should be noted, however, that when the track maintenance phase was examined, both types of track updates (Type III and Type IV) were performed on identically initialized tracks. This ensures that the effect of the style of track update was examined independently of the track initialization method.

In order to test the above track initialization methods for accelerating targets (but retaining the assumption of a CV target), the simulations were repeated for targets

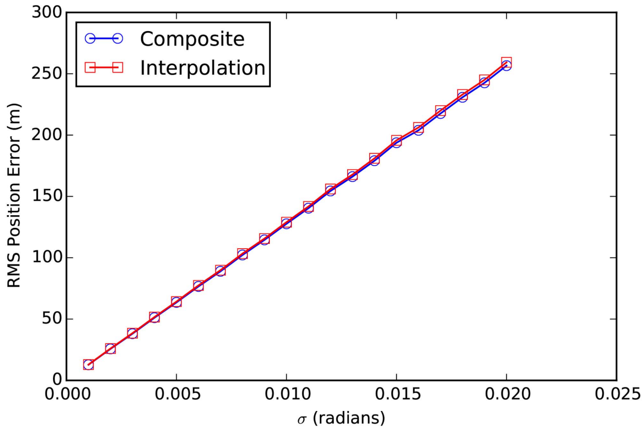

Fig. 10. RMS position error (over 10,000 Monte Carlo runs) of initial track state for y_2 = 8,000 and a target with 1 m/s^2 acceleration.

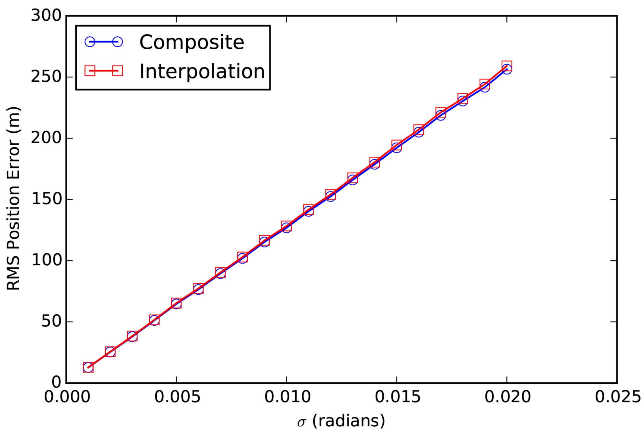

Fig. 11. RMS position error (over 10,000 Monte Carlo runs) of initial track state for y_2 = 8,000 and a target with 2 m/s^2 acceleration.

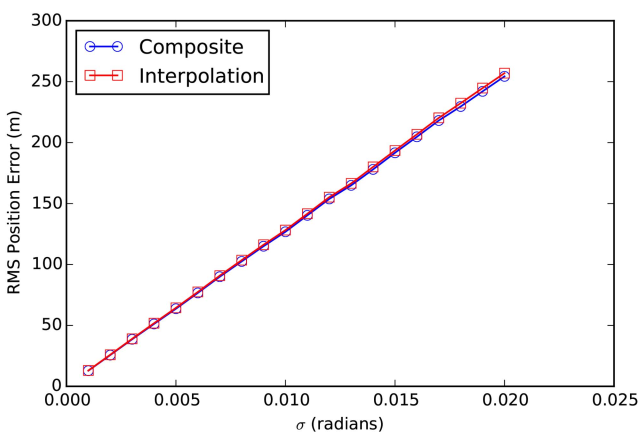

Fig. 12. RMS position error (over 10,000 Monte Carlo runs) of initial track state for y_2 = 8,000 and a target with 5 m/s^2 acceleration.

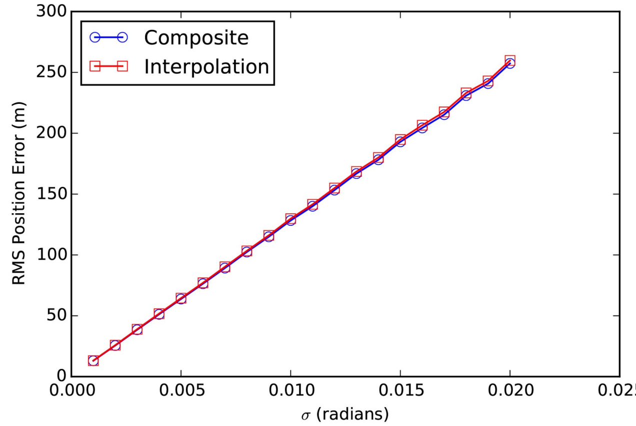

Fig. 13. RMS position error (over 10,000 Monte Carlo runs) of initial track state for y_2 = 8,000 and a target with 10 m/s^2 acceleration.

which had constant accelerations of 1 m/s^2, 2 m/s^2, 5 m/s^2 and 10 m/s^2.

Figures 10–13 show the RMSE position error for the various accelerating targets. There is no significant change in the RMSE position error over this range of accelerations. The RMSE velocity errors (not included here) show similar results.

Figures 14–17 show the NEES for the accelerating targets. When the acceleration is large enough, the errors from neglecting the acceleration component can have a significant impact on the statistical consistency for the smaller levels of measurement noise. When the measurement noise is large enough (or if the target was further away), the cross-range measurement error would mask the errors that are a result of neglecting the acceleration. In such cases (small levels of cross-range errors), the target model used in the initialization method would need to account for the acceleration. Using a constant acceleration model, however, would require more measurements in order to estimate the acceleration of the target in addition to the position and velocity.

5. CONCLUSIONS

The use of angular measurements for target localization and tracking has been widely studied, including the formation of fused composite measurements to avoid the need for nonlinear filtering. Previous research into the formation of composite Cartesian position measurements from LOS measurements demonstrated that the maximum likelihood (ML) estimate obtained via the ILS algorithm was able to provide a statistically efficient estimate using only two LOS measurements. This allowed the CRLB to be used as the measurement noise covariance for the purposes of target tracking with the fused composite measurements. This procedure required the measurements to be synchronized, however, which may be an unrealistic assumption for real systems.

This paper presented two methods of forming fused composite measurements from four *asynchronous* LOS measurements, and demonstrated that four LOS measurements are the minimum required from two asynchronous sensors to do so. In addition to forming a composite position and velocity estimate directly from the four asynchronous LOS measurements, an alternative

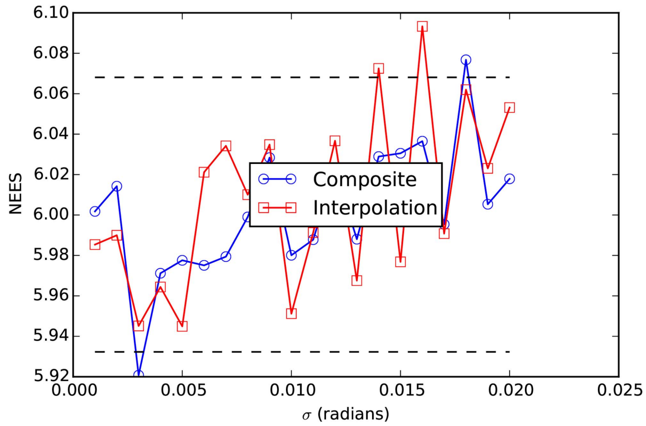

Fig. 14. Normalized estimation error squared (NEES) over 10,000 Monte Carlo runs, with y_2 = 8,000 and a target with 1 m/s^2 acceleration.

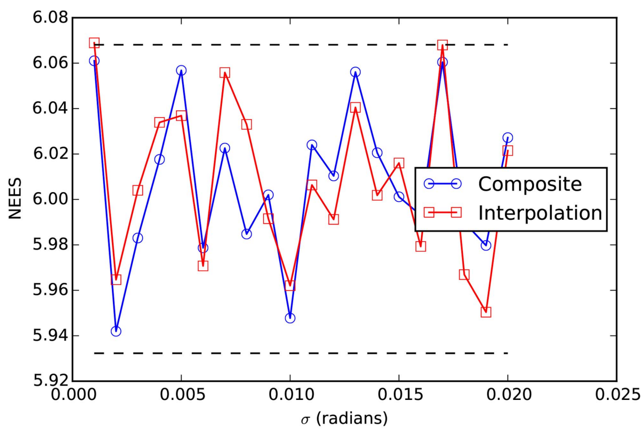

Fig. 15. Normalized estimation error squared (NEES) over 10,000 Monte Carlo runs, with y_2 = 8,000 and a target with 2 m/s^2 acceleration.

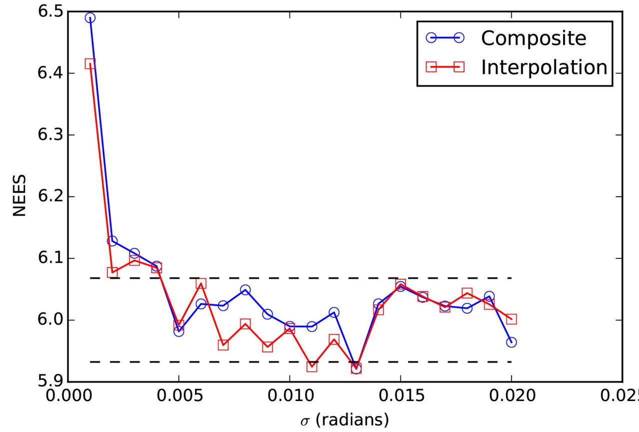

Fig. 16. Normalized estimation error squared (NEES) over 10,000 Monte Carlo runs, with y_2 = 8,000 and a target with 5 m/s^2 acceleration.

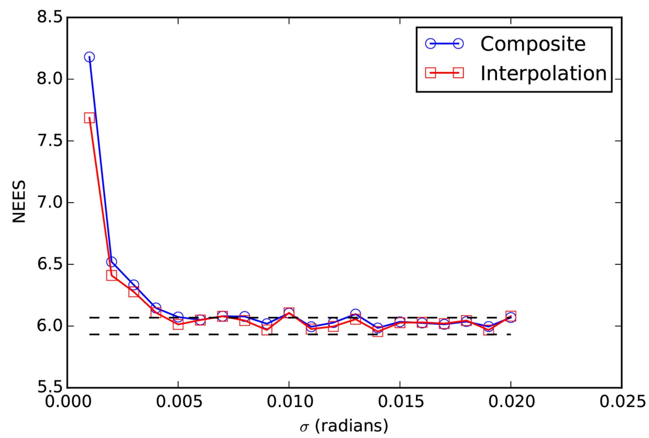

Fig. 17. Normalized estimation error squared (NEES) over 10,000 Monte Carlo runs, with y_2 = 8,000 and a target with 10 m/s^2 acceleration.

involving interpolating successive LOS measurements was presented. The resulting composite measurements were then compared to the ML method. Both methods provide a way to initialize tracks, and the difference in their accuracies were found to be statistically insignificant. Furthermore, both methods generally provide a statistically consistent error covariance. The interpolation method proved to provide a more consistent error covariance in the marginally observable case of a target trajectory which is coplanar with the line connecting the two sensors. The consistency of the error covariance could also break down for large target accelerations (in comparison to the cross-range error of the sensors). In such cases, the acceleration would need to be estimated as well, at the expense of requiring more measurements from the sensors.

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