# On Indistinguishability and Antisymmetry Properties in Multiple Target Tracking

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**The notion of indistinguishable targets is well established in advanced target tracking. If no specific target attributes are sensed, indistinguishability is often unavoidable and sometimes even desirable, for example, to enable "privacy by design" in public surveillance. Conceptually, this notion is rooted in quantum physics where functions of joint quantum particle states are considered that are either symmetric or antisymmetric under permutation of the particle labels. This symmetry dichotomy explains why quite fundamentally two disjunct classes of particles exist in nature: bosons and fermions. Besides symmetry, also *antisymmetry* has a place in multiple target tracking as we will show, leading to well-defined probability density functions describing the joint target states. Inbuilt antisymmetry implies a target tracking version of Pauli's *exclusion principle*: Real-world targets are "fermions" in the sense that cannot exist at the same time in the same state. This is of interest in dense tracking scenarios with resolution conflicts and split-off and may mitigate track coalescence phenomena, for example. We discuss the framework that is illustrated by an example.**

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## I. INTRODUCTION

*In grateful memory of Günther van Keuk (1940–2003), a pioneer in multiple target tracking.*

Since Donald B. Reid's seminal paper, multiple target tracking has been a topic of intensive research [1]–[5]. It provides backbone algorithms for multisensor fusion engines [6] that transform data streams from a variety of sensors along with context knowledge into situation pictures, the basis for decision making in an ever-increasing range of applications. Examples are manned–unmanned teaming and autonomous platform management, use cases in manufacturing, process control, or supply chain management, in health or elderly care, as well as in public security and defense. Situational awareness is basic not only to reaching goals efficiently, but also to reaching them in an ethically acceptable and responsible way [7].

Tracks represent the available knowledge on time-varying quantities of interest that characterize the state of the targets to be tracked. Quantitative performance measures describing the quality of this knowledge are part of the tracks. The information obtained by tracking algorithms also includes the history of the targets. Ideally, a one-to-one association between all the targets in the sensors' field of view and the produced tracks is to be established and to be preserved as long as possible. The achievable track quality depends not only on the performance of the sensors used, but also on the target properties, their kinematic behavior, and the environmental conditions within the scenario observed.

### A. Indistinguishable Targets

In the macrophysical world of target tracking, objects of interest, such as airplanes, vehicles, persons, ships, and so on, are mutually distinguishable physical objects in themselves. The information on them that is collected by sensors, however, covers a limited set of their properties only and is in many cases restricted to positional and kinematic properties. Let us call targets *identical* if two assumptions hold: (1) their *intrinsic* properties cannot be distinguished from each other by the measurements considered; and (2) they move according to the same dynamical model. Spatiotemporal target properties are *extrinsic* by definition.

From a systems engineering perspective, target tracking algorithms often have to obey certain nontechnical rules "by design" in order to make their use acceptable at all. Besides aspects formulated by rules of engagement in defense applications, surveillance systems for preserving public security are examples, where rule-constraint tracking systems are of growing interest. In particular, the "indistinguishability of the uninvolved" is a desired property in this context where multiple sensor assistance systems are to be designed that facilitate the assessment of the value of the additional security against the privacy lost by public surveillance. The proper and balanced


Fig. 1. Ellipses indicating imperfect knowledge on two mutually distinguishable (left) and indistinguishable (right) identical targets labeled by $i$ and $j$.

relation between the emerging surveillance technology for public security and the notion of an individual human subject entitled to "inalienable fundamental rights," for example, privacy, is of crucial importance.[1]

Within a conceptual framework that is inspired by classical mechanics, even identical objects in the previous sense can be distinguished from each other by their spatiotemporal behavior, since they move along well-defined trajectories. Let us consider, for example, a billiard game where all balls have the same color, their initial identity being known. Just by carefully watching, an observer could keep track of the balls as if they were individually colored. This changes even in classical mechanics in the case of "chaotic" dynamical systems according to sensitivity to initial conditions that are never known precisely. Even more so, this is valid in multiple target tracking problems where the temporal evolution has to be modeled stochastically and the measurements are inaccurate and ambiguous with respect to which object has produced which measurement, making a probabilistic description inevitable.

Fig. 1, left-hand side, illustrates the probabilistic representation of positional information on two well-separated identical targets. Even in case of imprecise positional information, each one of them occupies a clearly distinct spatial region, arbitrarily labeled by $i$ and $j$, thus allowing us to distinguish between these identical targets just as previously discussed. The right-hand side of the figure shows two identical targets in a situation where the probability density functions representing imprecise positional information are overlapping. It is no longer unambiguous in which region each target is to be expected. They have become *indistinguishable* [8], [9, Ch. 3].

More precisely speaking, our knowledge of indistinguishable targets remains unchanged if their labels are changed. In other words, the labels of indistinguishable targets have no longer a physical meaning. The joint probability density functions describing the kinematic states of indistinguishable targets must therefore obey symmetry restrictions: if any permutation is applied to the target labels, the density function has to remain invariant. In an early paper with Günter van Keuk [10, Sec. IV-B], the concept of symmetry has been used in Bayesian multiple hypothesis tracking.

### B. Bosons and Fermions in Quantum Physics

In quantum physics where the notion of individual particle trajectories is abandoned altogether, we are confronted with a similar situation. Here, a complex-valued function, the multiple particle *wave function* $\psi(\mathbf{x}_{1:n}, t)$, completely describes a quantum system composed of $n$ indistinguishable particles that at each instant of time $t$ are characterized by their joint state $\mathbf{x}_{1:n} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$. Knowledge of the wave function, together with the rules for the system's temporal evolution, exhausts all that can be known on the quantum system. By taking the absolute square of the *complex* wave function,

$$p(\mathbf{x}_{1:n}, t) = |\psi(\mathbf{x}_{1:n}, t)|^2, \tag{1}$$

a probability density function is obtained for calculating the probable outcome of each possible measurement on the system. It has to be invariant under any permutation taken from the set $S_n$ of all $n!$ permutations of the $n$ particle labels:

$$\forall \sigma \in S_n : \quad p(\mathbf{x}_{1:n}, t) = p(\mathbf{x}_{\sigma(1:n)}, t). \tag{2}$$

Since only the absolute square of wave functions has a physically interpretable meaning, multiple particle quantum systems are characterized by a *symmetry dichotomy*: the wave function for a collection of indistinguishable particles must be either *symmetric* or *antisymmetric* when two particle labels are exchanged, that is, when the wave functions involved remain invariant under any permutation of the particle labels up to a factor of $\pm 1$. If a wave function is initially symmetric (or antisymmetric), it will remain symmetric (or antisymmetric) as the quantum system evolves in time. The symmetry dichotomy also claims that *asymmetric* multiple identical particle wave functions are forbidden. Quantum particles are either *bosons*[2] or *fermions*[3] characterized by symmetric or antisymmetric wave functions, respectively. For further details, see any standard

[1]"to which a person is inherently entitled simply because she or he is a human being." *Human Rights*. In: Wikipedia, http://en.wikipedia.org/wiki/Human_rights, last accessed August 26, 2019.

[2]Named after the Indian physicist and polymath Satyendra Nath Bose (1894–1974) who provided the foundation for Bose–Einstein statistics and the theory of the Bose–Einstein condensate.

[3]Named after the Italian physicist Enrico Fermi (1901–1954), who first applied Pauli's exclusion principle to an ideal gas, employing a statistical formulation now known as Fermi–Dirac statistics.

textbook on quantum physics, such as [11, Ch. IX]. For historic aspects, see [12].

In the micro- and macrophysical world, the notions of *identity*, *individuality*, *distinguishability*, and their opposites are conceptually related, but to be distinguished carefully from each other in any philosophical reflections [8, Ch. 5], [13].

### C. Exclusion Principle in Target Tracking

A multiple target tracker extracts information on the kinematic properties of several moving targets from a time series of sensor data produced by a single sensor or multiple sensors; that is, target tracking provides information on the targets' position, velocity, and often also acceleration and related quantities.

As an example, let us consider a tracking problem with two targets, where probability density functions $p(\mathbf{x}_1, \mathbf{x}_2)$ represent the information available on the kinematic target states $\mathbf{x}_1$ and $\mathbf{x}_2$. In the case of indistinguishable targets, $p(\mathbf{x}_1, \mathbf{x}_2)$ is symmetric under permutation of the target labels: $p(\mathbf{x}_1, \mathbf{x}_2) = p(\mathbf{x}_2, \mathbf{x}_1)$. In many cases, $p(\mathbf{x}_1, \mathbf{x}_2)$ can be represented by a mixture with symmetric components: $p(\mathbf{x}_1, \mathbf{x}_2) = \sum_\nu p_\nu \; p_\nu(\mathbf{x}_1, \mathbf{x}_2)$ and weighting factors $p_\nu$. No objection can be made if we are representing the component densities $p_\nu(\mathbf{x}_1, \mathbf{x}_2)$ by the square of *real*-valued functions:

$$p_\nu(\mathbf{x}_1, \mathbf{x}_2) = \big(\psi_\nu(\mathbf{x}_1, \mathbf{x}_2)\big)^2. \tag{3}$$

One can easily see that the functions $\psi_\nu$ must be either

$$\text{symmetric} \quad \psi_\nu^+(\mathbf{x}_1, \mathbf{x}_2) = \psi_\nu^+(\mathbf{x}_2, \mathbf{x}_1) \quad \text{or} \tag{4}$$

$$\text{antisymmetric} \quad \psi_\nu^-(\mathbf{x}_1, \mathbf{x}_2) = -\psi_\nu^-(\mathbf{x}_2, \mathbf{x}_1) \tag{5}$$

under permutation of the target labels in order to guarantee that $p(\mathbf{x}_1, \mathbf{x}_2)$ represents two indistinguishable targets. Since considering functions $\psi_\nu^+$ does not add something substantially new to understanding the properties of symmetric densities $p^+$, we will be dealing with them as usual in the tracking literature. This is different, however, for densities $p^-$ that are given by the square of antisymmetric components $\psi_\nu^-$.

As shown later, Bayesian multiple target tracking is equivalent to iteratively calculating the probability densities $p^\pm$ of indistinguishable multiple targets. Of course, the "temporal propagation" of multiple target densities, driven by subsequent prediction and update steps, is mathematically quite different from the propagation of multiple particle wave functions in quantum physics. While the symmetric multiple target densities $p^\pm$ remain symmetric if the same dynamics model is assumed for both targets in the prediction step and under wide and realistic assumptions on the sensor models to be used for the filtering update, the antisymmetric $\psi^-$ components remain antisymmetric. Also in target tracking theory, we can therefore distinguish between *bosonic* and *fermionic* targets according to the symmetry properties of the functions $\psi_\nu^\pm$ describing them. As in quantum physics, this distinction is fundamental. Obviously, the macrophysical notion of bosonic and fermionic targets considered here is by no means related to the purely quantum physical concept of even or odd particle spin.

Real-world targets are fermions in the following sense: Due to the "fermionic" antisymmetry property of $\psi_\nu^-$, they cannot be characterized by the same state at the same instant of time:

$$p^-(\mathbf{x}, \mathbf{x}) = \sum_\nu \big(\psi_\nu^-(\mathbf{x}, \mathbf{x})\big)^2 = 0. \tag{6}$$

This is a target tracking version of the famous *exclusion principle*.[4]

### D. Contribution and Structure

In the tracking literature, indistinguishable targets have implicitly been considered as bosons; that is, no attention was given to antisymmetry. Attempts to broaden the methodological basis of point processes applied to target tracking, for example, do *not* use the concept of antisymmetry (see, e.g., the early and insightful paper by Sosho Mori and Chee-Yee Chong [14] or [15]). This is valid also for new trends in multitarget tracking such as labeled Random Finite Sets and message passing techniques to be mentioned [16]–[18]. Only a most recent paper, not yet published [19], points into the direction of "fermionic" multiple target tracking.

Since symmetric probability density functions are crucial building blocks for advanced trackers, see, for example, [3, pp. 239–244], also the notion of fermionic targets can quite naturally be introduced. In particular, the target tracking version of Pauli's exclusion principle leads us to multiple target trackers that are better adapted to real-world phenomena since targets simply cannot exist at the same place at the same time. It is an open question what type of phenomena to be tracked might best be modeled by bosonic targets. Two collectively moving groups that may merge and split off again are candidates of two bosons, while extended target tracking is fermionic in nature. In this sense, fermionic point targets might be called somewhat provocatively "extended" point targets.

After more precisely stating the notions of symmetry and antisymmetry as well as reviewing some basics of multiple target tracking in Section II, we rigorously discuss the problem of tracking two indistinguishable targets using a realistic sensor model with possibly missing, false, and unresolved measurements (Section III). Via a simulated example based on a tracking vignette with road moving targets, Section IV illustrates some characteristics of "fermionic" target tracking and compares

[4] It was formulated in 1925 by Wolfgang Pauli (1900–1958) at the University of Hamburg, Germany. Nominated by Albert Einstein (1879–1955), Pauli received the 1945 Nobel Prize in Physics for his "decisive contribution through his discovery of a new law of Nature, the exclusion principle or Pauli principle" [20].

them with "bosonic" target tracking and more standard approaches. Since our focus here is on the methodological approach, a more comprehensive qualitative discussion of the advantages of the proposed approach in comparison to alternative tracking methodologies, although desirable, goes beyond the scope of this publication and will be provided by subsequent work. An evident and practically relevant benefit of "fermionic" trackers to be stated right now is the mitigation of track coalescence phenomena in dense target situations. In Section V, we discuss the relevance of indistinguishable target tracking in surveillance systems for public security. "Indistinguishability of the uninvolved" seems to be a fundamental principle for security systems design to be recognized as a certifiable means for preserving informational self-determination. Conclusions and some physics-inspired remarks for generalizing the formalism conclude the paper.

At the 21st International Conference on Information Fusion, the general idea underlying this paper and its potential relevance to tracking closely spaced targets were sketched [21]. We here provide a more comprehensive view and coherently consider the quantum physical background, which has guided our approach. In its present form, this contribution reflects also a series of discussions that were stimulated by the preliminary publication. The author in particular wishes to thank three anonymous reviewers for their insightful and inspiring comments.

As Wolfgang Pauli made clear himself, the fundamental symmetry dichotomy, tightly connected with the notion of indistinguishability, that is visible and relevant also in multiple target tracking as shown in this paper, still calls for a deeper understanding.[5]

## II. BAYESIAN MULTIPLE IDENTICAL TARGET TRACKING

Tracking systems extract kinematic target information from a time series of data $Z_{k:1} = \{Z_k, Z_{k-1:1}\}$ produced by a single sensor or multiple sensors at certain instants of time $t_l, l = 1, \dots, k$, measuring positional and kinematic properties of the targets starting at an initial time $t_1$. The number of measurements $m_k$ in each data set $Z_k = \{\mathbf{z}_j\}_{j=1}^{m_k}$ produced at time $t_k$ can be equal to, less than, or larger than the number $n$ of targets to be tracked due to false, missing, and unresolved measurements. The targets' position, velocity, and possibly also acceleration are described by kinematic state vectors $\mathbf{x}_k^i$, $i = 1, \dots, n$, at instants of time $t_k$, the *joint state* being denoted by $\mathbf{x}_k^{1:n} = (\mathbf{x}_k^1, \dots, \mathbf{x}_k^n)$. Identical targets obey the same dynamical model.

[5] "Already in my original paper I stressed the circumstance that I was unable to give a logical reason for the exclusion principle or to deduce it from more general assumptions. I had always the feeling, and I still have it today, that this is a deficiency." [22].

The implications of antisymmetry in the formalism of multiple identical target tracking and its practical benefits are more clearly visible within the standard Bayesian framework where we assume independent targets along with a fixed and known number of targets than in more advanced tracking methodologies, such as Probalility Hypothesis Density and intensity filtering, where antisymmetry can be embedded as well.

In Bayesian context, the problem of tracking well-separated targets or well-separated groups consisting of not too many targets or that of tracking some well-separated targets or groups joining and separating after a while can be solved more or less rigorously, that is, by explicitly enumerating data interpretation hypotheses. Since it seems unreasonable to deal with large groups by keeping track of each individual group member, we should rather track the centroid and the boundary of the group in this case until it splits off into smaller components to be tracked individually; see [6, Sec. 8.2] and [23], for example.

Our general line of argumentation is valid for nonlinear, non-Gaussian sensor and evolution models where the resulting probability densities and $\psi$ functions can be calculated by numerical methods based on tensor decomposition methods, for example, those presented in [24]. For being able to discuss the impact of antisymmetry more analytically and in greater detail, however, we are assuming linear Gaussianity whenever to be justified and mathematically convenient.

### A. Antisymmetry in Mixture Densities

In the case of ambiguous sensor data, the time series $Z_{k:1}$ is to be interpreted by *data interpretation histories*, series of possible interpretation hypotheses of the sensor data sets at different instants of time. The conditional probability density function $p(\mathbf{x}_k^{1:n}|Z_{k:1})$ of the joint state $\mathbf{x}_k^{1:n}$ that contains all information on the state vectors available at time $t_k$ can thus be written as weighted sum of component densities $p_\nu$ related to these interpretation histories:

$$p(\mathbf{x}_k^{1:n}|Z_{k:1}) = \sum_\nu p_k^\nu \, p_\nu(\mathbf{x}_k^{1:n}|Z_{k:1}). \tag{7}$$

If at one particular instant of time $t_l$ the component densities $p_\nu(\mathbf{x}_l^{1:n})$ are symmetric under permutation of the target labels,

$$\forall \sigma \in S_n : \quad p_\nu(\mathbf{x}_l^{1:n}|Z_{k:1}) = p_\nu(\mathbf{x}_l^{\sigma(1:n)}|Z_{k:1}), \tag{8}$$

this property is preserved in the iterative calculation process of the densities that will become clear later. Symmetry in this sense can thus be imposed on the "noninformative" initial prior density as some structural information. As sketched in the introduction, the symmetric probability densities $p_\nu$ can either be considered in themselves, that is, instead as a square of symmetric functions, this bosonic case being denoted by $p_\nu^+(\mathbf{x}_k^{1:n}|Z_{k:1})$, or be written as the square of functions $\psi_\nu$ that are antisymmetric under permutation of the target labels:

$$p_\nu^-(\mathbf{x}_k^{1:n}|Z_{k:1}) = \Big(\psi_\nu(\mathbf{x}_k^{1:n}|Z_{k:1})\Big)^2. \quad (9)$$

With Dirac's[6] antisymmetrizing operator $\mathcal{A}$, see [11, p. 248],

$$\mathcal{A}f(\mathbf{x}_{1:n}) = \sum_{\sigma\in S_n}(-1)^\sigma f(\mathbf{x}_{\sigma(1:n)}), \quad (10)$$

where the symbol $(-1)^\sigma$ is 1 for even and $-1$ for odd permutations $\sigma$. Let $\psi_\nu$ be given by a weighted sum of Gaussians with positive and negative weighting factors:

$$\psi_\nu(\mathbf{x}_k^{1:n}|Z_{k:1}) = \sqrt{c_{k|k}^\nu}\,\mathcal{A}\mathcal{N}(\mathbf{x}_k^{1:n};\ \mathbf{x}_{k|k}^\nu,\ \mathbf{P}_{k|k}^\nu) \quad (11)$$

$$=: \psi(\mathbf{x}_k^{1:n}; \mathbf{x}_{k|k}^\nu, \mathbf{P}_{k|k}^\nu) \quad (12)$$

that are characterized by joint state expectation vectors $\mathbf{x}_{k|k}^\nu$, corresponding covariance matrices $\mathbf{P}_{k|k}^\nu$, and a properly defined normalization constant (see Section A.1 in Appendix A):

$$1/c_{k|k}^\nu = \int d\mathbf{x}_k^{1:n}\ \Big(\psi(\mathbf{x}_k^{1:n}; \mathbf{x}_{k|k}^\nu, \mathbf{P}_{k|k}^\nu)\Big)^2. \quad (13)$$

Under these modeling assumptions, the fermionic component densities $p_\nu^-$ are therefore given by correctly normalized, well-defined Gaussian mixture densities with possibly *negative* weighting factors that sum up to 1. More general non-Gaussian representations are possible.

With the symmetrizing operator $\mathcal{S}$,

$$\mathcal{S}f(\mathbf{x}_{1:n}) = \sum_{\sigma\in S_n} f(\mathbf{x}_{\sigma(1:n)}), \quad (14)$$

let the bosonic components be given by

$$p_\nu^+(\mathbf{x}_k^{1:n}|Z_{k:1}) = \frac{1}{n!}\,\mathcal{S}\mathcal{N}(\mathbf{x}_k^{1:n};\ \mathbf{x}_{k|k}^\nu,\ \mathbf{P}_{k|k}^\nu). \quad (15)$$

With these definitions, the overall densities $p^\pm(\mathbf{x}_k^{1:n}|Z_{1:k})$ are symmetric under permutation of the target labels. The symmetrizing and antisymmetrizing operators $\mathcal{S}$ and $\mathcal{A}$ are projectors into disjunct function subspaces.

## B. Fermionic Prediction

Let $\mathbf{F}'_{k|k-1}$ and $\mathbf{D}'_{k|k-1}$ denote the evolution and plant noise covariance matrices describing the temporal evolution of the identical targets as usual in the tracking literature. With $\mathbf{F}_{k|k-1} = \mathbf{1}_n \otimes \mathbf{F}'_{k|k-1}$ and $\mathbf{D}_{k|k-1} = \mathbf{1}_n \otimes \mathbf{D}'_{k|k-1}$, where $\mathbf{1}_n$ denotes the $n$-dimensional unity matrix and the Kronecker product is used, a Gauss–Markov transition density for $n$ identical independently moving targets is defined by

$$p(\mathbf{x}_k^{1:n}|\mathbf{x}_{k-1}^{1:n}) = \mathcal{N}(\mathbf{x}_k^{1:n};\ \mathbf{F}_{k|k-1}\mathbf{x}_{k-1}^{1:n},\ \mathbf{D}_{k|k-1}). \quad (16)$$

Since all identical targets obey the same evolution model, the multiple identical target transition density has the following property:

$$\forall\sigma\in S_n:\quad p(\mathbf{x}_k^{1:n}|\mathbf{x}_{k-1}^{1:n}) = p(\mathbf{x}_k^{\sigma(1:n)}|\mathbf{x}_{k-1}^{\sigma(1:n)}). \quad (17)$$

While the bosonic prediction update is quite straightforward, the fermionic version of it requires some care. The square root of the transition density is given by (see Section A.2 in Appendix A)

$$\pi(\mathbf{x}_k^{1:n}|\mathbf{x}_{k-1}^{1:n}) = |8\pi\mathbf{D}_{k|k-1}|^{1/4}\ \mathcal{N}(\mathbf{x}_k^{1:n};\ \mathbf{F}_{k|k-1}\mathbf{x}_{k-1}^{1:n},\ 2\mathbf{D}_{k|k-1}). \quad (18)$$

For modeling the prediction step in the tracking process, we consider predictive $\psi$ functions defined by

$$\psi(\mathbf{x}_k^{1:n}|Z_{k-1:1}) = \sum_\nu p_k^\nu\ \psi(\mathbf{x}_k^{1:n}; \mathbf{x}_{k|k-1}^\nu, \mathbf{P}_{k|k-1}^\nu) \quad (19)$$

with mixture components given by

$$\psi(\mathbf{x}_k^{1:n}; \mathbf{x}_{k|k-1}^\nu, \mathbf{P}_{k|k-1}^\nu) = \sqrt{c_{k|k}^\nu}\,\mathcal{A}\mathcal{N}(\mathbf{x}_k^{1:n};\ \mathbf{x}_{k|k-1}^\nu,\ \mathbf{P}_{k|k-1}^\nu) \quad (20)$$

with properly defined normalizing constants $c_{k|k-1}^\nu$ and the standard, though "relaxed" Kalman prediction step:

$$\mathbf{x}_{k|k-1}^\nu = \mathbf{F}_{k|k-1}\mathbf{x}_{k-1|k-1}^\nu, \quad (21)$$

$$\mathbf{P}_{k|k-1}^\nu = \mathbf{F}_{k|k-1}\mathbf{P}_{k-1|k-1}^\nu\mathbf{F}_{k|k-1}^\top + 2\mathbf{D}_{k|k-1}. \quad (22)$$

The predicted fermionic density is thus given by

$$p^-(\mathbf{x}_k^{1:n}|Z_{k-1:1}) = \sum_\nu p_k^\nu\ \Big(\psi(\mathbf{x}_k^{1:n}; \mathbf{x}_{k|k-1}^\nu, \mathbf{P}_{k|k-1}^\nu)\Big)^2.$$

## C. Intrinsic Symmetry in Sensor Models

Likelihood functions represent imperfect and ambiguous information on the target states $\mathbf{x}_k^{1:n}$ that is provided by a set of sensor data $Z_k$ at time $t_k$ as well as context knowledge on the sensor performance and the sensing environment. For identical targets, the likelihood functions necessarily have to be symmetric under permutation of the target labels, since otherwise the targets could be distinguished from each other via sensor data processing.

Likelihood functions $\ell(\mathbf{x}_k^{1:n}; Z_k)$ are up to a multiplicative constant determined by the conditional densities $p(Z_k|\mathbf{x}_k^{1:n})$:

$$\ell(\mathbf{x}_k^{1:n}; Z_k) \propto p(Z_k|\mathbf{x}_k^{1:n}). \quad (23)$$

The potential origin of ambiguous sensor data $Z_k$ is explained by a set of data interpretation hypotheses $h_k \in H_k$, which are assumed to be exhaustive and mutually exclusive, yielding a representation by a weighted

[6] Paul Adrien Maurice Dirac (1902–1984) shared the 1933 Nobel Prize in Physics with Erwin Schrödinger (1887–1961).

sum:

$$\ell(\mathbf{x}_k^{1:n}; Z_k) \propto \sum_{h_k \in H_k} p(h_k)\, p(Z_k|\mathbf{x}_k^{1:n}, h_k). \quad (24)$$

Following well-established and fairly general modeling assumptions for the sensors considered [6, Sec. 7.1], the likelihood functions can be rearranged as a sum of partial sums over classes $H_k^\mu$ of data interpretation hypotheses that are similar in the sense that they differ only in a permutation of the target labels:

$$\ell(\mathbf{x}_k^{1:n}; Z_k) = \sum_\mu \ell_\mu(\mathbf{x}_k^{1:n}; Z_k) \quad (25)$$

$$\text{with} \quad \ell_\mu(\mathbf{x}_k^{1:n}; Z_k) \propto \sum_{h_k \in H_k^\mu} p(h_k)\, p(Z_k|\mathbf{x}_k^{1:n}, h_k). \quad (26)$$

As a result, the component likelihood functions $\ell_\mu$ related to $H_k^\mu$ are symmetric under permutation of the target labels:

$$\forall \sigma \in S_n: \quad \ell_\mu(\mathbf{x}_k^{1:n}; Z_k) = \ell_\mu(\mathbf{x}_k^{\sigma(1:n)}; Z_k). \quad (27)$$

This can be shown by assuming false measurements that are Poisson distributed in number with a spatial false measurement density $\rho_\mathrm{F}$ and uniformly distributed in the measurement space, missing measurements occurring according to a detection probability $P_\mathrm{D}$, and the measurements $\mathbf{z}_k \in Z_k$ being mutually independent. Moreover, let a *resolved* measurement $\mathbf{z}_k^j$ related to target *i* be characterized by a Gaussian likelihood

$$p(\mathbf{z}_k^j|\mathbf{x}_k^i) = \mathcal{N}(\mathbf{z}_k^j;\, \mathbf{H}_k \mathbf{x}_k^i,\, \mathbf{R}_k^j) \quad (28)$$

with measurement and error covariance matrices $\mathbf{H}_k$ and $\mathbf{R}_k^j$.

Inherently, antisymmetry and the exclusion principle it implies are only relevant for targets that may move closely spaced. Due to the finite resolution capabilities of real-world sensors, such targets are expected to transition from being resolved to unresolved and back again. It is thus inevitable to model the sensors' resolution capability appropriately and to take this phenomenon explicitly into account. In practical applications, only a small number of targets are expected to be jointly unresolved.

Let an *unresolved* measurement $\mathbf{z}_k^u$ produced by a group of *n* closely spaced targets be modeled as a measurement of the group centroid that is characterized by the Gaussian likelihood

$$p(\mathbf{z}_k^u|\mathbf{x}_k^{1:n}) = \mathcal{N}(\mathbf{z}_k^u;\, \mathbf{H}_g \mathbf{x}_k^{1:n},\, \mathbf{R}_g) \quad (29)$$

with $\mathbf{R}_g$ denoting the measurement error of unresolved measurements and a measurement matrix given by

$$\mathbf{H}_g = (1, \ldots, 1) \otimes \mathbf{H}_k. \quad (30)$$

The probability $P_u(\mathbf{x}_k^{1:n})$ of *n* targets being jointly unresolved is modeled by pseudo-measurement "zero" of the distances between the targets [6, Sec. 7.1], where the sensor resolution in the measured quantities such as range and cross range, $\alpha_\mathrm{r}$ and $\alpha_\mathrm{xr}$, can be considered as standard deviations entering a related pseudo-measurement error covariance matrix $\mathbf{A}_u$:

$$P_u(\mathbf{x}_k^{1:n}) = |2\pi \mathbf{A}_u|^{1/2}\, \mathcal{N}(0;\, \mathbf{H}_d \mathbf{x}_k^{1:n},\, \mathbf{A}_u), \quad (31)$$

where the corresponding pseudo-measurement matrix $\mathbf{H}_d$ that describes mutual distances is given by

$$\mathbf{H}_d = \begin{pmatrix} 1 & -1 & 0 & \ldots \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & 1 & -1 \\ -1 & 0 & \ldots & 1 \end{pmatrix} \otimes \mathbf{H}_k. \quad (32)$$

Both Gaussians related to unresolved measurements are evidently symmetric under permutation of the target labels:

$$\forall \sigma \in S_n: \quad \begin{aligned} \mathcal{N}(\mathbf{z}_k^u;\, \mathbf{H}_g \mathbf{x}_k^{1:n},\, \mathbf{R}_g) &= \mathcal{N}(\mathbf{z}_k^u;\, \mathbf{H}_g \mathbf{x}_k^{\sigma(1:n)},\, \mathbf{R}_g), \\ \mathcal{N}(0;\, \mathbf{H}_d \mathbf{x}_k^{1:n},\, \mathbf{R}_u) &= \mathcal{N}(0;\, \mathbf{H}_d \mathbf{x}_k^{\sigma(1:n)},\, \mathbf{R}_u). \end{aligned}$$

In order to pinpoint the effects of antisymmetry, a fully detailed discussion of this fairly general approach in the limiting case of two closely spaced targets is provided in Section III.

### D. Fermionic Filtering

The data update of the fermionic density follows from Bayes' rule; that is, it is provided by normalizing the product of the sensor likelihood $\ell(Z_k; \mathbf{x}_k^{1:n})$ and the predicted density $p^-(\mathbf{x}_k^{1:n}|Z_{1:k-1})$:

$$p^-(\mathbf{x}_k^{1:n}|Z_{k:1}) = c_{k|k}\, \ell(\mathbf{x}_k^{1:n}; Z_k)\, p^-(\mathbf{x}_k^{1:n}|Z_{k-1:1}) \quad (33)$$

$$\text{with} \quad 1/c_{k|k} = \int d\mathbf{x}_k^{1:n}\, p(Z_k|\mathbf{x}_k^{1:n})\, p^-(\mathbf{x}_k^{1:n}|Z_{1:k-1}).$$

We can therefore write the fermionic density function of the joint state as a mixture density:

$$p^-(\mathbf{x}_k^{1:n}|Z_{1:k}) = c_{k|k} \sum_{\mu,\nu} \ell_\mu(\mathbf{x}_k^{1:n}; Z_k) \times \left(\psi\left(\mathbf{x}_k^{1:n}; \mathbf{x}_{k|k-1}^\nu, \mathbf{P}_{k|k-1}^\nu\right)\right)^2. \quad (34)$$

If it is possible to rewrite the symmetric component likelihood functions $\ell_\mu$ as squares of symmetric functions, the fermionic filtering update consists in updating the antisymmetric component $\psi$ functions and squaring them. To keep the discussion simple, let us consider a tracking problem of reduced complexity that is still rich enough to be practically relevant.

## III. EXAMPLE WITH POSSIBLY UNRESOLVED TARGETS

While applicable for *n* targets, the effects of antisymmetry in identical target tracking can more easily be analyzed in the case of two targets that may move closely spaced for a while. Depending on the sensor-to-target geometry, the finite sensor resolution may even play a dominant role in target tracking.

In order to preserve antisymmetry of the fermionic $\psi$ functions in the filtering update, the likelihood functions need to be modified appropriately. To do so, let us be guided by some sort of "correspondence principle" in the sense that for well-separated fermionic targets the effect of fermionically modified likelihood functions is the same as that for bosonic targets. If there is no need for any linear Gaussianity as in case of direct numerical calculation [24] where the square roots can be drawn directly, no modification is necessary.

### A. Components of the Likelihood Function

For two targets moving in a cluttered environment, five different classes $H_k^m, m = 1, \ldots, 5$, of data interpretation hypotheses exist [6, Sec. 7.1]. The likelihood function for the bosonic and femionic filtering update has thus a sum representation:

$$\ell^{\pm}(\mathbf{x}_k^{1:n}; Z_k) \propto \sum_{i=1}^{5} \ell_i^{\pm}(\mathbf{x}_k^{1:2}; Z_k), \tag{35}$$

where the five component likelihood functions are symmetric under permutation of the target labels and correspond to the following data interpretation classes.

*1) $H_k^1$—Both targets were resolvable, but not detected; all $m_k$ measurements in $Z_k$ are false (one interpretation):* The component likelihood $\ell_1$ is the same for bosonic and fermionic tracking and given by

$$\ell_1^{\pm}(\mathbf{x}_k^{1:2}; Z_k) = \rho_F^2(1 - P_D)^2\big(1 - P_u(\mathbf{x}_k^{1:2})\big). \tag{36}$$

*2) $H_k^2$—Both targets were neither resolvable nor detected as a group; all measurements in $Z_k$ are assumed to be false (one interpretation hypothesis):* Also here, there is no difference between the bosonic and fermionic cases:

$$\ell_2^{\pm}(\mathbf{x}_k^{1:2}; Z_k) = \rho_F(1 - P_D^u)P_u(\mathbf{x}_k^{1:2}). \tag{37}$$

*3) $H_k^3$—Both targets were not resolvable but detected as a group with probability $P_D^u$, $\mathbf{z}_k^j \in Z_k$ representing the centroid measurement; all remaining returns are false ($m_k$ data interpretations):* Up to constant factors, the corresponding component likelihood is equivalent to joint centroid and distance measurements; that is, the single unresolved group measurement $\mathbf{z}_k^j$ provides under this hypothesis a measurement of the full joint position of the targets:

$$\ell_3^{\pm}(\mathbf{x}_k^{1:2}; Z_k) = \rho_F P_D^u P_u(\mathbf{x}_k^{1:2}) \sum_{j=1}^{m_k} \mathcal{N}\big(\mathbf{z}_k^j;\ \mathbf{H}_g\mathbf{x}_k^{1:2},\ \mathbf{R}_g\big) \tag{38}$$

$$= \rho_F P_D^u\, |2\pi\mathbf{R}_u|^{1/2} \sum_{j=1}^{m_k} \mathcal{N}\big(\mathbf{z}_k^{j,1:2};\ \mathbf{H}_u\mathbf{x}_k^{1:2},\ \mathbf{R}_u\big), \tag{39}$$

$\mathbf{z}_k^{j,1:2} = (\mathbf{z}_k^j, \mathbf{0})$, $\mathbf{H}_u = \mathrm{diag}[\mathbf{H}_g, \mathbf{H}_d]$, and $\mathbf{R}_u = \mathrm{diag}[\mathbf{R}_g, \mathbf{A}_u]$.

*4) $H_k^4$—Both objects were resolvable but only one object was detected, $\mathbf{z}_k^j$ is the measurement, $m_k - 1$ measurements are false ($2m_k$ interpretations):* With the abbreviation

$$\lambda_4\big(\mathbf{z}_k^j; \mathbf{H}_k\mathbf{x}_k^{1:2}, \mathbf{R}_k^j\big) = \mathcal{N}\big(\mathbf{z}_k^j;\ \mathbf{H}_k\mathbf{x}_k^1,\ \mathbf{R}_k^j\big) + \mathcal{N}\big(\mathbf{z}_k^j;\ \mathbf{H}_k\mathbf{x}_k^2,\ \mathbf{R}_k^j\big), \tag{40}$$

the bosonic component likelihood is given by

$$\ell_4^{+}(\mathbf{x}_k^{1:2}; Z_k) = \rho_F P_D(1 - P_D)\,\big(1 - P_u(\mathbf{x}_k^{1:2})\big) \times \sum_{j=1}^{m_k} \lambda_4\big(\mathbf{z}_k^j; \mathbf{H}_k\mathbf{x}_k^{1:2}, \mathbf{R}_k^j\big). \tag{41}$$

For applying this component likelihood in the filtering update where the antisymmetric structure of $\psi$ functions is to be preserved, we need a representation by appropriate "squares." According to the introductory remarks, let us make an "ansatz":

$$\ell_4^{-}(\mathbf{x}_k^{1:2}; Z_k) = \rho_F P_D(1 - P_D)\,\big(1 - P_u(\mathbf{x}_k^{1:2})\big) \times \sum_{j=1}^{m_k} \lambda_4\big(\mathbf{z}_k^j; \mathbf{H}_k\mathbf{x}_k^{1:2}, 2\mathbf{R}_k^j\big)^2. \tag{42}$$

*5) $H_k^5$—Both objects were resolvable and detected, $\mathbf{z}_k^i$ and $\mathbf{z}_k^j$ are the measurements, $m_k - 2$ measurements are false ($m_k(m_k - 1)$ interpretations):* With the abbreviation

$$\lambda_5\big(\mathbf{x}_k^{1:2}; \mathbf{z}_k^{ij}, \mathbf{R}_k^{ij}\big) = \mathcal{S}\,\mathcal{N}\big(\mathbf{z}_k^{ij};\ \mathbf{H}_k\mathbf{x}_k^{1:2},\ \mathbf{R}_k^{ij}\big), \tag{43}$$

the bosonic component likelihood is given by

$$\ell_5^{+}(\mathbf{x}_k^{1:2}; Z_k) = P_D^2\big(1 - P_u(\mathbf{x}_k^{1:2})\big) \sum_{i=1}^{m_k-1} \sum_{j=1}^{m_k-i} \lambda_5\big(\mathbf{x}_k^{1:2}; \mathbf{z}_k^{ij}, \mathbf{R}_k^{ij}\big), \tag{44}$$

while we assume for the fermionic component

$$\ell_5^{-}(\mathbf{x}_k^{1:2}; Z_k) = P_D^2\big(1 - P_u(\mathbf{x}_k)\big) \sum_{i=1}^{m_k-1} \sum_{j=1}^{m_k-i} \times \Big(\lambda_5\big(\mathbf{z}_k^{ij}; \mathbf{H}_k^{ij}\mathbf{x}_k^{1:2}, 2\mathbf{R}_k^{ij}\big)\Big)^2. \tag{45}$$

Note the "relaxed" measurement error covariance matrix in the fermionic versions of the component likelihood functions $\ell_4^-$ and $\ell_5^-$.

Each component likelihood is symmetric under permutation of the target labels. If an unresolved group is assumed, two measurements are to be processed: a real measurement of the group centroid and a pseudo-measurement "zero" of the distance between the objects. We can thus speak of a piece of *negative sensor information*, as the lack of a second target measurement conveys information on the target position, since in the case of a resolution conflict, the relative target distances must be smaller than the sensor resolution.

## B. Fermionic Filtering Update: Discussion

The general structure of the filtering update in case of two targets becomes visible even in the absence of clutter, $\rho_F = 0$, and in case of perfect detection, $P_D = 1$. This means that at a given instant of time $t_k$ either two resolved measurements $\mathbf{z}_k^1$ and $\mathbf{z}_k^2$ or a single unresolved measurement $\mathbf{z}_k$ has to be processed. Let the predictive $\psi$ function be given by

$$\psi\left(\mathbf{x}_k^{1:2}|Z_{k-1:1}\right) = \psi\left(\mathbf{x}_k^{1:2}; \mathbf{x}_{k|k-1}, \mathbf{P}_{k|k-1}\right) \quad (46)$$

or by a weighted sum of such components.

*1) Unresolved measurement:* In this case, the bosonic and fermionic updates use the same component likelihood. Up to a constant, the square root of the likelihood is given by (see Section A.2 in Appendix A)

$$\lambda_3(\mathbf{x}_k^{1:2}; \mathbf{z}_k) \propto \mathcal{N}\left(\mathbf{z}_k^{u,1:2}; \mathbf{H}_u \mathbf{x}_k^{1:2}, 2\mathbf{R}_u\right). \quad (47)$$

The filtering update by a measurement that is assumed to be unresolved yields a $\psi$ function $\psi(\mathbf{x}_k^{1:2}; \mathbf{x}_{k|k}, \mathbf{P}_{k|k})$ characterized by a standard Kalman update based on the "relaxed" measurement error covariance matrix $2\mathbf{R}_u$:

$$\mathbf{x}_{k|k}^{1:2} = \mathbf{x}_{k|k-1}^{1:2} + \mathbf{W}_k(\mathbf{z}_k^{u,1:2} - \mathbf{H}_u \mathbf{x}_{k|k-1}^{1:2}), \quad (48)$$

$$\mathbf{P}_{k|k}^{1:2} = \mathbf{P}_{k|k-1}^{1:2} - \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^\top \quad (49)$$

with innovation covariance and gain matrices given by $\mathbf{S}_k^u = \mathbf{H}_u \mathbf{P}_{k|k-1}^{1:2} \mathbf{H}_u^\top + 2\mathbf{R}_u$ and $\mathbf{W}_k^u = \mathbf{P}_{k|k-1}^{1:2} \mathbf{H}_u^\top \mathbf{S}_k^{u^{-1}}$.

*2) Resolved measurements:* In this case, we obtain for the fermionic update (see Section A.3 in Appendix A)

$$\psi\left(\mathbf{x}_k^{1:2}|Z_{1:k}\right) \propto \lambda_5\left(\mathbf{x}_k^{1:2}; \mathbf{z}_k^{ij}, 2\mathbf{R}_k^{ij}\right) \psi\left(\mathbf{x}_k^{1:2}; \mathbf{x}_{k|k-1}, \mathbf{P}_{k|k-1}\right) \quad (50)$$

$$= p_k^{12}\, \psi\left(\mathbf{x}_k^{1:2}; \mathbf{x}_{k|k}^{12}, \mathbf{P}_{k|k}^{12}\right) + p_k^{21}\, \psi\left(\mathbf{x}_k^{1:2}; \mathbf{x}_{k|k}^{21}, \mathbf{P}_{k|k}^{21}\right), \quad (51)$$

where $\mathbf{x}_{k|k}^{ij}$ and $\mathbf{P}_{k|k}^{ij}$ result from the standard Kalman update equations with measurement vectors $\mathbf{z}_k^{ij} = (\mathbf{z}_k^i, \mathbf{z}_k^j)$. The weighting factors result from the corresponding innovation:

$$p_k^{ij} = \mathcal{N}\left(\mathbf{z}_k^{ij}; \mathbf{H}_k^{ij} \mathbf{x}_{k|k-1}, \mathbf{S}_k^{ij}\right) \quad (52)$$

with $\mathbf{S}_k^{ij} = \mathbf{H}_k^{ij} \mathbf{P}_{k|k-1} \mathbf{H}_k^{ij\top} + \mathbf{R}_k^{ij}$. Via symmetrized moment matching [10, Sec. IV-B], an increasing number of mixture components by the fermionic update can be avoided:

$$\psi\left(\mathbf{x}_k^{1:2}|Z_{1:k}\right) \approx \psi\left(\mathbf{x}_k^{1:2}; \mathbf{x}_{k|k}, \mathbf{P}_{k|k}\right). \quad (53)$$

# IV. EXAMPLE: GMTI TRACKING OF ROAD MOVING VEHICLES

Tracking of road moving targets using data from airborne GMTI (ground moving target indicator) radar is a relevant problem. Since here the state space has only one spatial dimension, the impact of antisymmetry can easily be visualized.

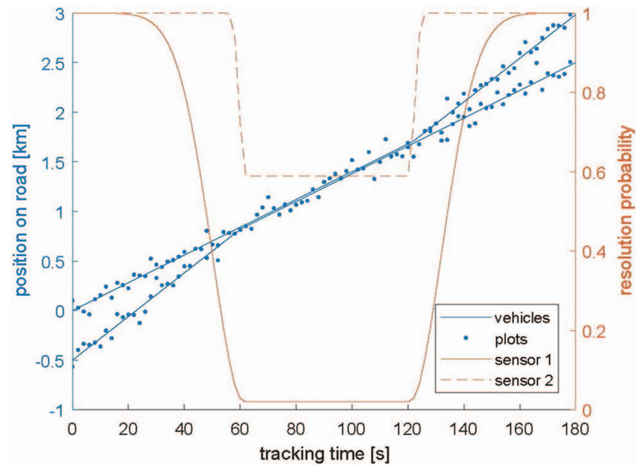


Fig. 2. Two road moving vehicles observed with GMTI radar.

## A. Description of a Characteristic Vignette

Let us therefore consider a straight road given by the $x$-axis of the chosen coordinate system with a road map error of 5 m. See [6, Sec. 9.1] for details and generalizations to winding roads. As a function of time, Fig. 2 shows the position of vehicle 1 moving uniformly with the speed $v_1 = 14$ m/s. At time $t_1 = 120$ s, it smoothly accelerates with $a = 2\,\text{m/s}^2$ over 4 s and continues to move uniformly with $v_2 = 22$ m/s. Vehicle 2 approaches vehicle 1 with the initial speed $v_2$. At time $t_2 = 58$ s, it decelerates with $-a$ over 4 s and follows vehicle 1 at a distance of 20 m until vehicle 1 is accelerating.

Let this vignette be observed by a typical GMTI radar positioned at $s_1 = (1, 40)$ km. For the sake of simplicity, we neglect the phenomenon of GMTI Doppler blindness [6, Sec. 7.2]. Moreover, we assume for resolved and unresolved measurements the same standard deviations of the measurement errors in range and cross range that are given by $\sigma_r = 10$ m and $\sigma_{xr} = 70$ m, respectively, while the sensor resolution parameters are $\alpha_r = 15$ m and $\alpha_{xr} = 100$ m.

Fig. 2 also shows the variation of the resolution probabilities in time and a time series of GMTI plots that are simulated according to these assumptions. Apparently, the vehicles are unresolved in the intermediate period of the vignette. The measurement and resolution capabilities of GMTI sensors strongly depend on the chosen sensor-to-target geometry. This is clearly indicated by the resolution probability of a second GMTI radar located at $s_2 = (40, 0)$ km (dashed line).

## B. Comparison of Fermionic and Bosonic Densities

For the first sensor-to-target geometry previously discussed, we focus on four instants of time, shortly before the vehicles are becoming unresolvable (after 36 s), after processing a fairly long sequence of unresolved measurements (100 s), during the process of splitting off (140 s), and well after the vehicles have split off again (170). Figs. 3 (36, 100 s) and 4 (140, 170 s) show the

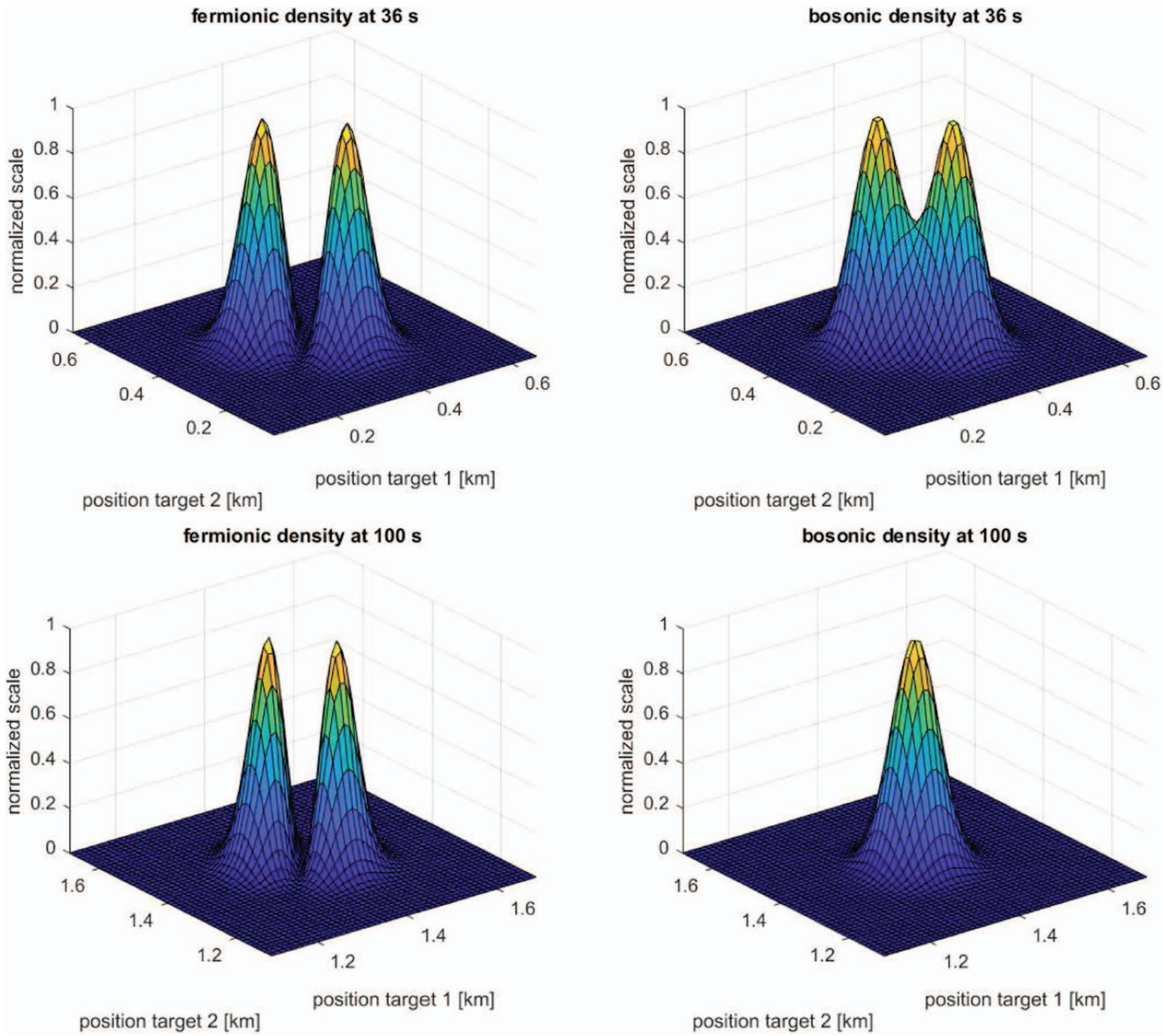


Fig. 3. Spatial projection of $p^{\pm}(\mathbf{x}_k^{1:2}|Z_{k:1})$ at two different instants of time (36, 100 s).

spatial projections of the fermionic (left) and bosonic (right) joint densities representing the positional information on them for these instants of time. For about 30 s from the beginning of the vignette, the vehicles are well separated and characterized by two distinct Gaussian peaks that are the same in the fermionic and bosonic cases.

At time $t_1 = 36$ s, however, the bosonic peaks are close to merging, while the peaks of the fermionic density are separated by a notch along the line where the vehicle positions are identical. This notch, which might be called the Pauli notch, is even more pronounced at time $t_2 = 100$ s, when the bosonic peaks are completely merged for quite a long time. With "a smiling wink of the eye," one might be tempted to speak of a Bose–Einstein condensate of the two tracks. At time $t_3 = 140$ s, the bosonic tracks are beginning to be separated again, while at time $t_4 = 170$ s, when the vehicles are well separated again, the fermionic and the bosonic densities look identical.

Fig. 5 (left-hand side) shows the corresponding $\psi$ function in a combined surface and contour plot where the Pauli notch is clearly visible. This phenomenon resembles the clutter notch in GMTI tracking [6, Sec. 7.2] that also "forbids" certain state characteristics. The Pauli notch vanishes when the vehicles become well separated again as shown for $t_3 = 170$ s. The square of $\psi$ function yields the fermionic density at this time (Fig. 4, left-hand side), which is essentially the same as in the bosonic case and in the beginning of the vignette.

In our simulations, we have observed that both fermionic and bosonic multiple identical target trackers mitigate the phenomenon of track coalescence, while fermionic trackers react significantly more agile to target split-off.

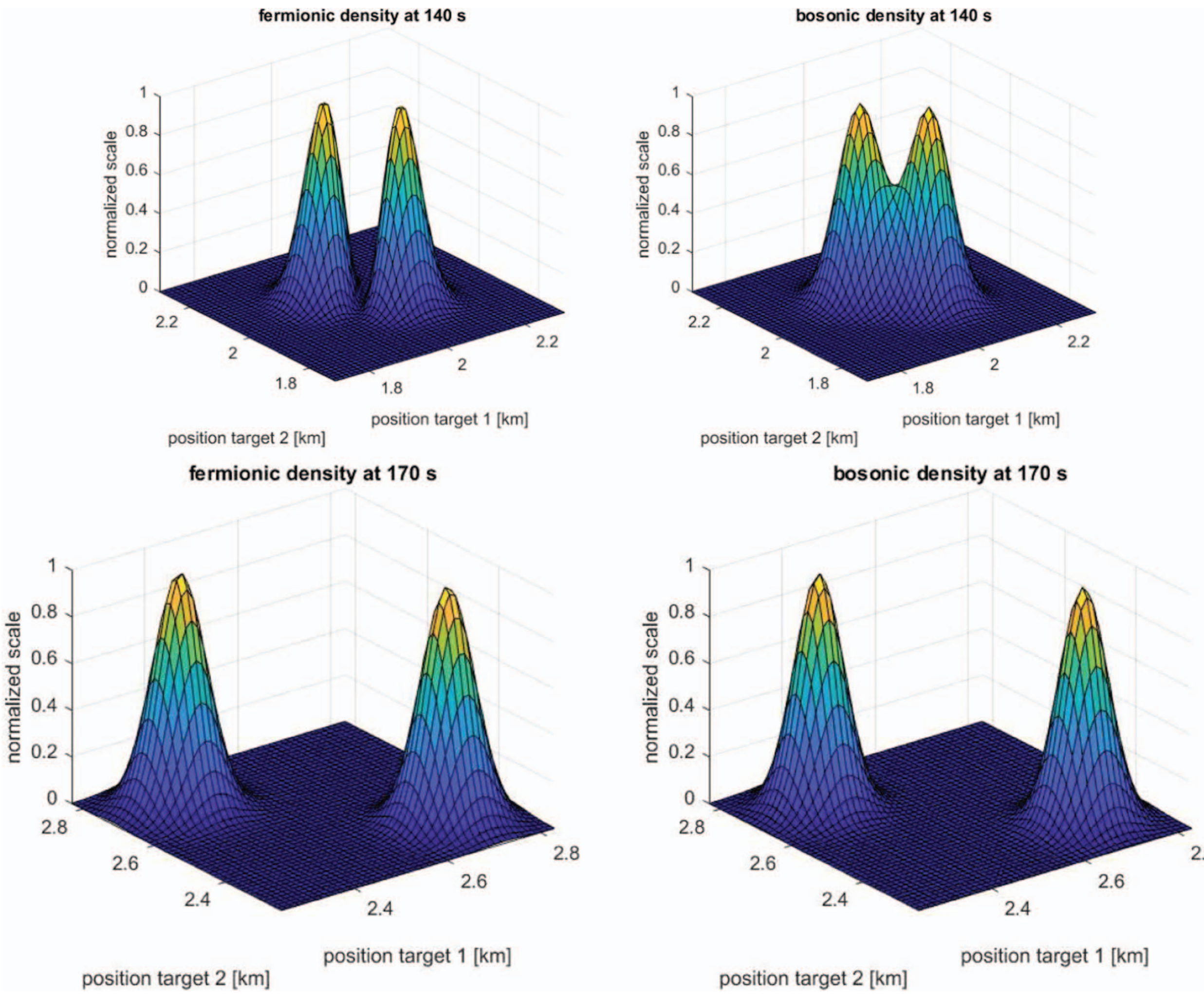


Fig. 4. Spatial projection of $p^{\pm}(\mathbf{x}_k^{1:2}|Z_{k:1})$ at two different instants of time (134, 170 s).

## V. INDISTINGUISHABILITY AND PUBLIC SECURITY

Since security of public life is a basic human desire and a fundamental prerequisite of liberal societies, its satisfaction raises an important question: How can public security be improved by morally and legally conformable and societally acceptable multiple sensor surveillance systems in public spaces? Perhaps rather

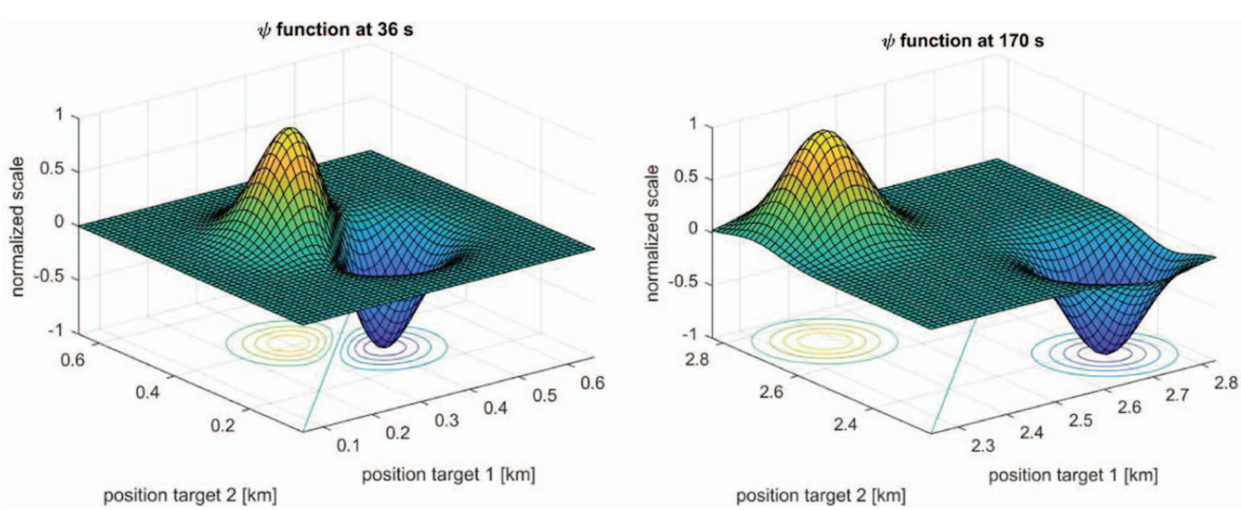


Fig. 5. Fermionic $\psi$ function for closely spaced and well-separated vehicles (36, 170 s).

unexpectedly, bosonic and fermionic multiple target tracking, that is, *indistinguishable* target tracking, seems to play a key role seen from a systems engineering perspective whenever the problem of reconciling the values of greater security with the values of the liberality, freedom, personal dignity, or privacy that an individual foregoes is to be solved.

In the context of public surveillance, the tracking approach proposed here guarantees "indistinguishability of the uninvolved," a notion that seems to play the role of a quite fundamental systems design principle. By considering persons to be tracked as indistinguishable targets, such security systems will be able to preserve the anonymity of the vast majority of persons until a certain level of suspicion is reached that may finally justify the identification of an individual, for example, by using the output of biometric sensors. From a systems engineering point of view, we conclude this paper discussing a prototypical realization that addresses security threats by hazardous materials in public infrastructures.

At the Nuclear Security Summit 2016,[7] radiological terrorism was identified as one of the greatest challenges to international security. Compared to nuclear weapons, improvised radiological dispersion devices (IRDDs) are relatively easy to produce, for which radioactive isotopes are used in many facilities, and often susceptible to theft. With the explicit constraint of not compromising the informational self-determination, an experimental public security system was developed to detect IRDDs in person streams and to make the security personnel aware of potential suspects. This research was part of a research project, which investigated the vulnerability of the transnational high-speed train systems [25]. While maintaining an open transport concept as far as possible, an analysis of the infrastructure usually available in and around railway stations shows that there are always areas suitable for continuous radiological monitoring. For details, see [26].

A spatially distributed network of gamma sensors records and classifies gamma radiation emitted by the materials used for building IRDDs. Any effective shielding by the perpetrators is impracticable. Such sensors provide data about the existence of a radiological hazard, the materials involved, the intensity, indications whether the material is incorporated for medical purposes or extracorporeal, and other attributes derivable from gamma spectra. The reliable localization of the source of gamma radiation, however, is not possible by considering spectrometers only.

The assignment of a radiological threat detected and classified to an individual is possible in a multiple sensor approach that exploits besides the spectra from spatially distributed gamma sensors also the temporal dimension by tracking the persons while they are moving within the surveillance area. For tracking purposes, time-of-flight (ToF) cameras, cheap mass products, are used that are located in the ceiling above the surveillance area. These sensors provide additionally depth information in addition to the images. Person streams thus appear as "hilly landscapes" characterized by the moving heads of the people. Each individual can thus be tracked with high precision and without the risk of occlusions, even in dense crowds.

The demonstration of the experimental system shown in Fig. 6 shows persons walking around gamma sensors that in practical realizations may well be hidden in the walls or in the floor. The association of positive signatures provided by the gamma sensors with an individual and its track over time is produced by a track-while-classify (TwC) algorithm such as that described in [27]. Indistinguishable target tracking is essential in the TwC step that treats persons as fermionic targets. In other words, the overall system preserves in a certifiable sense personal privacy by the "indistinguishability of the uninvolved" principle that we would like to see recognized as a generally used principle of systems design in public surveillance applications.

The key benefit of indistinguishable target tracking in public security applications lies less in the fact that "better" tracks in a certain respect are produced, for example, in terms of accuracy or continuity, but to guarantee that no "uninvolved" person can be distinguished from another as long as it is not "uninvolved" any more, that is, until a sufficient level of "suspicion" has been accumulated, thus establishing *privacy by design*. In a crowd of persons, fermionic trackers may also provide a certain gain in track continuity as discussed in the example of the previous section.

## VI. CONCLUSIONS AND WAY AHEAD

Based on the fundamental observation that real-world targets cannot exist at the same time at the same place, we have introduced Pauli's exclusion principle into multiple identical target tracking. Symmetry in target tracking, either in their fermionic variant or in their bosonic variant, inherently implies a multiple hypothesis structure where all measurements are associated with all targets that should conceptually be distinguished from classical enumeration of data interpretation hypotheses.

- Antisymmetry can seamlessly be embedded into the joint probability functions describing the kinematic properties of identical targets. Preliminary simulations indicate benefits in situation where targets may move closely spaced.
- In particular, antisymmetry leads to Gaussian sum representations with normalized weighting factors that are possibly negative. Such densities do

[7]Nuclear Security Summit, Washington, DC, USA, 2016, http://www.nee2016.org, last accessed August 26, 2019.

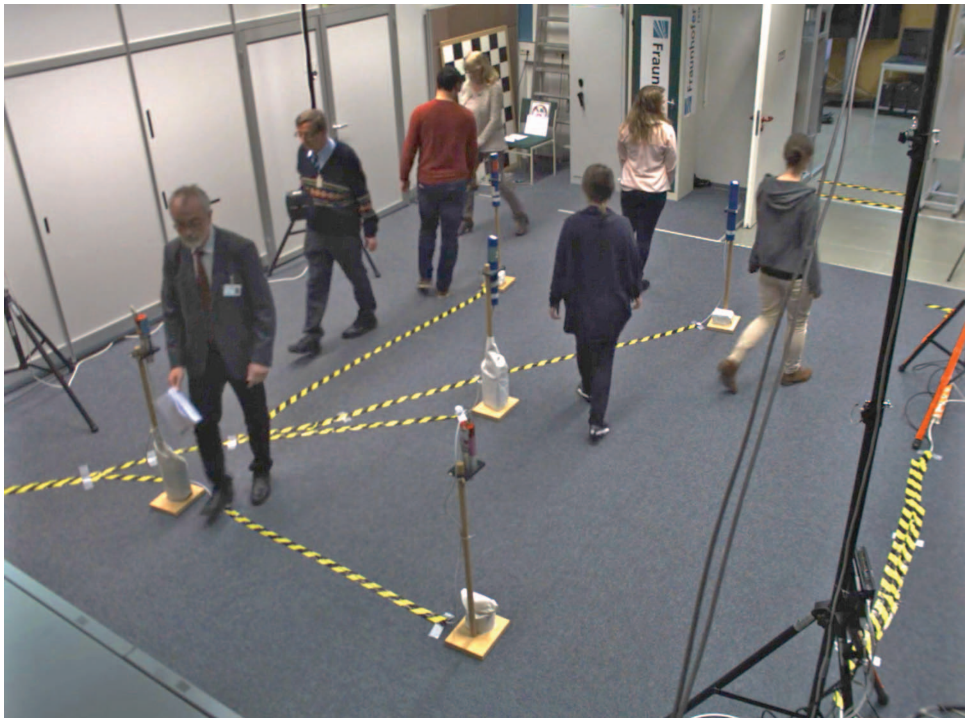

Fig. 6. Lab view of demonstrating IRDD localization in person streams using five gamma sensors on stabs and ToF cameras at the ceiling (invisible).

occur in target tracking for several reasons (see, e.g., [6, Sec. 7.4]).

- Extensive simulations will have to explore the properties and benefits of fermionic trackers quantitatively. In particular, the width of the Pauli notches has to be characterized and to be related to the targets' properties.
- Suitable approximations have to be developed as well and to be evaluated in view of practical implementations. Many-particle quantum physics has much more to offer to the tracking community as it would seem.
- Symmetry and antisymmetry can be embedded into group and extended target trackers, where the kinematics is described by random vectors and their shape by random matrices [6, Sec. 8.2]. While group targets might be dealt with as bosonic targets, extended targets are fermions.
- Antisymmetry is potentially present in *every* identical target tracking problem. Alternative methodologies are based on symmetric point processes [3, pp. 19, 240]. There are results for anti/skew-symmetric or "determinantal" point processes that are relevant to target tracking [19].
- Finally, symmetry and antisymmetry properties seem to be linked to "spooky action at a distance," first observed in tracking by Dietrich Fränken, Michael Schmidt, and Martin Ulmke [28]. Apparently, entanglement is not restricted to the microphysical world. The physics literature may stimulate progress in understanding this paradox in target tracking [29].

## APPENDIX

### *A.1 Normalizing $\psi$ Functions*

With $\Pi$ defined by $\mathbf{x}_k^{1:2} = \Pi \mathbf{x}_k^{2:1}$, we obtain

$$\int \mathrm{d}\mathbf{x}_k^{1:2} \left( \mathcal{N}(\mathbf{x}_k^{1:2};\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k}) - \mathcal{N}(\mathbf{x}_k^{1:2};\, \Pi\mathbf{x}_{k|k},\, \Pi\mathbf{P}_{k|k}\Pi^\top)\right)^2 = \frac{2}{\sqrt{|4\pi\mathbf{P}_{k|k}|}} - 2\,\mathcal{N}\big(\mathbf{x}_{k|k};\, \Pi\mathbf{x}_{k|k},\, \mathbf{P}_{k|k} + \Pi\mathbf{P}_{k|k}\Pi^\top\big). \tag{A.1}$$

### *A.2 Square Roots of Gaussians*

According to the product formula for Gaussians, see [6, A.5], for example, we obtain

$$\big(\mathcal{N}(\mathbf{z};\, \mathbf{x},\, 2\mathbf{P})\big)^2 = \mathcal{N}(\mathbf{z};\, \mathbf{z},\, 4\mathbf{P})\, \mathcal{N}(\mathbf{x};\, \mathbf{z},\, \mathbf{P}). \tag{A.2}$$

### *A.3 Fermionic Filtering Update*

Since $\mathbf{z}_k^1$ and $\mathbf{z}_k^2$ are independent of each other,

$$\begin{aligned}
&\lambda_5\big(\mathbf{z}_k^{1:2};\, \mathbf{H}_k^{1:2}\mathbf{x}_k^{1:2},\, \mathbf{R}_k^{1:2}\big)\, \psi\big(\mathbf{x}_k^{1:2};\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big)\\
&= \mathcal{N}\big(\mathbf{z}_k^{1:2};\, \mathbf{H}_k^{1:2}\mathbf{x}_k^{1:2},\, \mathbf{R}_k^{1:2}\big)\, \mathcal{N}\big(\mathbf{x}_k^{1:2};\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big)\\
&- \mathcal{N}\big(\mathbf{z}_k^{2:1};\, \mathbf{H}_k^{2:1}\mathbf{x}_k^{2:1},\, \mathbf{R}_k^{2:1}\big)\, \mathcal{N}\big(\mathbf{x}_k^{2:1};\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big)\\
&+ \mathcal{N}\big(\mathbf{z}_k^{2:1};\, \mathbf{H}_k^{2:1}\mathbf{x}_k^{1:2},\, \mathbf{R}_k^{2:1}\big)\, \mathcal{N}\big(\mathbf{x}_k^{1:2};\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big)\\
&- \mathcal{N}\big(\mathbf{z}_k^{1:2};\, \mathbf{H}_k^{1:2}\mathbf{x}_k^{2:1},\, \mathbf{R}_k^{1:2}\big)\, \mathcal{N}\big(\mathbf{x}_k^{2:1};\, \mathbf{x}_{k|k-1},\, \mathbf{P}_{k|k-1}\big).
\end{aligned}$$

From the product formula [6, A.5], the update equations result.

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