Evaluation of Fusion Algorithms for Passive Localization of Multiple Transient Emitters

WENBO DOU
JEMIN GEORGE
LANCE M. KAPLAN
RICHARD W. OSBORNE, III
YAAKOV BAR-SHALOM

The problem of localizing an unknown number of stationary transient emitters using passive sensors in the presence of missed detections and false alarms is investigated. Each measurement is based on one detection by a passive sensor and consists of a time of arrival and a bearing. It is assumed that measurements within a short time interval have to be associated before estimation. Both a Bernoulli measurement model and a Poisson measurement model are considered for each target. These two measurement models lead to two different proposed problem formulations: one is an *S*-dimensional (*S*-D) assignment problem and the other is a cardinality selection problem. The former can be solved by the Lagrangian relaxation algorithm reliably when the number of sensors is small. The sequential *m*-best 2-D (SEQ[*m*(2-D)]) assignment algorithm, which is resistant to the ghosting problem due to the estimation of the emitter signal's emission time, is developed to solve the problem when the number of sensors becomes large. Simulation results show that the SEQ[*m*(2-D)] assignment algorithm is efficient for real-time processing with reliable associations and estimates. In the cardinality selection formulation, a list of measurements is modeled as either realizations of a random variable with a uniform–Gaussian mixture (UGM) density or a Poisson point process (PPP). Because of an efficient way of incorporating false alarm rate, the UGM formulation is shown to be a useful alternative to the PPP formulation. Simulation studies show that both UGM and PPP formulations, which are based on the expectation–maximization algorithm, require the right initial estimates to yield reliable localization results.

Manuscript received March 20, 2016; revised August 19, 2016 and January 18, 2018; released for publication July 8, 2019.

Refereeing of this contribution was handled by Ramona Georgescu.

Authors' addresses: W. Dou, R. W. Osborne, III, and Y. Bar-Shalom are with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269, USA (E-mail: wenbo.dou@uconn.edu, richard.osborne@engineer.uconn.edu, ybs@engr.uconn.edu).

J. George and L. M. Kaplan are with the U.S. Army Research Laboratory, 2800 Powder Mill Rd., Adelphi, MD 20783, USA (E-mail: jemin.george.civ, lance.m.kaplan.civ@mail.mil).

This research was supported by ARO Grant W991NF-10-1-0369.

1557-6418/19/$17.00 © 2019 JAIF

I. INTRODUCTION

This paper considers the problem of multiple transient emitter (target) localization using a group of passive sensors. One particular application is to utilize a network of acoustic gunfire detection systems on a group of soldiers to localize adversaries in a battlefield [12], [20]. It is assumed that the targets are stationary during the time window of interest but the number of targets is unknown. The sensors can measure line of sight (LOS) angles to the targets by detecting their emitted acoustic signals and record the times of arrival of the detected signals. Missed detections and false alarms are present due to the imperfection of the sensors. Furthermore, the association between the measurements and the targets is unknown; that is, each sensor does not know from which target (or clutter) a particular measurement originates. Before estimating the position of any target, one has to associate the measurements from all the sensors. Therefore, the quality of data association is critical to the overall localization performance.

The problem of data association has been studied extensively in tracking multiple targets. Methods including multiple hypothesis tracking [6], joint probabilistic data association filter [11], and probability hypothesis density filter [16] are recursive algorithms that require persistent measurements and provide solutions to a dynamic data association problem. Therefore, they cannot be employed to solve the static data association problem considered in the situation of multiple transient emitter localization.

There are two different philosophies—hard data association and soft data association (see [4, Sec. 2.4.3])—in solving the static data association problem considered in this paper. Hard data association either assigns a measurement to one and only one target or condemns it as a false alarm; in other words, the probability of a measurement coming from a target is either 0 or 1 (discrete). In contrast, soft data association assigns the event that a measurement originates from a target to a (continuous) probability, which can be any value between 0 and 1.

The hard data association for *S* lists of measurements with one list from each sensor,[1] assuming a Bernoulli measurement model that the number of measurements from each target received at each sensor is a Bernoulli random variable with parameter equal to the probability of detection, leads to an *S*-dimensional (*S*-D) assignment problem, which can be formulated as a discrete

[1] In a multisensor localization application, as in this paper, the number of lists is the same as the number of sensors.

constrained optimization problem aiming to find out the set of S-tuples of measurements that minimizes the overall association cost. The number of possible S-tuple sets for T targets and S sensors in the absence of missed detections and false alarms is $(T!)^{S-1}$, from which it can be seen that S-D assignment problem is nondeterministic polynomial-time (NP) hard with $S \geq 3$. Therefore, it is of great interest and importance to find robust suboptimal algorithms.

The Lagrangian relaxation-based approach [8], which is termed as the S-D algorithm in this paper, provides a measure of how close the final solution is to the (unknown) optimal solution in terms of the association cost. The application of the S-D assignment algorithm on a multiple shooter localization problem using a small number of sensors was presented in [19]. Although it does not explore the entire space of the S-tuple sets, it needs to calculate the cost of candidate S-tuples. The cost calculation involves finding the maximum likelihood (ML) estimate of the target locations and can take most of the computational time. The number of candidate S-tuples for T targets and S sensors in the absence of missed detections and false alarms is T^S, which increases exponentially with the number of sensors. Since more sensors generate more accurate estimates in the fusion center, computationally efficient algorithms are required when a large number of sensors are deployed.

The S_0-D + SEQ(2-D) algorithm [23], which performs the S-D assignment algorithm on S_0 lists of measurements before applying the modified auction algorithm [21] for 2-D assignments on the remaining lists sequentially $S - S_0$ times, is a more efficient algorithm than the S-D assignment. The number of candidate associations increases quadratically (rather than combinatorially/exponentially) with the number of sensors. Because of the ghosting problem [4], the S_0-D step requires, in general, at least three lists to achieve reliable association. However, since in the present problem one also has arrival times, one can use $S_0 = 2$.

The problem of multiple shooter localization using a single sensor [13] or using multiple sensors [14] is formulated as a cardinality (number of targets) selection problem that assumes a Poisson measurement model that the number of measurements from each target received at each sensor is a Poisson random variable with parameter equal to the probability of detection. The measurements at a single sensor from all targets and the clutter are modeled as a Poisson point process (PPP) [7]. For each possible selected cardinality, one solves a subproblem based on the learning expectation–maximization (EM) algorithm [9] to select the best cardinality based on an information criterion [1], [22]. During every iteration of the EM algorithm, each measurement will be assigned a probability of having originated from a target, which is an example of the soft data association.

In this paper, we discuss two classes of algorithms, each for a specific measurement model in the multiple passive transient emitter localization problem. For the Bernoulli measurement model, the SEQ[m(2-D)] algorithm [2], the m-best version of the fastest sequential algorithm SEQ(2-D), is shown to be able to yield associations as good as the S-D assignment. The ghosting effect for a pair of sensors is no longer present due to the estimation of the signal emission time, which makes SEQ[m(2-D)] practical. For the Poisson measurement model, we discuss both uniform–Gaussian mixture (UGM) [5] and PPP modeling of the lists of measurements for the cardinality selection formulation. In the previous work on PPP [14], both the range and bearing measurements are assumed available and the initialization in the EM-based algorithm uses a finite set including target locations that are close to the truth. Since the range measurement and prior information for a "good" initialization is not always available in the real world, this paper considers bearing and time of arrival measurements and presents some measurement-driven initialization approaches for the EM-based algorithms. In the UGM formulation, the probability of detection (assumed not known) and the expected number of false alarms per sensor (which can be known or unknown) are incorporated into the mixture coefficients and the maximization step in the EM algorithm is developed such that the constraint that the resulting probability of detection is not larger than unity is always satisfied.

The remaining sections of this paper are organized as follows. Section II describes the problem of localizing an unknown number of transient emitters. Section III assumes a Bernoulli measurement model for each target, formulates an S-D assignment problem, and presents two assignment algorithms. Sections IV and V present the UGM and PPP formulations both of which assume a Poisson measurement model for each target. Simulation results are shown and analyzed in Section VI and the conclusions are drawn in Section VII. For the convenience of the reader, the list of notations used in this paper is given in Table I.

II. PROBLEM DESCRIPTION

Consider a scenario where there are N targets located in $\mathbb{R}^2$. The target locations (fixed) are denoted as

$$\mathbf{T} = (\mathrm{T}_1, \mathrm{T}_2, \ldots, \mathrm{T}_N) = \left(\begin{bmatrix} T_{x_1} \\ T_{y_1} \end{bmatrix}, \begin{bmatrix} T_{x_2} \\ T_{y_2} \end{bmatrix}, \ldots, \begin{bmatrix} T_{x_N} \\ T_{y_N} \end{bmatrix} \right) \tag{1}$$

and the emission times are denoted as

$$\mathbf{t}^{\mathrm{e}} = (t_1^{\mathrm{e}}, t_2^{\mathrm{e}}, \ldots, t_N^{\mathrm{e}}). \tag{2}$$

The number of targets and their locations are unknown quantities of interest, to be estimated. A total number of N_{s} stationary sensors with known locations at

$$\mathbf{S} = (\mathrm{S}_1, \mathrm{S}_2, \ldots, \mathrm{S}_{N_{\mathrm{s}}}) = \left(\begin{bmatrix} S_{x_1} \\ S_{y_1} \end{bmatrix}, \begin{bmatrix} S_{x_2} \\ S_{y_2} \end{bmatrix}, \ldots, \begin{bmatrix} S_{x_{N_{\mathrm{s}}}} \\ S_{y_{N_{\mathrm{s}}}} \end{bmatrix} \right) \tag{3}$$

are able to observe transient acoustic events that occurred at target locations at the emission times and

TABLE I
List of Notations

Notation	Definition
S	Dimension of the assignment problem
$\mathbf{T}$	Set of target position vectors
T_i	Position vector of target i
$\mathbf{t}_\mathrm{e}$	Set of emission times
t_i^e	Signal emission time of target i
S_ℓ	Position vector of sensor ℓ
n_ℓ	Number of measurements at sensor ℓ
$\mathbf{z}_{\ell j}$	jth measurement at sensor ℓ
N	Number of targets
N_s	Number of sensors
N_fa	Expected number of false alarms per sensor
T_0	The clutter
Φ	The range of the sensor field view
$\mathbf{Z}_\ell$	Augmented measurement list at sensor ℓ
$\mathbf{z}_{\ell 0}$	Dummy measurement at sensor ℓ
$Z_{j_1 j_2 \cdots j_{N_\mathrm{s}}}$	An N_s-tuple of measurements, one from each sensor
$c_{j_1 j_2 \cdots j_{N_\mathrm{s}}}$	Cost of associating $Z_{j_1 j_2 \cdots j_{N_\mathrm{s}}}$ with a target
$\rho_{j_1 j_2 \cdots j_{N_\mathrm{s}}}$	Binary variable denoting whether $Z_{j_1 j_2 \cdots j_{N_\mathrm{s}}}$ is an association in the final assignment
$p_{\mathrm{d}\ell}$	Detection probability for sensor ℓ
m	Number of top solutions to be kept in the SEQ[m(2-D)] assignment algorithm
$\mathbf{K}$	Set of all $\mathbf{k}_{\ell j}$
$\mathbf{k}_{\ell j}$	Association variable of $\mathbf{z}_{\ell j}$ in the UGM formulation
π_i	Mixing coefficient of the UGM
$\boldsymbol{\kappa}$	Set of all $\kappa_{\ell j}$
$\kappa_{\ell j}$	Association variable of $\mathbf{z}_{\ell j}$ in the PPP formulation

measure the bearings to these targets and the time of arrival of the observed acoustic signals. For events and measurements that are separated significantly in time, there is no data association ambiguity, so it is assumed that only measurements falling within a certain time window of interest need to be associated. Let n_ℓ denote the number of such measurements (one measurement is defined as a vector consisting of both a bearing and a time of arrival due to one acoustic signal in this context) obtained by the ℓth sensor within the time window.

The jth measurement (a direction of arrival and time of arrival) received by the ℓth sensor, if it corresponds to the event at t_i^e from the ith target, is

$$\mathbf{z}_{\ell j}(\mathrm{T}_i, t_i^\mathrm{e}) = \mathbf{h}_\ell\,(\mathrm{T}_i, t_i^\mathrm{e}) + \mathbf{w}_{\ell j}, \quad i = 1, \ldots, N; \quad \ell = 1, \ldots, N_\mathrm{s}; \quad j = 1, \ldots, n_\ell \tag{4}$$

where $\mathbf{w}_{\ell j}$ is a zero-mean white Gaussian measurement noise with known covariance matrix R_ℓ and

$$\mathbf{h}_\ell\,(\mathrm{T}_i, t_i^\mathrm{e}) = \begin{bmatrix}\theta_{\ell i}\\ t_{\ell i}\end{bmatrix} = \begin{bmatrix}\arctan\left[\frac{T_{y_i}-S_{y_\ell}}{T_{x_i}-S_{x_\ell}}\right]\\ t_i^\mathrm{e} + \dfrac{\sqrt{(T_{x_i}-S_{x_\ell})^2+(T_{y_i}-S_{y_\ell})^2}}{c}\end{bmatrix} \tag{5}$$

where t_i^e is the unknown emission time of the acoustic signal from T_i and c is the known speed of sound.

To incorporate false alarms, we denote a clutter target (with index 0) as T_0. A false measurement detected by the ℓth sensor consists of a bearing θ_0, which is uniformly distributed in the field of view of the ℓth sensor, and its arrival time t_0, which is uniformly distributed in the interval $[0, W]$. The number of false alarms from each sensor is assumed to be a Poisson random variable[2] with mean

$$N_\mathrm{fa} = \lambda_\mathrm{fa}\Phi W \tag{6}$$

where Φ is the range of field of view and is assumed to be the same for each sensor and λ_fa can be interpreted as the temporal–spatial density.

The probability density function (pdf) of measurement j from sensor ℓ—the likelihood function [3] of the target location and its emission time based on the measurement[3]—is

$$p(\mathbf{z}_{\ell j}|\mathrm{T}_0) = p(\theta_0)p(t_0) = \frac{1}{\Phi W} \triangleq \Lambda(\mathrm{T}_0; \mathbf{z}_{\ell j}) \tag{7}$$

$$p(\mathbf{z}_{\ell j}|\mathrm{T}_i, t_i^\mathrm{e}) = |2\pi R_\ell|^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2}\left[\mathbf{z}_{\ell j} - \mathbf{h}_\ell(\mathrm{T}_i, t_i^\mathrm{e})\right]' R_\ell^{-1}\left[\mathbf{z}_{\ell j} - \mathbf{h}_\ell(\mathrm{T}_i, t_i^\mathrm{e})\right]\right\} \triangleq \Lambda(\mathrm{T}_i, t_i^\mathrm{e}; \mathbf{z}_{\ell j}), \qquad i = 1, \ldots, N \tag{8}$$

where (7) is the pdf of a measurement from the clutter (a false alarm) and (8) is the pdf of a measurement from a true target.

The problem is to estimate N and $\mathbf{T} = \{\mathrm{T}_i, i = 1, \ldots, N\}$ given the complete set of observations $\mathbf{Z} = \{\mathbf{z}_{\ell j}, \ell = 1, \ldots, N_\mathrm{s}; j = 1, \ldots, n_\ell\}$ in the presence of missed detections and false alarms and without the knowledge of the true data association.

III. THE S-D ASSIGNMENT ALGORITHM

A. Formulation

The S-D assignment problem formulation assumes a Bernoulli measurement model that the number of measurements from a real target received by a sensor is a Bernoulli random variable. Note that the number of false alarms is modeled as a Poisson random variable.

An augmented list of measurements at the ℓth sensor is defined as

$$\mathbf{Z}_\ell \triangleq \{\mathbf{z}_{\ell 0}, \ldots, \mathbf{z}_{\ell n_\ell}\} \tag{9}$$

where $\mathbf{z}_{\ell 0}$ is a dummy measurement[4] representing missed detections. An association of N_s measurements (N_s-tuple) consisting of one measurement from each augmented list will be denoted as

$$Z_{j_1 j_2 \cdots j_{N_\mathrm{s}}} = \{\mathbf{z}_{1 j_1}, \mathbf{z}_{2 j_2}, \ldots, \mathbf{z}_{N_\mathrm{s} j_{N_\mathrm{s}}}\} \tag{10}$$

where $j_\ell \in \{0, 1, \ldots, n_\ell\}$ represents the index of the measurement from the augmented list $\mathbf{Z}_\ell$, which is included in the association.[5]

[2]While for targets we consider two measurement models (Bernoulli and Poisson), for clutter only a Poisson model is considered.
[3]If the source is clutter, it has no emission time, only an arrival time.
[4]Please see [20, Fig. 2] for the illustration of dummy measurement and N_s-tuple.
[5]Recall that $j_\ell = 0$ represents the dummy measurement, so (10) need not contain N_s "real" measurements; i.e., missed detections are allowed in the association.

Assuming that the measurements in $Z_{j_1 j_2 \dots j_{N_s}}$ originated from the same target at the location T_i and emission time t_i^{e}, the cost of this association will be given by the (physically dimensionless) negative log-likelihood ratio

$$c_{j_1 j_2 \dots j_{N_s}} = -\ln \frac{\Lambda(\mathrm{T}_i, t_i^{\mathrm{e}}; Z_{j_1 j_2 \dots j_{N_s}})}{\Lambda(\mathrm{T}_0; Z_{j_1 j_2 \dots j_{N_s}})} \tag{11}$$

where the numerator is calculated based on (8) and the denominator (the likelihood that they are all false) is calculated using (7).

Assuming the measurements are (conditioned on the true target locations) independent across the sensors, i.e., uncorrelated measurement noises, the likelihood function that the measurements in $Z_{j_1 j_2 \dots j_{N_s}}$ originated from the same target at the location T_i and emission time t_i^{e} is

$$\Lambda(\mathrm{T}_i, t_i^{\mathrm{e}}; Z_{j_1 j_2 \dots j_{N_s}}) = \prod_{\ell=1}^{N_s} (1 - p_{\mathrm{d}\ell})^{1-u(j_\ell)} \cdot (p_{\mathrm{d}\ell}\, p(\mathbf{z}_{\ell j_\ell} | \mathrm{T}_i, t_i^{\mathrm{e}}))^{u(j_\ell)} \tag{12}$$

where $p_{\mathrm{d}\ell}$ is the probability of detection for the ℓth sensor (assumed the same for each real target) and the indicator function $u(j_\ell)$ is

$$u(j_\ell) \triangleq \begin{cases} 0, & \text{if } j_\ell = 0 \\ 1, & \text{otherwise} \end{cases}. \tag{13}$$

Since the target location T_i and the emission time t_i^{e} are unknown, we replace them by their ML estimates $\hat{\mathrm{T}}_i$ and $\hat{t}_i^{\mathrm{e}}$ that are obtained by maximizing (12), that is,

$$\hat{\mathrm{T}}_i, \hat{t}_i^{\mathrm{e}} = \arg\max_{\mathrm{T}_i, t_i^{\mathrm{e}}} \Lambda(\mathrm{T}_i, t_i^{\mathrm{e}}; Z_{j_1 j_2 \dots j_{N_s}}). \tag{14}$$

Therefore, (11) is modified to a *generalized* negative log-likelihood ratio given by

$$c_{j_1 j_2 \dots j_{N_s}} = -\ln \frac{\Lambda(\hat{\mathrm{T}}_i, \hat{t}_i^{\mathrm{e}}; Z_{j_1 j_2 \dots j_{N_s}})}{\Lambda(\mathrm{T}_0; Z_{j_1 j_2 \dots j_{N_s}})}. \tag{15}$$

The likelihood that all the measurements in $Z_{j_1 j_2 \dots j_{N_s}}$ are false alarms is

$$\Lambda(\mathrm{T}_0; Z_{j_1 j_2 \dots j_{N_s}}) = \prod_{\ell=1}^{N_s} \left(\frac{1}{\Phi W} \right)^{u(j_\ell)}. \tag{16}$$

The assignment problem is formulated as

$$\min_{\rho_{j_1 j_2 \cdots j_{N_s}}} \sum_{j_1=0}^{n_1} \sum_{j_2=0}^{n_2} \cdots \sum_{j_{N_s}=0}^{n_{N_s}} c_{j_1 j_2 \dots j_{N_s}} \rho_{j_1 j_2 \dots j_{N_s}} \tag{17}$$

subject to

$$\sum_{j_2=0}^{n_2} \sum_{j_3=0}^{n_3} \cdots \sum_{j_{N_s}=0}^{n_{N_s}} \rho_{j_1 j_2 \dots j_{N_s}} = 1, \quad j_1 = 1, 2, \dots, n_1 \tag{18}$$

$$\sum_{j_1=0}^{n_1} \sum_{j_3=0}^{n_3} \cdots \sum_{j_{N_s}=0}^{n_{N_s}} \rho_{j_1 j_2 \dots j_{N_s}} = 1, \quad j_2 = 1, 2, \dots, n_2 \tag{19}$$

$$\vdots \qquad\qquad \vdots$$

$$\sum_{j_1=0}^{n_1} \sum_{j_2=0}^{n_2} \cdots \sum_{j_{N_s-1}=0}^{n_{N_s-1}} \rho_{j_1 j_2 \dots j_{N_s}} = 1, \quad j_{N_s} = 1, 2, \dots, n_{N_s} \tag{20}$$

where $\rho_{j_1 j_2 \dots j_{N_s}} \in \{0, 1\}$ and $\rho_{j_1 j_2 \dots j_{N_s}} = 1(0)$ means $Z_{j_1 j_2 \dots j_{N_s}}$ is (not) an association in the final assignment.

Note that if $c_{j_1 j_2 \dots j_{N_s}} > 0$, then $Z_{j_1 j_2 \dots j_{N_s}}$ will not be an association in the final assignment since the overall cost will be smaller for the decision that all the real measurements in $Z_{j_1 j_2 \dots j_{N_s}}$ are false (cost = 0) than for the decision that they are from the same real target.

The ℓth constraint set in (18)–(20)

$$\sum_{j_1=0}^{n_1} \cdots \sum_{j_{\ell-1}=0}^{n_{\ell-1}} \sum_{j_{\ell+1}=0}^{n_{\ell+1}} \cdots \sum_{j_{N_s}=0}^{n_{N_s}} \rho_{j_1 j_2 \dots j_{N_s}} = 1, \quad j_\ell = 1, 2, \dots, n_\ell \tag{21}$$

enforces that each measurement (except the dummy) is associated with a single measurement from each other list, yielding a "target." Once the minimization problem (17) is solved, based on the assumption that each target is associated with one and only one measurement in each sensor list (including the dummy measurement), the number of associations will be equal to the number of targets (some will be real and some false). Associations with less than τ real measurements will be considered as from the clutter. The remaining associations will be deemed from real targets. The corresponding locations and emission times will be the ML estimates as obtained in (14).

B. The Optimization via Lagrangian Relaxation

The optimization problem (17) is NP hard when $N_s \geq 3$. One suboptimal algorithm is the Lagrangian relaxation-based S-D assignment algorithm as shown in Fig. 1, which solves the original problem as a series of relaxed 2-D subproblems. The rth ($r = N_s, N_s - 1, \dots, 3$) constraint set is successively relaxed and appended to the cost with Lagrange multipliers $\mathbf{u}_r$. At stage $r = 3$, one has a 2-D problem, which can be optimally[6] solved using the modified auction algorithm.

The constraint sets are then reimposed one at a time ($r = 3, 4, \dots, N_s$), and the corresponding Lagrange multipliers are updated to $\mathbf{u}_r^{\mathrm{new}}$; at each stage, the cost J_r of the resulting feasible solution is computed, until all constraint sets are met. The duality gap—difference between the cost J_2^* from the maximally relaxed problem

[6] Up to the rounding error, i.e., quasi-optimally.

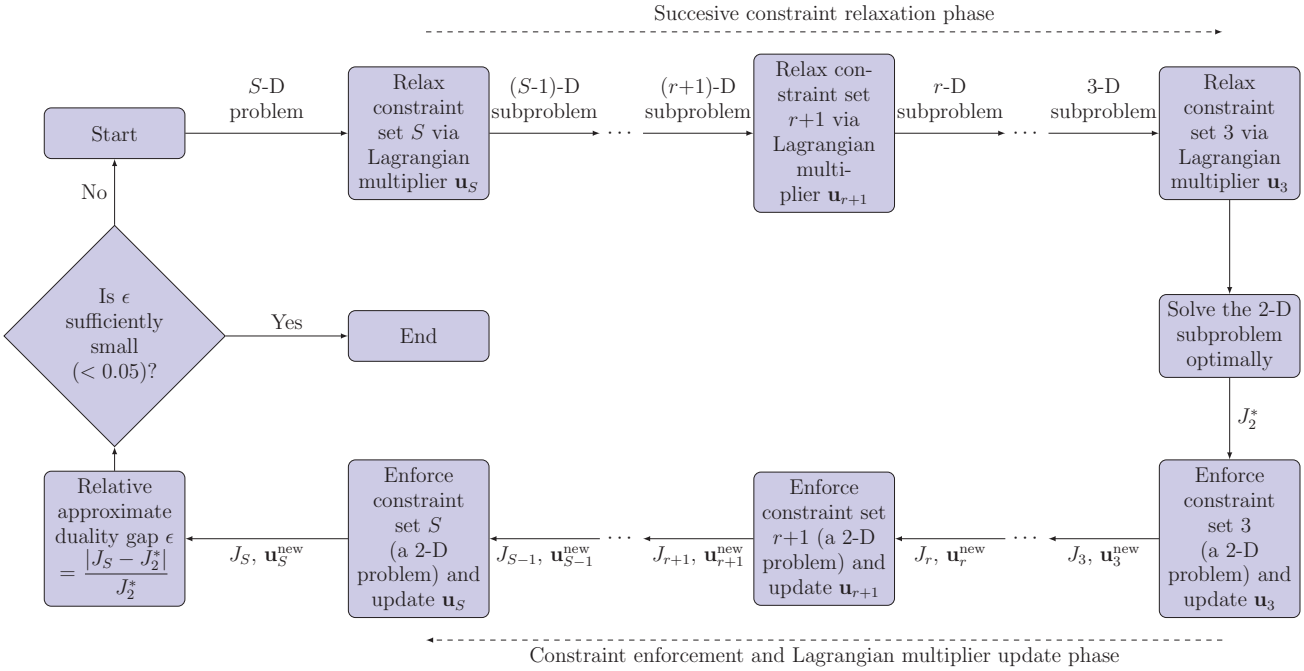


Fig. 1. Flow chart of the Lagrangian relaxation-based S-D assignment algorithm.

and J_S from the fully constrained one—is calculated and the iterations continue until this gap is small enough (usually 5% of the cost from the fully constrained one). See [8] and [21] for the detailed description.

C. The SEQ[m(2-D)] Assignment Algorithm

When $N_s = 2$, (17) becomes a 2-D assignment problem. By using Murty's ranking algorithm [18], one can find the top m best assignments instead of only the best one. The SEQ[m(2-D)] assignment algorithm can be described as follows. Initially, one selects two lists of measurements and obtains the top m best 2-D assignments with each assignment being a set of 2-tuples. Next, for each of these one continues to solve an m-best 2-D assignment, which yields a set of 3-tuples, between any one of the previous m association results and a third list of measurement. After this second step, one has m^2 assignments available, out of which the top m solutions in terms of the association cost will be selected for the next step. This procedure (shown in Fig. 2) is repeated until all the N_s lists of measurements are processed and the final assignment will be a set of N_s-tuples.

Note that it is possible to have an association $Z_{j_1 j_2 \cdots j_{N_s}}$ with $c_{j_1 j_2 \cdots j_{N_s}} > 0$ in the final assignment once the SEQ[m(2-D)] algorithm terminates. Such associations will be discarded before any association with less than τ real measurements is removed.

IV. UGM FORMULATION

If one assumes a Poisson measurement model that the number of measurements from a real target received by a sensor is a Poisson random variable, then one can model a list of measurements as realizations of a random variable with a UGM density [5] or a PPP. The UGM formulation is presented in this section and the PPP formulation will be presented in Section V.

A. Formulation

Assume (temporarily) the number of targets, N, is given. Since the association between a measurement $\mathbf{z}$ (without subscript, for simplicity) and the targets is unknown, we introduce an $(N+1)$-dimensional random binary-valued association vector

$$\mathbf{k} = [k_0, k_1, \ldots, k_N] \tag{22}$$

to indicate the target from which the measurement $\mathbf{z}$ originates. In our formulation, the random variable $\mathbf{z}$ is observed. The random variable $\mathbf{k}$ is not observed, thus is called a latent variable.[7] The entries k_i of the vector $\mathbf{k}$ satisfy the following conditions:

$$\sum_{i=0}^{N} k_i = 1 \tag{23}$$

$$k_i \in \{0, 1\} \quad \forall\, i \tag{24}$$

that is, there are $N+1$ possible values for the vector $\mathbf{k}$. Let us use $\mathbf{e}_i$ to denote the $(N+1)$-dimensional vector with 1 in its ith entry and zeros elsewhere. The event $\{\mathbf{k} = \mathbf{e}_1\}$, which is the same as the event $\{k_0 = 1\}$, means that $\mathbf{z}$ is

[7] Latent variables are random variables whose values we do not observe or measure.

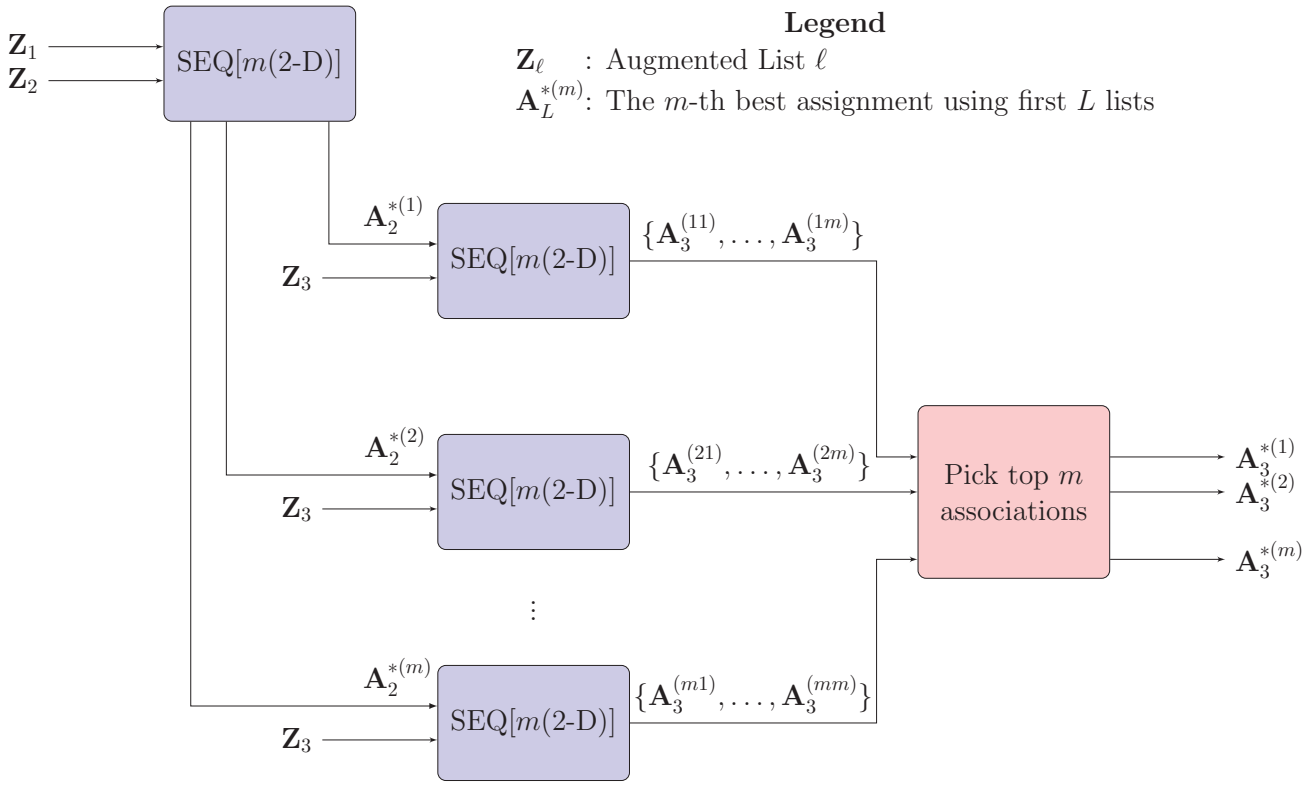


Fig. 2. Initial iteration of the SEQ[m(2-D)] assignment algorithm.

a clutter-originated measurement. The event $\{\mathbf{k} = \mathbf{e}_{i+1}\}$ with $i > 0$, which is the same as $\{k_i = 1\}$, means that the measurement $\mathbf{z}$ originated from the ith target.

The prior probability that $\mathbf{z}$ originated from the ith target given that the acoustic signal has been detected (assuming that a detected acoustic signal originates equally likely from all the targets) is

$$p(\mathbf{k} = \mathbf{e}_{i+1}) = p(k_i = 1) \triangleq \pi_i = \frac{p_{\mathrm{d}}(\mathrm{T}_i)}{\sum_{i=0}^{N} p_{\mathrm{d}}(\mathrm{T}_i)}, \quad i = 0, 1, \ldots, N \tag{25}$$

where $p_{\mathrm{d}}(\mathrm{T}_i)$ is the probability of detection for the real target i ($i \neq 0$) and is assumed to be the same at each sensor and

$$p_{\mathrm{d}}(\mathrm{T}_0) = N_{\mathrm{fa}}. \tag{26}$$

With abuse of notation, (26) is the expected number of false alarms at each sensor. The probabilities π_i, therefore, satisfy the following two conditions:

$$0 \leq \pi_i \leq 1 \tag{27}$$

$$\sum_{i=0}^{N} \pi_i = 1. \tag{28}$$

Because of (23) and (24), the prior probability in (25) can be equivalently expressed, in the form of a probability mass function, as

$$p(\mathbf{k} = \mathbf{e}_{i+1}) = p(k_i = 1) = \pi_i = \prod_{i=0}^{N} \pi_i^{k_i} \tag{29}$$

where the last equality holds because only the exponent k_i is equal to 1 while all other exponents are equal to 0, and thus do not affect the product.

From (7) and (8), the conditional pdf of a measurement $\mathbf{z}$ (without subscript, for simplicity) obtained by the ℓth sensor given that it is associated with the ith target is

$$p_\ell(\mathbf{z}|k_0 = 1, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \frac{1}{\Phi W} \tag{30}$$

$$p_\ell(\mathbf{z}|k_i = 1, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \mathcal{N}\left(\mathbf{z}; \mathbf{h}_\ell(\mathrm{T}_i, t_i^{\mathrm{e}}), R_\ell\right), \quad i = 1, \ldots, N. \tag{31}$$

For notational simplicity, let us denote[8]

$$g_{\ell i}(\mathbf{z}) = p_\ell(\mathbf{z}|k_i = 1, \mathbf{T}, \mathbf{t}^{\mathrm{e}}), \quad i = 0, 1, \ldots, N. \tag{32}$$

In a similar way as we derived (29), using (23) and (24) we have

$$p_\ell(\mathbf{z}|\mathbf{k} = \mathbf{e}_{i+1}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = p_\ell(\mathbf{z}|k_i = 1, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = g_{\ell i}(\mathbf{z}) = \prod_{i=0}^{N} \left(g_{\ell i}(\mathbf{z})\right)^{k_i}. \tag{33}$$

[8] $g_{\ell i}(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})$ will be used when the conditioning needs to be explicitly indicated.

The joint density of a measurement $\mathbf{z}$ from sensor ℓ and its association vector $\mathbf{k}$ is therefore

$$p_\ell(\mathbf{z}, \mathbf{k} = \mathbf{e}_{i+1}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = p_\ell(\mathbf{z}|\mathbf{k} = \mathbf{e}_{i+1}, \mathbf{T}, \mathbf{t}^{\mathrm{e}})p(\mathbf{k} = \mathbf{e}_{i+1}) = \prod_{i=0}^{N} (\pi_i\, g_{\ell i}(\mathbf{z}))^{k_i} = \pi_i\, g_{\ell i}(\mathbf{z}) \tag{34}$$

where the first equality holds because of the conditional probability definition, the second equality holds as a result of direct substitutions of $p(\mathbf{k} = \mathbf{e}_{i+1})$ from (29) and $p_\ell(\mathbf{z}|\mathbf{k} = \mathbf{e}_{i+1}, \mathbf{T}, \mathbf{t}^{\mathrm{e}})$ from (33), and the last equality holds because only the exponent k_i is equal to 1 and other exponents are zero. The marginal density of $\mathbf{z}$ is then obtained by summing the joint density over all the $N + 1$ values of $\mathbf{k}$ as

$$p_\ell(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \sum_{i=0}^{N} p_\ell(\mathbf{z}, \mathbf{k} = \mathbf{e}_{i+1}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \sum_{i=0}^{N} \pi_i\, g_{\ell i}(\mathbf{z}) \tag{35}$$

where the first equality holds because of the total probability theorem and the second equality holds as a result of substitution of $p_\ell(\mathbf{z}, \mathbf{k} = \mathbf{e}_{i+1}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})$ from (34). Therefore, the marginal density of one measurement is a mixture (termed as "uniform–Gaussian" mixture in this paper) of one uniform density and N Gaussian densities with the parameters π_i being the mixing coefficients. The conditional density of $\mathbf{k}$ given $\mathbf{z}$ is obtained using Bayes' theorem as

$$p_\ell(\mathbf{k}|\mathbf{z}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \frac{p_\ell(\mathbf{z}, \mathbf{k}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})}{p_\ell(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})} = \frac{\prod_{i=0}^{N} (\pi_i\, g_{\ell i}(\mathbf{z}))^{k_i}}{\sum_{i=0}^{N} \pi_i\, g_{\ell i}(\mathbf{z})} \tag{36}$$

which is equivalent to

$$P(k_i = 1|\mathbf{z}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \frac{\pi_i\, g_{\ell i}(\mathbf{z})}{\sum_{i=0}^{N} \pi_i\, g_{\ell i}(\mathbf{z})}. \tag{37}$$

Let (with $\mathbf{k}$ indexed as in (4))

$$\mathbf{K} = \{\mathbf{k}_{\ell j}, \ell = 1, 2, \ldots, N_{\mathrm{s}};\ j = 1, 2, \ldots, n_\ell\} \tag{38}$$

be the corresponding set of association vectors (or latent variables) for $\mathbf{Z}$ and

$$\boldsymbol{\pi} = \{\pi_i, i = 0,\ 1,\ \ldots,\ N\}. \tag{39}$$

From (34), the conditional independence of measurements across all the sensors yields the joint density of $\mathbf{Z}$ and $\mathbf{K}$

$$p(\mathbf{Z}, \mathbf{K}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \prod_{\ell=1}^{N_{\mathrm{s}}} \prod_{j=1}^{n_\ell} \prod_{i=0}^{N} (\pi_i\, g_{\ell i}(\mathbf{z}_{\ell j}))^{[k_{\ell j}]_i} \tag{40}$$

where $\left[k_{\ell j}\right]_i$ is the ith component of the association vector $\mathbf{k}_{\ell j}$. The marginal density of $\mathbf{Z}$ is obtained by summing the joint density (40) over all possible values of $\mathbf{K}$ as

$$p\,(\mathbf{Z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \prod_{\ell=1}^{N_{\mathrm{s}}} \prod_{j=1}^{n_\ell} \left(\sum_{i=0}^{N} \pi_i\, g_{\ell i}(\mathbf{z}_{\ell j}) \right) \tag{41}$$

and the posterior density (actually probability mass function (pmf) since $\mathbf{K}$ is discrete) of $\mathbf{K}$ conditioned on $\mathbf{Z}$ is

$$p(\mathbf{K}|\mathbf{Z}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \prod_{\ell=1}^{N_{\mathrm{s}}} \prod_{j=1}^{n_\ell} \frac{\prod_{i=0}^{N} (\pi_i g_{\ell i}(\mathbf{z}_{\ell j}))^{[k_{\ell j}]_i}}{\sum_{i=0}^{N} \pi_i\, g_{\ell i}(\mathbf{z}_{\ell j})}. \tag{42}$$

B. The EM Algorithm

We are interested in finding the ML estimates of $\mathbf{T}$ and $\mathbf{t}^{\mathrm{e}}$ that maximize $p\,(\mathbf{Z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})$ or $\ln p\,(\mathbf{Z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})$. However, it is difficult to obtain these estimates since the data association between $\mathbf{Z}$ and $\mathbf{T}$ is unknown; that is, $\mathbf{K}$ is not observed. Mathematically, setting the derivatives of $\ln p\,(\mathbf{Z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})$ with respect to $\mathbf{T}$ and $\mathbf{t}^{\mathrm{e}}$ does not lead to a closed-form solution, which suggests an iterative approach.

The EM algorithm [9] is a two-step iterative optimization technique to find the ML estimate from incomplete data. In this context, $\{\mathbf{Z}, \mathbf{K}\}$ are the complete data set and the observed data $\mathbf{Z}$ are the incomplete data available since the association variables in $\mathbf{K}$ are unknown.

Each iteration of the EM algorithm has an expectation step (E step) and a maximization step (M step). In the E step, we use temporary estimates of $\mathbf{T}$ and $\mathbf{t}^{\mathrm{e}}$ to find the posterior distribution in (42) to "learn" about $\mathbf{K}$. We then use this posterior distribution of $\mathbf{K}$ to find the expectation of the joint density of $\mathbf{Z}$ and $\mathbf{K}$ in (40). In the M step, we maximize the expectation obtained in the E step to obtain updated estimates of $\mathbf{T}$ and $\mathbf{t}^{\mathrm{e}}$ [5].

C. Optimization

1) Initialization: The EM algorithm is an iterative method. The first step is to initialize the parameters $\mathbf{T}$, $\mathbf{t}^{\mathrm{e}}$, and $\boldsymbol{\pi}$. Here, we assume that the mixing coefficients π_i are scalar quantities that need to be estimated along with T_i and t_i^{e}.

The EM algorithm guarantees that $p\,(\mathbf{Z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})$ increases at each iteration. However, a poor initialization can cause convergence to a local maximum as opposed to the global one. As shown later, because of the relationship between π_i and $p_{\mathrm{d}}(\mathrm{T}_i)$ in (25), the iterative procedures depend on whether N_{fa} is known. In either case, $p_{\mathrm{d}}(\mathrm{T}_i)$ is initialized to be 1 and the initial values of π_i will be calculated according to (25). In this paper, three initialization approaches for $\mathbf{T}$ and $\mathbf{t}^{\mathrm{e}}$ are considered and will be discussed in Section VI.D.

2) E Step: Let $\mathbf{T}^{(n-1)}$, $\mathbf{t}^{\mathrm{e},(n-1)}$, and $\boldsymbol{\pi}^{(n-1)}$ denote the estimates from the previous step. In the expectation step, we compute $p(\mathbf{K}|\mathbf{Z}, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ and evaluate the expected value of $\ln p(\mathbf{Z}, \mathbf{K}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})$ conditioned on $p(\mathbf{K}|\mathbf{Z}, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$, which is given by

$$\begin{aligned} &Q(\mathbf{T}, \mathbf{t}^{\mathrm{e}}\big|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \\ &\quad = \mathbb{E}[\ln(p(\mathbf{Z}, \mathbf{K}\,|\mathbf{T}, \mathbf{t}^{\mathrm{e}}))|p(\mathbf{K}\,|\mathbf{Z}, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})] \\ &\quad = Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) + Q_\pi(\boldsymbol{\pi}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \end{aligned} \tag{43}$$

where

$$Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \sum_{i=0}^{N} \ln\left(g_{\ell i}(\mathbf{z}_{\ell j})\right) w_{\ell ji}^{(n-1)} \tag{44}$$

$$Q_\pi(\boldsymbol{\pi} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \sum_{i=0}^{N} \ln(\pi_i)\, w_{\ell ji}^{(n-1)} \tag{45}$$

where

$$w_{\ell ji}^{(n-1)} = \frac{\pi_i^{(n-1)} g_{\ell i}^{(n-1)}(\mathbf{z}_{\ell j})}{\sum_{i=0}^{N} \pi_i^{(n-1)} g_{\ell i}^{(n-1)}(\mathbf{z}_{\ell j})} \tag{46}$$

$$g_{\ell i}^{(n-1)}(\mathbf{z}_{\ell j}) = p_\ell(\mathbf{z}_{\ell j} | k_i = 1, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \tag{47}$$

$w_{\ell ji}^{(n-1)}$ is the posterior probability that the measurement $\mathbf{z}_{\ell j}$ originates from the ith target, given that the target locations are $\mathbf{T}^{(n-1)}$ and the emission times are $\mathbf{t}^{\mathrm{e},(n-1)}$.

3) M Step: In the maximization step, we maximize $Q(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ over all feasible $\mathbf{T}$, $\mathbf{t}^{\mathrm{e}}$, and $\boldsymbol{\pi}$. Inspection of (43) reveals that $Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ depends only on the locations $\mathbf{T}$ and $Q_\pi(\boldsymbol{\pi} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ depends only on detection probabilities through mixing coefficients. Therefore, maximization of $Q(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ can be done by maximizing $Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ and $Q_\pi(\boldsymbol{\pi} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ separately.

We define $Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ as

$$Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{i=0}^{N} Q_{\mathrm{T}_i}(\mathrm{T}_i | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \tag{48}$$

where

$$Q_{\mathrm{T}_i}(\mathrm{T}_i | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \ln\left(g_{\ell i}(\mathbf{z}_{\ell j})\right) w_{\ell ji}^{(n-1)}. \tag{49}$$

Note that $Q_{\mathrm{T}_0}(\mathrm{T}_0 | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ is a constant, and there is no functional relation between T_{i_1} and T_{i_2} for $i_1 \neq i_2$ Therefore, each target location T_i can be obtained separately by maximizing $Q_{\mathrm{T}_i}(\mathrm{T}_i | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$.

Next we maximize $Q_\pi(\boldsymbol{\pi} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ with respect to π_i, while accounting for the constraint that the mixing coefficients sum to 1. This can be achieved using a Lagrange multiplier λ and maximizing the following quantity:

$$Q_\pi^L(\boldsymbol{\pi} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \sum_{i=0}^{N} \ln(\pi_i)\, w_{\ell ji}^{(n-1)} + \lambda \left(\sum_{i=0}^{N} \pi_i - 1 \right) \tag{50}$$

which gives

$$\pi_i^{(n)} = \frac{\sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} w_{\ell ji}^{(n-1)}}{\sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \sum_{i=0}^{N} w_{\ell ji}^{(n-1)}}. \tag{51}$$

When N_{fa} is unknown, one can set $p_{\mathrm{d}}(\mathrm{T}_i)$ and N_{fa} based on (51) as follows:

$$j = \arg\max_{i, i \neq 0} \pi_i \tag{52}$$

$$p_{\mathrm{d}}(\mathrm{T}_i) = \frac{\pi_i}{\pi_j}, \quad i \neq j \tag{53}$$

$$N_{\mathrm{fa}} = \frac{\pi_0}{\pi_j} \tag{54}$$

which guarantees the constraints

$$p_{\mathrm{d}}(\mathrm{T}_i) \leq 1, \quad i > 0. \tag{55}$$

However, when N_{fa} is known, it is not always possible to find $p_{\mathrm{d}}(\mathrm{T}_i) \leq 1$ such that (51) holds. For instance, the following may not hold for π_0 from (51) and a given N_{fa}:

$$\pi_0 = \frac{N_{\mathrm{fa}}}{\sum_{i=1}^{N} p_{\mathrm{d}}(\mathrm{T}_i) + N_{\mathrm{fa}}} \geq \frac{N_{\mathrm{fa}}}{N + N_{\mathrm{fa}}}. \tag{56}$$

We need to maximize $Q_\pi(\boldsymbol{\pi} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ with respect to $p_{\mathrm{d}}(\mathrm{T}_i)$ subject to (55).

The Karush–Kuhn–Tucker (KKT) conditions [15] give rise to the following proposition (see Appendix A for proof):

Proposition 1 *Let*

$$\mathcal{S} = \left\{ i \,\middle|\, \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} w_{\ell ji}^{(n-1)} N_{\mathrm{fa}} > \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} w_{\ell j0}^{(n-1)} \right\} \tag{57}$$

which can be an empty set, and its cardinality is denoted by $|\mathcal{S}|$. The optimal values of $p_{\mathrm{d}}(\mathrm{T}_i)$ are given by

$$p_{\mathrm{d}}^{(n)}(\mathrm{T}_i) = \begin{cases} 1, & \text{if } i \in \mathcal{S} \\ \dfrac{\sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} w_{\ell ji}^{(n-1)} (|\mathcal{S}| + N_{\mathrm{fa}})}{\sum_{k>0, k \in \mathcal{S}} \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} w_{\ell jk}^{(n-1)} + \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} w_{\ell j0}^{(n-1)}}, & \text{if } i \notin \mathcal{S} \end{cases}. \tag{58}$$

Therefore, by (25)

$$\pi_i^{(n)} = \frac{p_{\mathrm{d}}^{(n)}(\mathrm{T}_i)}{\sum_{i=0}^{N} p_{\mathrm{d}}^{(n)}(\mathrm{T}_i)} \tag{59}$$

which can be verified to be identical to (51) when the set $\mathcal{S}$ is empty.

The EM algorithm is terminated when the likelihood function (41) converges, that is,

$$\left| \ln p\left(\mathbf{Z} | \mathbf{T}^{(n)}, \mathbf{t}^{\mathrm{e}(n)}\right) - \ln p\left(\mathbf{Z} \,\middle|\, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right) \right| \leq \epsilon \tag{60}$$

where ϵ is a small number (e.g., 10^{-3}).

One can use a fixed $\boldsymbol{\pi}$ throughout the EM iterations and skip the update process in (51) under the assumption

that both the detection probabilities and the false alarm density are known.

D. Use of the Information Criterion for Cardinality Selection

So far, we have assumed that the number of targets is given. Since the number of targets is unknown, we can use the above described procedure to estimate the parameters $\boldsymbol{\pi}$ and $\mathbf{T}$ given a specific cardinality, i.e., the number of targets N. Now we are faced with a cardinality selection problem, or a model selection problem where the dimensionality of a model is the number of targets. One of the most widely used criteria for model selection problems is the Bayesian information criterion (BIC) [22].

Let $M^k = \{\hat{\boldsymbol{\pi}}^k, \hat{\mathbf{T}}^k, \hat{N}^k\}$ denote the set of estimated parameters based on the kth cardinality. According to BIC, we choose the model for which the following is largest:

$$\ln p\left(\mathbf{Z}\,|M^k\right) - \frac{1}{2}d^k \ln(N_{\mathbf{z}}) \tag{61}$$

where from (41)

$$p\left(\mathbf{Z}\,|M^k\right) = \prod_{\ell=1}^{N_{\mathrm{s}}} \prod_{j=1}^{n_\ell} \left(\sum_{i=0}^{\hat{N}^k} \hat{\pi}_i^k \, g_{\ell i}(\mathbf{z}_{\ell j}|\hat{\mathbf{T}}^k) \right) \tag{62}$$

and $N_{\mathbf{z}}$ is the total number of measurements across all the sensors; d^k is the total number of parameters to be estimated based on the kth cardinality. In our case, d^k is $4\hat{N}^k + 1$ ($2\hat{N}^k$ position coordinates for a problem in 2-D, $\hat{N}^k$ emission times, $\hat{N}^k + 1$ UGM coefficients including the expected number of false alarms) if it is assumed the detection probabilities and the false alarm density are unknown. If the detection probabilities and the false alarm density are assumed to be known, then d^k is $3\hat{N}^k$.

V. PPP MODEL

A. Formulation

Assume the number of targets, N, is given. Let $\mathbf{w} = \{w_i, i = 1, \ldots, N\}$, where

$$w_i = p_{\mathrm{d}}(\mathrm{T}_i) \tag{63}$$

are the detection probabilities. The number of measurements n_ℓ and $\{\mathbf{z}_{\ell j},\, j = 1, 2, \ldots, n_\ell\}$ obtained at the ℓth sensor are jointly modeled as a realization of a PPP. The measurement set at the ℓth sensor is denoted as

$$\boldsymbol{\psi}_\ell = \left\{n_\ell,\, \mathbf{z}_{\ell 1},\, \mathbf{z}_{\ell 2},\, \ldots,\, \mathbf{z}_{\ell n_\ell}\right\}. \tag{64}$$

In this case, the points $\mathbf{z}_{\ell j}$ occur in the space $\mathbb{S} = \{(\theta, t) : \theta \in [-\pi, \pi), t \in [0, W]\}$ and their order is irrelevant. The PPP is fully parameterized by its spatial intensity function

$$\mu_\ell(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \sum_{i=0}^{N} p_{\mathrm{d}}(\mathrm{T}_i)\, g_{\ell i}(\mathbf{z}) \tag{65}$$

where, similarly to (26),

$$p_{\mathrm{d}}\left(\mathrm{T}_0\right) = N_{\mathrm{fa}}. \tag{66}$$

The number of points in the PPP is a Poisson random variable with rate $\int_{\mathbb{S}} \mu_\ell(\mathbf{z})d\mathbf{z}$; that is, the probability mass function of n_ℓ is

$$p(n_\ell) = \frac{\left(\int_{\mathbb{S}} \mu_\ell(\mathbf{z})d\mathbf{z}\right)^{n_\ell}}{n_\ell\,!} \exp\left\{ -\int_{\mathbb{S}} \mu_\ell(\mathbf{z})d\mathbf{z} \right\}. \tag{67}$$

The n_ℓ points are defined as independent and identically distributed (i.i.d.) samples of a random variable with probability density function

$$p(\mathbf{z}) = \frac{\mu_\ell(\mathbf{z})}{\int_{\mathbb{S}} \mu_\ell(\mathbf{z})d\mathbf{z}} = \frac{\sum_{i=0}^{N} p_{\mathrm{d}}(\mathrm{T}_i)\, g_{\ell i}(\mathbf{z})}{\sum_{i=0}^{N} p_{\mathrm{d}}(\mathrm{T}_i)}. \tag{68}$$

The joint pmf–pdf of $\boldsymbol{\psi}^\ell$ is

$$p\left(\boldsymbol{\psi}^\ell\right) = \exp\left(-\int_{\mathbb{S}} \mu_\ell(\mathbf{z}\,|\mathbf{T}, \mathbf{t}^{\mathrm{e}})\, d\mathbf{z} \right) \prod_{j=1}^{n_\ell} \mu_\ell(\mathbf{z}_{\ell j}\,|\mathbf{T}, \mathbf{t}^{\mathrm{e}}). \tag{69}$$

The factorial term $n_\ell\,!$ in (67) is canceled out because there are $n_\ell\,!$ permutations of an ordered list of measurements.

Let $\boldsymbol{\Psi}$ denote the set of all measurement sets (from the N_{s} sensors), i.e.,

$$\boldsymbol{\Psi} = \left\{\boldsymbol{\psi}_1,\, \boldsymbol{\psi}_2,\, \ldots,\, \boldsymbol{\psi}_{N_{\mathrm{s}}}\right\}. \tag{70}$$

The independence of the N_{s} measurement sets yields

$$p\left(\boldsymbol{\Psi}\,|\mathbf{T}, \mathbf{t}^{\mathrm{e}}\right) = \prod_{\ell=1}^{N_{\mathrm{s}}} p\left(\boldsymbol{\psi}_\ell\,|\mathbf{T}, \mathbf{t}^{\mathrm{e}}\right). \tag{71}$$

Since the intensity function is a mixture of uniform or Gaussian pdf and the association is unknown, we model the latent association variables as conditionally independent random variables

$$\kappa_{\ell j} \in \{0, 1, 2, \ldots, N\} \tag{72}$$

that identify which component spawned the jth measurement in the ℓth sensor. Here, $\kappa_{\ell j} = 0$ indicates that the measurement is generated by the clutter. The set of latent variables for the ℓth sensor is denoted as

$$\boldsymbol{\kappa}_\ell = \{\kappa_{\ell 1}, \ldots, \kappa_{\ell n_\ell}\} \tag{73}$$

such that the full set is

$$\boldsymbol{\kappa} = \{\boldsymbol{\kappa}_1, \ldots, \boldsymbol{\kappa}_{N_{\mathrm{s}}}\}. \tag{74}$$

The latent association variables may be regarded as "marks" associated with each of the points in the PPP. Define a mark space

$$M \triangleq \{0, 1, 2, \ldots, N\}. \tag{75}$$

Now

$$\boldsymbol{\psi}_\ell^{\mathrm{M}} = \left\{n_\ell, (\mathbf{z}_{\ell 1}, \kappa_{\ell 1}), \ldots, (\mathbf{z}_{\ell n_\ell}, \kappa_{\ell n_\ell})\right\} \tag{76}$$

denotes a realization of the marked PPP for the ℓth sensor, where "M" indicates that the associations are known

("marked"). Based on the marking theorem [17], the intensity function of $\boldsymbol{\psi}_\ell^{\mathrm{M}}$ is

$$\mu_\ell^{\mathrm{M}}(\mathbf{z}, \kappa|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \mu_\ell(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) p_\ell(\kappa|\mathbf{z}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) \tag{77}$$

where $p_\ell(\kappa|\mathbf{z}, \mathbf{T}, \mathbf{t}^{\mathrm{e}})$ denotes the conditional probability of κ given $\mathbf{z}$ (without subscript here, for simplicity) with $\kappa = i$ indicating the probability of $\mathbf{z}$ originating from target i. Using the same reasoning as for the derivation of (25), we have the prior probability of κ

$$p_\ell(\kappa|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \frac{w_\kappa}{\sum_{i=0}^{N} w_i}. \tag{78}$$

Given that a point $\mathbf{z}$ in the PPP is associated with the κth mixture component, the conditional intensity becomes

$$\mu_\ell(\mathbf{z}|\kappa) = w_\kappa \, g_{\ell\kappa}(\mathbf{z}) \tag{79}$$

and the conditional density of $\mathbf{z}$ given κ is

$$p_\ell(\mathbf{z}|\kappa, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = g_{\ell\kappa}(\mathbf{z}). \tag{80}$$

Using Bayes' theorem

$$p_\ell(\kappa|\mathbf{z}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \frac{p_\ell(\mathbf{z}|\kappa, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) p_\ell(\kappa|\mathbf{T}, \mathbf{t}^{\mathrm{e}})}{p_\ell(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})} = \frac{w_\kappa g_{\ell\kappa}(\mathbf{z})}{\mu_\ell(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})}. \tag{81}$$

Substituting (81) into (77) yields

$$\mu_\ell^{\mathrm{M}}(\mathbf{z}, \kappa|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) = w_\kappa g_{\ell\kappa}(\mathbf{z}). \tag{82}$$

The joint probability density function of $\boldsymbol{\psi}_\ell^{\mathrm{M}}$ is, similarly to (69), given by

$$p\left(\boldsymbol{\psi}_\ell^{\mathrm{M}}\right) = \exp\left(\sum_{\kappa=0}^{N} - \int_{\mathbb{S}} \mu_\ell^{\mathrm{M}}(\mathbf{z}, \kappa|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) \, d\mathbf{z}\right) \cdot \prod_{j=1}^{n_\ell} \mu_\ell^{\mathrm{M}}(\mathbf{z}_{\ell j}, \kappa_{\ell j}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}). \tag{83}$$

Now let the complete data from (76) be

$$\boldsymbol{\Psi}^{\mathrm{M}} = \left\{\boldsymbol{\psi}_1^{\mathrm{M}}, \, \boldsymbol{\psi}_2^{\mathrm{M}}, \, \ldots, \, \boldsymbol{\psi}_{N_{\mathrm{s}}}^{\mathrm{M}}\right\}. \tag{84}$$

The conditional independence of the N_{s} measurement sets yields the pmf–pdf for the complete data

$$p\left(\boldsymbol{\Psi}^{\mathrm{M}}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}\right) = \exp\left(-N_{\mathrm{s}} \sum_{i=0}^{N} w_i\right) \cdot \prod_{\ell=1}^{N_{\mathrm{s}}} \prod_{j=1}^{n_\ell} w_{\kappa_{\ell j}} g_{\ell\kappa_{\ell j}}(\mathbf{z}_{\ell j}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) \tag{85}$$

where we have used the fact

$$\sum_{\kappa=0}^{N} \left(\int w_\kappa g_{\ell\kappa}(\mathbf{z}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}) \, d\mathbf{z}\right) = \sum_{i=0}^{N} w_i. \tag{86}$$

Dividing (85) by (71) leads to the density of the marks conditioned on the observed measurements and the unknown parameters

$$p(\boldsymbol{\kappa}|\mathbf{Z}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \prod_{\ell=1}^{N_{\mathrm{s}}} \prod_{j=1}^{n_\ell} p_\ell(\kappa_{\ell j} \,|\, \mathbf{z}_{\ell j}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) \tag{87}$$

where

$$p_\ell(\kappa_{\ell j} \,|\, \mathbf{z}_{\ell j}, \mathbf{T}, \mathbf{t}^{\mathrm{e}}) = \frac{w_{\kappa_{\ell j}} g_{\ell\kappa_{\ell j}}(\mathbf{z}_{\ell j}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})}{\mu_\ell(\mathbf{z}_{\ell j}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})}. \tag{88}$$

In this PPP formulation, the goal is find the ML estimate of $\mathbf{T}$ by maximizing (71), which can also be solved using the EM algorithm.

B. Optimization

1) Initialization: In this paper, three initialization approaches are considered and will be discussed in Section VI.D.

2) E Step: Let $\mathbf{w}^{(n-1)}$, $\mathbf{T}^{(n-1)}$, and $\mathbf{t}^{\mathrm{e},(n-1)}$ denote the estimates from the previous step. In the expectation step, we use them to find $p(\boldsymbol{\kappa}|\mathbf{Z}, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$ and compute the expected value of $p\left(\boldsymbol{\Psi}^{\mathrm{M}}|\mathbf{T}, \mathbf{t}^{\mathrm{e}}\right)$ conditioned on $p(\boldsymbol{\kappa}|\mathbf{Z}, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})$; that is, we evaluate

$$\begin{aligned} &Q(\mathbf{T}, \mathbf{t}^{\mathrm{e}}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \\ &\quad = \mathbb{E}\left[\ln\left(p(\boldsymbol{\Psi}^{\mathrm{M}}|\mathbf{T}, \mathbf{t}^{\mathrm{e}})\right) \left| p(\boldsymbol{\kappa}\,|\mathbf{Z}, \, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})\right.\right] \\ &\quad = Q_w(\mathbf{w}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) + Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \end{aligned} \tag{89}$$

where

$$Q_w(\mathbf{w}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = -N_{\mathrm{s}} \sum_{i=0}^{N} w_i + \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \sum_{i=0}^{N} \ln(w_i) \, \alpha_{\ell ji}^{(n-1)} \tag{90}$$

$$Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \sum_{i=0}^{N} \ln\left(g_{\ell i}(\mathbf{z}_{\ell j})\right) \alpha_{\ell ji}^{(n-1)} \tag{91}$$

and

$$\begin{aligned} \alpha_{\ell ji}^{(n-1)} &= p_\ell(\kappa_{\ell j} = i \,|\, \mathbf{z}_{\ell j}, \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \\ &= \frac{w_i^{(n-1)} g_{\ell i}^{(n-1)}(\mathbf{z}_{\ell j})}{\sum_{i=0}^{N} w_i^{(n-1)} g_{\ell i}^{(n-1)}(\mathbf{z}_{\ell j})}. \end{aligned} \tag{92}$$

The weight $\alpha_{\ell ji}^{(n-1)}$ is the probability that the point $\mathbf{z}_{\ell j}$ is generated by the ith target given $\mathbf{T}^{(n-1)}$ and $\mathbf{t}^{\mathrm{e},(n-1)}$.

3) M Step: The M step maximizes $Q\left(\mathbf{T}, \mathbf{t}^{\mathrm{e}}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right)$ over all feasible values for $\mathbf{T}$ and $\mathbf{t}^{\mathrm{e}}$. Inspection of (90) reveals that $Q_w\left(\mathbf{w}\,|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right)$ depends only on the values of $p_{\mathrm{d}}(T_i)$ because $w_i = p_{\mathrm{d}}(T_i)$ for $i = 1, \ldots, N$. Likewise, $Q_T\left(\mathbf{T}, \mathbf{t}^{\mathrm{e}}|\mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right)$ depends only on the target locations and emission times through $g_{\ell i}(\mathbf{z}_{\ell j})$.

Therefore, maximization of $Q\left(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right)$ is accomplished by maximizing $Q_w\left(\mathbf{w} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right)$ and $Q_T\left(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right)$ separately.

The value of $Q_T\left(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}\right)$ in (91) decomposes as

$$Q_T(\mathbf{T}, \mathbf{t}^{\mathrm{e}} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{i=0}^{N} Q_{T_i}(T_i | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) \tag{93}$$

where

$$Q_{T_i}(T_i | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)}) = \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \ln\left(g_{\ell i}\left(\mathbf{z}_{\ell j}\right)\right) \alpha_{\ell j i}^{(n-1)}. \tag{94}$$

For $i = 0$, $Q_{T_i}\left(T_i | \mathbf{T}^{(n-1)}\right)$ is constant with respect to T_i since the density is assumed known for the clutter. When $i \neq 0$, $g_{\ell i}\left(\mathbf{z}_{\ell j}\right)$ depends only on T_i through $\mathbf{h}_\ell(T_i, t_i^{\mathrm{e}})$; thus, $T_i^{(n)}$ is determined by maximizing (94) separately for each value of i.

The values of $w_i^{(n)}$ are determined by maximizing (90) given the fact that $p_{\mathrm{d}}(T_i) = w_i$ for $k = 1, \ldots, N$ and the assumption that they are scalar quantities. The detection probabilities are also constrained to less than or equal to 1. By setting up the Lagrange multipliers, it is easy to see that the KKT conditions are satisfied when

$$w_i{}^{(n)} = \min\left\{1, \frac{1}{N_{\mathrm{s}}} \sum_{\ell=1}^{N_{\mathrm{s}}} \sum_{j=1}^{n_\ell} \alpha_{\ell j i}^{(n-1)}\right\}. \tag{95}$$

The EM algorithm is terminated when the likelihood function (71) converges, that is,

$$|\ln p(\mathbf{\Psi} | \mathbf{T}^{(n)}, \mathbf{t}^{\mathrm{e}(n)}) - \ln p(\mathbf{\Psi} | \mathbf{T}^{(n-1)}, \mathbf{t}^{\mathrm{e},(n-1)})| \leq \epsilon. \tag{96}$$

One can use a fixed $\mathbf{w}$ throughout the EM iterations and skip the update process in (95) under the assumption that both the detection probabilities and the false alarm density are known.

C. Use of the Information Criterion for Cardinality Selection

The EM algorithm will eventually converge to $\hat{\mathbf{w}}^k$ and $\hat{\mathbf{T}}^k$ given the number of targets $\hat{N}^k$. Let the set $M_p^k = \{\hat{\mathbf{w}}^k, \hat{\mathbf{T}}^k, \hat{N}^k\}$ denote the estimation result based on the kth cardinality. The BIC selects the set M_p^k that minimizes

$$-2 \ln p\left(\mathbf{\Psi} \left| M_p^k\right.\right) + d \ln N_{\mathbf{z}} \tag{97}$$

where from (71)

$$p\left(\mathbf{\Psi} | M_p^k\right) = \exp\left(-N_{\mathrm{s}} \sum_{i=0}^{\hat{N}^k} w_i^k\right) \prod_{\ell=1}^{N_{\mathrm{s}}} \prod_{j=1}^{n_\ell} \sum_{i=0}^{\hat{N}^k} w_i^k \, g_{\ell i}(\mathbf{z}_{\ell j} | \hat{\mathbf{T}}^k) \tag{98}$$

and d is the total number of parameters to be estimated.

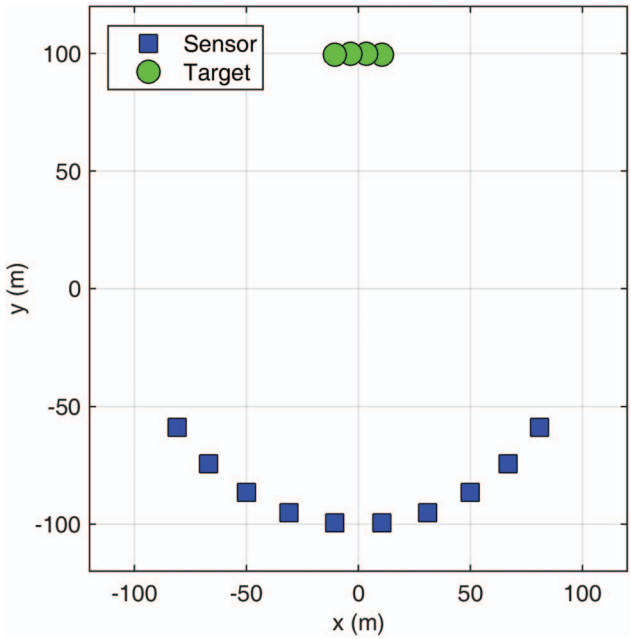


Fig. 3. Overhead view of a ten-sensor four-target scenario.

VI. SIMULATION RESULTS

A. Scenario

Assume there are four targets ($N = 4$). The emission times of the acoustic events at the target locations are 0.2, 0.25, 0.3, and 0.35 s, respectively. The speed of the acoustic signal is assumed to be 342 m/s. The measurement noise covariance matrix is

$$R_\ell = \begin{bmatrix} 7.6 \times 10^{-5} & 0 \\ 0 & 2.5 \times 10^{-5} \end{bmatrix} \tag{99}$$

i.e., the bearing standard deviation amounts to $\sqrt{76}$ mrad $= 0.5^{\mathrm{o}}$ and the time of arrival measurement standard deviation amounts to 5 ms, assumed to be the same for all targets. The time window W is chosen to be 1 s and the density of the false alarms is set to be 0.32 $\mathrm{s}^{-1}\mathrm{rad}^{-1}$ such that the expected number of false alarms at each sensor is 1. The field view of each sensor is from 0 to π. Fig. 3 shows one example using ten sensors to localize these four targets. In the simulation, the targets and the sensors are arranged in the way such that the angle between two LOS from two neighboring targets to any sensor is 2^{o}, which is four times the standard deviation of LOS measurement noise.

B. Performance Metrics

The performance metrics of interest for N_{MC} Monte Carlo runs include

1) φ_{over}: fraction of N_{MC} runs for which N (the number of targets) has been overestimated;
2) $\bar{M}_{\mathrm{over}}$: average magnitude of estimation error for N from $N_{\mathrm{MC}}\varphi_{\mathrm{over}}$ runs;

TABLE II
SEQ[m(2-D)] Assignment Performance Using Different m for Known p_d

p_d	0.7			0.8			0.9		
m	1	2	4	1	2	4	1	2	4
φ_{over}	1.6%	1.2%	0.9%	0.9%	0.8%	0.7%	0.6%	0.6%	0.4%
$\bar{M}_{over}$	1	1	1	1	1	1	1	1	1
T_{over}^{RMSE} (m)	2.86	5.13	3.01	1.97	1.63	1.72	2.19	2.19	1.98
θ_{over}^{RMSE} (°)	0.289	0.460	0.298	0.255	0.251	0.262	0.272	0.272	0.274
φ_{under}	3.2%	2.3%	2.1%	0.2%	0	0	0	0	0
$\bar{M}_{under}$	1	1	1	1.5	N.A.	N.A.	N.A.	N.A.	N.A.
T_{under}^{RMSE} (m)	2.58	2.69	2.69	4.50	N.A.	N.A.	N.A.	N.A.	N.A.
θ_{under}^{RMSE} (°)	0.296	0.299	0.298	0.411	N.A.	N.A.	N.A.	N.A.	N.A.
φ_{exact}	95.2%	96.5%	97.0%	98.9%	99.2%	99.3%	99.4%	99.4%	99.6%
T_{exact}^{RMSE} (m)	2.92	2.81	2.76	3.23	2.59	2.59	1.99	1.96	1.96
θ_{exact}^{RMSE} (°)	0.329	0.320	0.313	0.329	0.282	0.282	0.239	0.236	0.236
T_{all}^{RMSE} (m)	2.91	2.85	2.76	3.23	2.58	2.58	1.99	1.96	1.96
θ_{all}^{RMSE} (°)	0.328	0.322	0.313	0.328	0.282	0.282	0.239	0.236	0.236
t (s)	0.068	0.142	0.310	0.077	0.166	0.353	0.084	0.186	0.394

3) T_{over}^{RMSE}: root mean squared error (RMSE) of the target location estimate from $N_{MC}\varphi_{over}$ runs;
4) θ_{over}^{RMSE}: RMSE of the bearing estimate from $N_{MC}\varphi_{over}$ runs;
5) φ_{under}: fraction of N_{MC} runs for which N has been underestimated;
6) $\bar{M}_{under}$: average magnitude of estimation error for N from $N_{MC}\varphi_{under}$ runs;
7) T_{under}^{RMSE}: RMSE of the target location estimate from $N_{MC}\varphi_{under}$ runs;
8) θ_{under}^{RMSE}: RMSE of the bearing estimate from $N_{MC}\varphi_{under}$ runs;
9) φ_{exact}: fraction of N_{MC} runs for which N has been correctly estimated;
10) T_{exact}^{RMSE}: RMSE of the target location estimate from $N_{MC}\varphi_{exact}$ runs;
11) θ_{exact}^{RMSE}: RMSE of the bearing estimate from $N_{MC}\varphi_{exact}$ runs;
12) T_{all}^{RMSE}: RMSE of the target location estimate from all N_{MC} runs;
13) θ_{all}^{RMSE}: RMSE of the bearing estimate from all N_{MC} runs.
14) t: average processing time in a single run.

For all the simulations in this paper, $N_{MC} = 1000$ unless otherwise specified. In the overestimation cases, RMSE is calculated by mapping the best (yielding the minimum RMSE) subset of estimated targets to true targets. In the underestimation cases, RMSE is calculated by mapping the estimated targets to the best subset of true targets. The bearing estimate is examined here, because in some applications (for instance, shooter localization), bearing accuracy is more critical than location accuracy.

C. Assignment Algorithms

The multidimensional assignment problem (17), which is solved by the two assignment algorithms—the S-D assignment algorithm and the SEQ[m(2-D)] algorithm, assumes a Bernoulli measurement model that the number of measurements from a real target received by a sensor is a Bernoulli random variable whose parameter equal to the probability of detection p_d. For the evaluation of these two assignment algorithms in this section, the target measurements are generated according to this Bernoulli measurement model; specifically, one measurement from each target is generated for each sensor with a probability p_d or nothing with a probability $1 - p_d$. The false alarms are generated for each sensor according to the Poisson model (6) and (7).

Note that the values of the probability of detection, p_d, and the expected number of false alarms, N_{fa}, are required to generate the target measurements. The assignment algorithms do not need to know the value of N_{fa} but need to know the value of p_d. However, the assignment algorithms are shown to be robust to incorrect p_d (see Table IV).

Table II shows the effects of the algorithm parameter m, the number of best assignments to be kept at each step, and p_d, probability of detection, on the performance of the SEQ[m(2-D)] assignment algorithm in a scenario using ten sensors to locate four targets. The true probability of detection is assumed to be known. Once the assignment algorithm is finished, any association with less than three ($\tau = 3$) real measurements is discarded. For each p_d, keeping more (larger m) top assignments at each 2-D step makes it more likely to find the best assignment; therefore, a larger m gives better estimates for the number of targets, the locations of the targets, and the directions to the targets (for counterfire). In fact, if the association between the measurements and

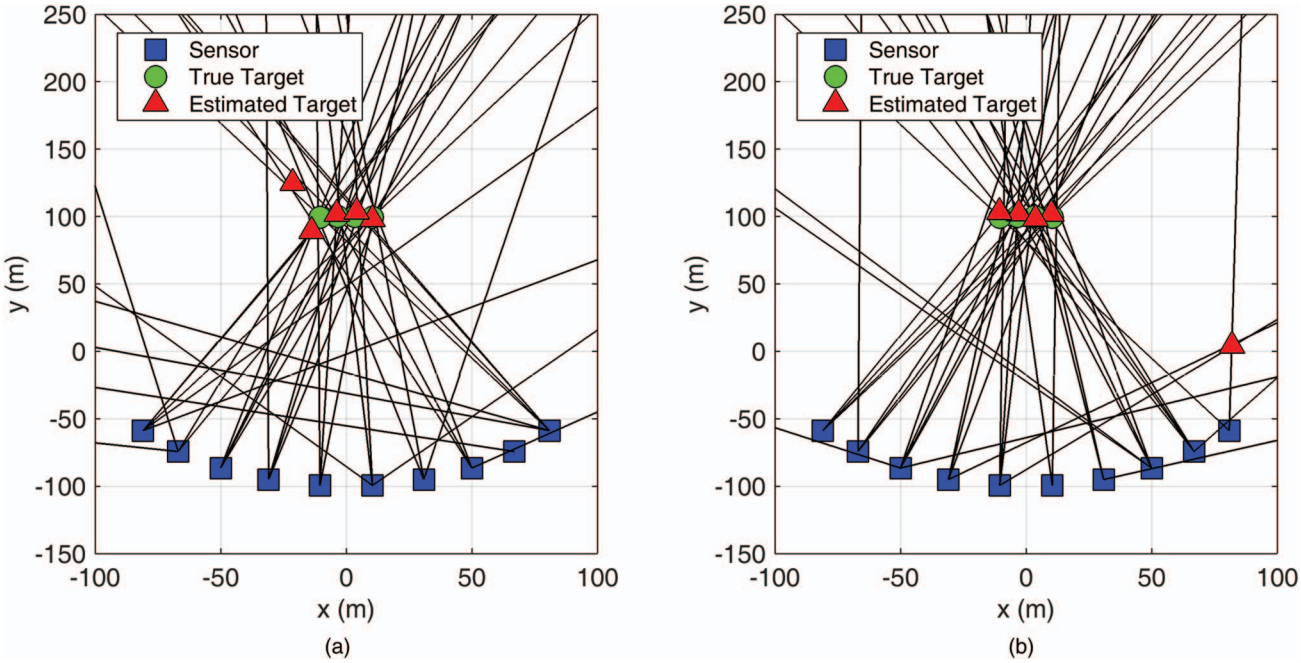


Fig. 4. Two cardinality overestimation situations.

the targets is known, the values of $\theta_{\text{all}}^{\text{RMSE}}$ would be 2.50, 2.15, and 1.91 m in the three scenarios with p_{d} varying from 0.7 to 0.9. The sequential algorithm with $m = 4$ yields very good association accuracy since the values of $\theta_{\text{all}}^{\text{RMSE}}$ are very close to these lower bounds. The performance gain is at the expense of a higher computational cost, which, however, is acceptable for real-time applications. For a fixed value of m, the algorithm performs better when the probability of detection increases. Lower p_{d} makes it more likely for the final association to have fewer real measurements, which could fail in the real threshold test; therefore, there are more cases when the number of targets is underestimated at $p_{\text{d}} = 0.7$. There are two situations when the number of targets is overestimated. The first one is "target splitting"; that is, one real target is perceived as two (or more) targets that are close to each other as shown in Fig. 4a, where the targets at location $(-10, 99)$ are split into the targets at locations $(-14, 89)$ and $(-21, 125)$. The second one is when three (or more) false measurements are perceived to be from a real target as shown in Fig. 4b at location $(82, 4)$. Both situations are more likely to occur at a lower p_{d}, so there are slightly fewer overestimation cases as p_{d} increases, as shown in Table II. When p_{d} is higher, there could be more real measurements available, which, if correctly associated, can lead to more accurate location and direction estimates. This is also at the expense of a slightly longer processing time.

Table III shows how the performance of the SEQ[m(2-D)] assignment algorithm with $m = 4$ varies with the number of sensors for three levels of known p_{d} in the scenario with four real targets. For the threshold test, any associations with less than three real measurements are discarded. For each p_{d}, it is observed that using a smaller number of sensors leads to more cardinality underestimation cases and fewer overestimation cases. This is so because the chance that only one sensor detects a real target is higher when a smaller number of sensors are used. On the other hand, a larger number of sensors in the presence of false alarms make it more likely to associate false measurements into a ghost like the situation in Fig. 4b. However, with a larger number of sensors, the decrease in the occurrence of overestimation cases is more significant than the increase in the underestimation cases. In addition, deploying more sensors could give rise to more measurements for a target, which in return generate more accurate location and direction estimates. In general, it is more beneficial to use a larger number of sensors.

The calculation of the cost (11) requires the knowledge of p_{d}. In practical scenarios, the actual value of p_{d} may not be available, in which case one has to use an estimated value of p_{d}. Table IV shows that the performance of the SEQ[m(2-D)] assignment algorithm is almost insensitive to the mismatch between the assumed value and the true value of p_{d} when they are close (up to 0.1 difference) in the scenario where ten sensors are used to locate four targets.

Fig. 5 shows the performance of the SEQ[m(2-D)] assignment algorithm for a wider range of true p_{d}. The quality of cardinality, location, and bearing estimates is almost independent of the assumed p_{d} value when the true p_{d} is from 0.6 to 0.9. When the true p_{d} is as low as 0.5, the quality of those estimates will vary with the assumed p_{d} value. A good location or bearing estimate requires at least three real measurements to be associated correctly; however, the probability that at least three out of ten sensors have detected the same target is only around

TABLE III
SEQ[m(2-D)] Assignment Performance Using Different N_s for Known p_d With $m = 4$

p_d	0.7			0.8			0.9		
N_s	6	8	10	6	8	10	6	8	10
φ_{over}	0.1%	0.4%	0.9%	0.2%	0.6%	0.7%	0.4%	0.5%	0.4%
$\bar{M}_{over}$	1	1	1	1	1	1	1	1	1
T_{over}^{RMSE} (m)	4.37	4.50	3.01	3.90	2.87	1.72	4.21	2.98	1.98
θ_{over}^{RMSE} (°)	0.453	0.386	0.298	0.361	0.294	0.262	0.302	0.289	0.274
φ_{under}	28.2%	7.8%	2.1%	7.8%	1%	0	0.4%	0	0
$\bar{M}_{under}$	1.12	1.01	1	1.04	1	N.A.	1	N.A.	N.A.
T_{under}^{RMSE} (m)	5.95	3.89	2.69	4.93	4.11	N.A.	4.64	N.A.	N.A.
θ_{under}^{RMSE} (°)	0.402	0.344	0.298	0.347	0.328	N.A.	0.330	N.A.	N.A.
φ_{exact}	71.7%	91.8%	97.0%	92%	98.4%	99.3%	99.2%	99.5%	99.6%
T_{exact}^{RMSE} (m)	7.26	4.37	2.76	5.66	3.41	2.59	4.42	2.76	1.96
θ_{exact}^{RMSE} (°)	1.03	0.442	0.313	0.369	0.310	0.282	0.314	0.266	0.236
T_{all}^{RMSE} (m)	6.99	4.35	2.76	5.62	3.42	2.58	4.42	2.76	1.96
θ_{all}^{RMSE} (°)	0.931	0.437	0.313	0.368	0.310	0.282	0.314	0.266	0.236
t (s)	0.132	0.212	0.310	0.158	0.274	0.353	0.183	0.277	0.394

TABLE IV
SEQ[m(2-D)] Assignment Performance Using Different Assumed Values for Unknown p_d

True p_d	0.7			0.8			0.9		
Assumed p_d	0.6	0.7	0.8	0.7	0.8	0.9	0.8	0.9	0.95
φ_{over}	1.0%	0.9%	0.8%	0.8%	0.7%	0.5%	0.5%	0.4%	0.2%
$\bar{M}_{over}$	1	1	1	1	1	1	1	1	1
T_{over}^{RMSE} (m)	3.12	3.01	2.14	2.40	1.72	1.71	1.97	1.98	2.12
θ_{over}^{RMSE} (°)	0.314	0.298	0.252	0.315	0.262	0.249	0.263	0.274	0.270
φ_{under}	1.7%	2.1%	2.2%	0	0	0.1%	0	0	0
$\bar{M}_{under}$	1	1	1	N.A.	N.A.	1	N.A.	N.A.	N.A.
T_{under}^{RMSE} (m)	2.80	2.69	2.79	N.A.	N.A.	2.91	N.A.	N.A.	N.A.
θ_{under}^{RMSE} (°)	0.311	0.298	0.317	N.A.	N.A.	0.325	N.A.	N.A.	N.A.
φ_{exact}	97.3%	97.0%	97.0%	99.2%	99.3%	99.4%	99.5%	99.6%	99.8%
T_{exact}^{RMSE} (m)	2.76	2.76	2.80	2.60	2.59	2.60	1.96	1.96	1.96
θ_{exact}^{RMSE} (°)	0.313	0.313	0.318	0.284	0.282	0.282	0.236	0.236	0.236
T_{all}^{RMSE} (m)	2.76	2.76	2.80	2.60	2.58	2.59	1.96	1.96	1.96
θ_{all}^{RMSE} (°)	0.313	0.313	0.317	0.284	0.282	0.282	0.236	0.236	0.236
t (s)	0.307	0.310	0.316	0.353	0.353	0.350	0.394	0.394	0.392

0.80 at true $p_d = 0.5$. In other words, at true $p_d = 0.5$ in about 200 simulation runs either there is at least one target missing or there is at least a false association, which could cause very different performances at different assumed p_d values. It gets worse at lower p_d values such as 0.4 and 0.3. Therefore, given a fixed number of sensors, there is a lower bound on the true p_d, below which it is very difficult to achieve good location and bearing estimates.

Fig. 6 shows the dependence of the performance of the SEQ[m(2-D)] assignment algorithm on the expected number of false alarms per sensor when ten sensors are used to localize four targets with known p_d values at 0.7, 0.8, and 0.9. In the final assignment, the least number of real measurements required to be associated with a real target is 3. As expected, the performance gets worse when the false alarm rate is higher. When the true p_d is 0.8 or 0.9, the localization results are reliable even for $N_{fa} = 4$ (the total expected number of false alarms is larger than the total expected number of real measurements). When the expected number of false alarms per sensor is very large ($N_{fa} = 8$), there are more false targets in the final assignment, leading to a worse performance. Setting a higher number for the required minimum number of real measurements reduces the number of false targets but will also miss real targets in a scenario, especially for a low p_d value.

Fig. 7a shows the processing time (averaged over 100 Monte Carlo runs) of the S-D assignment algorithm and the SEQ[m(2-D)] ($m = 4$) assignment algorithm in

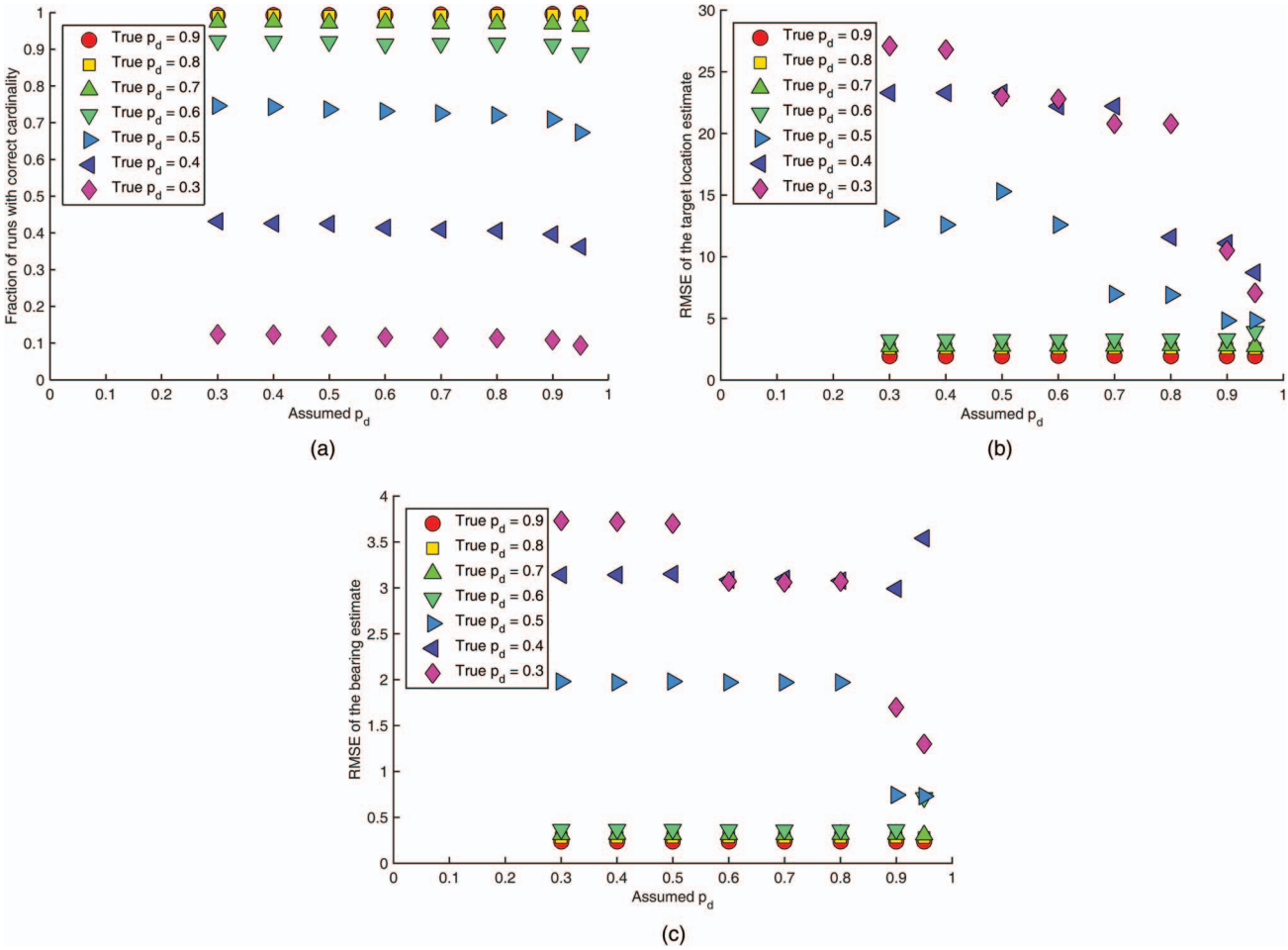


Fig. 5. The performance (in terms of φ_{exact}, $T_{\text{all}}^{\text{RMSE}}$, and $\theta_{\text{all}}^{\text{RMSE}}$) of the SEQ[$m$(2-D)] assignment algorithm using different assumed values of unknown p_{d} for true p_{d} values ranging from 0.3 to 0.9.

scenarios with the probability of detection $p_{\text{d}} = 0.9$ and the expected number of false alarms (per sensor) $N_{\text{fa}} = 1$. When the number of targets, N, is 4, the processing time using the S-D assignment algorithm with seven sensors is around 20 s, which is too long as far as real-time applications are concerned. This is due to the fact that the processing time of the S-D assignment algorithm scales exponentially with the number of sensors while the sequential processing time scales quadratically with the number of sensors in the worst case. In terms of the localization performance, the S-D assignment algorithm is also shown to be inferior to the sequential m-best assignment algorithm.

We must note that both algorithms are very different suboptimal solutions to the problem (17). The sequential m-best assignment algorithm makes efficient use of the modified auction algorithm in a sequential manner. As a greedy algorithm, it solves a locally optimal solution based on two lists of measurements followed by considering one more list of measurements at a time until a final solution is obtained for all lists of measurements. It uses a suitably large value of the parameter m with the hope that the optimal solution is kept in the search space at all times and the final solution is close to optimal.

In contrast, the S-D assignment algorithm is a much more sophisticated, iterative technique as shown in Fig. 1. However, it was shown in [8] that the S-D assignment algorithm could be underperforming in challenging localization scenarios where the association graph is strongly connected and the number of candidate associations is huge. In a scenario with $p_{\text{d}} = 0.9$ where five targets and seven sensors are placed such that the worst angular intertarget separations as seen by the seven sensors are 2, 3.5, 5, 6.5, 5, 3.5, and 2 standard deviations of the bearing measurement noise, the association accuracy of the S-D assignment algorithm is shown to be 74%. In our simulation scenario, the worst angular intertarget separation is four standard deviations for every sensor, which poses a similar challenging situation in terms of the density of the association graph (or the number of candidate associations). Therefore, the somewhat inferior localization performance of the S-D assignment algorithm at $p_{\text{d}} = 0.9$ is not unexpected.

As suggested in [8], the algorithmic parameters are selected such that the algorithm is terminated if the relative approximate duality gap is less than 5% or the number of iterations exceeds 100. Although the chance is very small, it is still possible that the algorithm already

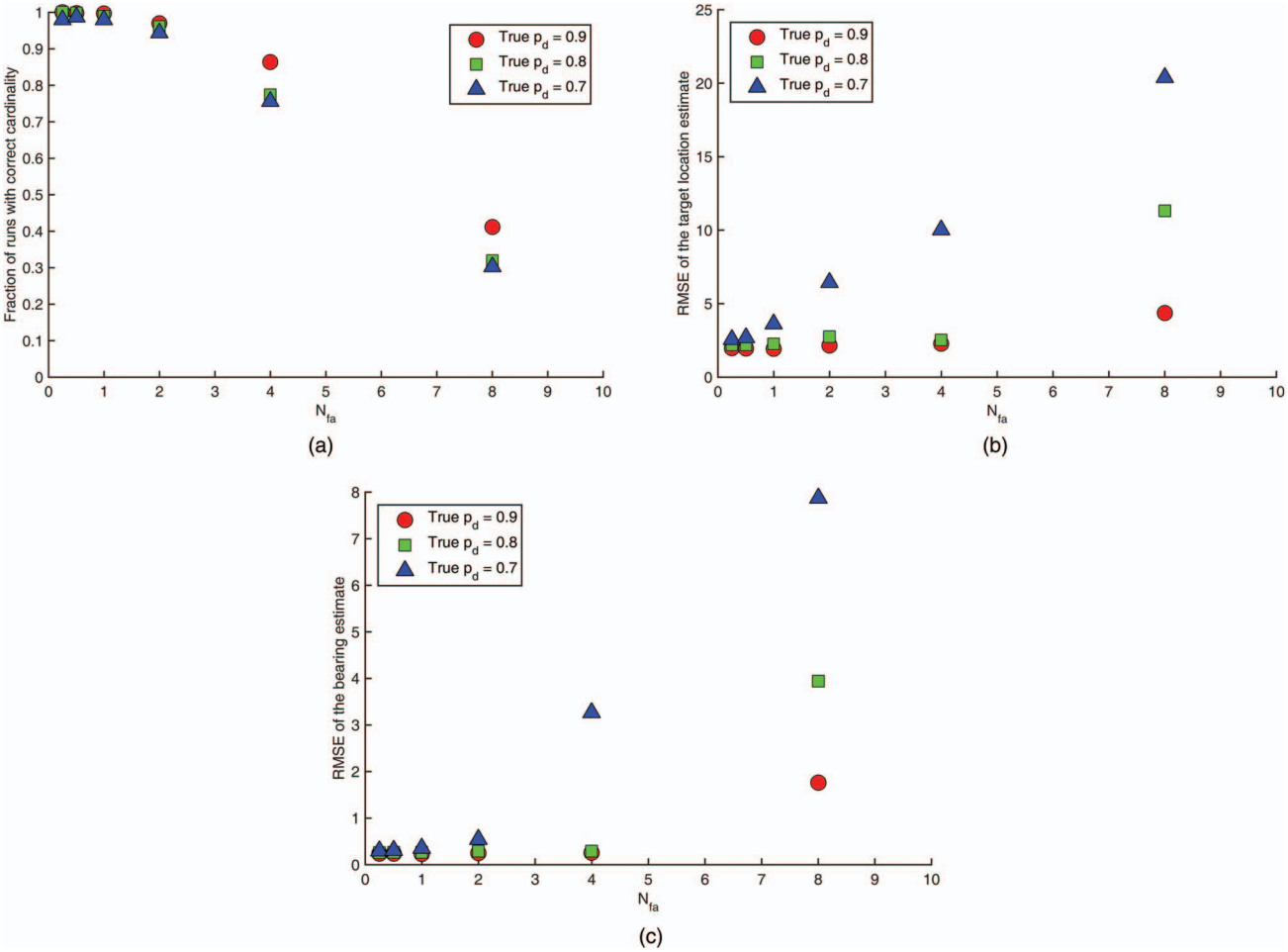


Fig. 6. The performance (in terms of φ_{exact}, $T_{\text{all}}^{\text{RMSE}}$, and $\theta_{\text{all}}^{\text{RMSE}}$) of the SEQ[$m$(2-D)] assignment algorithm in scenarios with different known expected number of false alarms (0.25, 0.5, 1, 2, 4, and 8) for known p_{d} values at 0.7, 0.8, and 0.9.

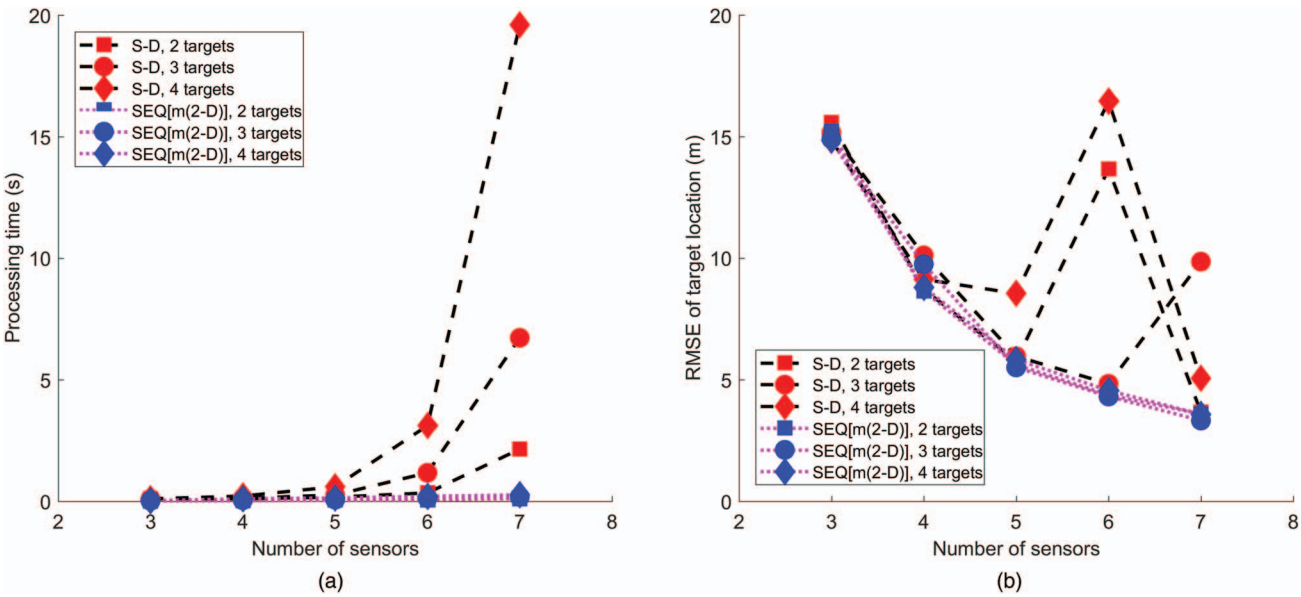


Fig. 7. Performance comparison between the S-D assignment algorithm and the SEQ[m(2-D)] assignment algorithm for $p_{\text{d}} = 0.9$ and $N_{\text{fa}} = 1$.

TABLE V
Comparison Between S-D Assignment and Sequential m-Best (SmB) 2-D Assignment Performance for $p_d = 0.9$ (Assumed Unknown)

	S-D			SmB($m = 4$)		
Assumed p_d	0.8	0.9	0.99	0.8	0.9	0.99
φ_{over}	1.8%	2.0%	3.3%	0.4%	0.4%	0.2%
$\bar{M}_{over}$	1	1	1	1	1	1
T_{over}^{RMSE} (m)	14.88	14.24	6.93	3.68	3.55	3.64
θ_{over}^{RMSE} (°)	0.714	0.640	0.459	0.309	0.306	0.241
φ_{under}	9.7%	1.9%	0.8%	0.4%	0.3%	0.3%
$\bar{M}_{under}$	1.06	1	1	1	1	1
T_{under}^{RMSE} (m)	30.67	18.14	8.72	3.08	2.91	2.91
θ_{under}^{RMSE} (°)	1.17	0.792	0.518	0.272	0.286	0.286
φ_{exact}	88.5%	96.1%	95.9%	99.2%	99.3%	99.5%
T_{exact}^{RMSE} (m)	10.47	9.44	4.87	4.91	4.42	4.43
θ_{exact}^{RMSE} (°)	1.14	0.832	0.348	0.325	0.318	0.319
T_{all}^{RMSE} (m)	13.14	9.74	4.98	4.90	4.42	4.42
θ_{all}^{RMSE} (°)	1.14	0.828	0.354	0.325	0.318	0.319
t (s)	0.549	0.568	0.541	0.166	0.162	0.160

TABLE VI
Comparison Between S-D Assignment and Sequential m-Best (SmB) 2-D Assignment Performance for $p_d = 0.8$ (Assumed Unknown)

	S-D			SmB($m = 4$)		
Assumed p_d	0.7	0.8	0.9	0.7	0.8	0.9
φ_{over}	0.7%	0.6%	0.9%	0.4%	0.4%	0.4%
$\bar{M}_{over}$	1	1	1	1	1	1
T_{over}^{RMSE} (m)	33.13	4.21	15.96	6.74	6.74	6.74
θ_{over}^{RMSE} (°)	0.882	0.302	0.829	0.421	0.421	0.421
φ_{under}	31.1%	25.4%	11.8%	7.6%	7.5%	7.5%
$\bar{M}_{under}$	1.24	1.23	1.03	1.03	1.03	1.03
T_{under}^{RMSE} (m)	31.40	30.12	20.52	5.46	5.49	5.88
θ_{under}^{RMSE} (°)	1.53	1.58	0.989	0.366	0.367	0.380
φ_{exact}	68.2%	74.0%	87.3%	92%	92.1%	92.1%
T_{exact}^{RMSE} (m)	12.69	12.86	18.51	5.11	5.11	5.11
θ_{exact}^{RMSE} (°)	1.10	1.06	1.10	0.359	0.359	0.359
T_{all}^{RMSE} (m)	19.08	17.49	18.67	5.14	5.14	5.16
θ_{all}^{RMSE} (°)	1.22	1.18	1.09	0.360	0.360	0.361
t (s)	0.415	0.420	0.413	0.142	0.141	0.150

terminates at the 100th iteration and the relative approximate duality gap has not been reduced to 5%. Even if the relative approximate duality gap is less than 5% long before the 100th iteration, it is still possible that the algorithm stops at a local minimum of the objective function (17) and, although it has a similar association cost to the optimal solution, it yields different target locations. In addition, the more false alarms the sensors detect, the more likely that the algorithm terminates at a local minimum.

As the number of sensors increases, the association graph becomes more dense and the number of candidate associations explodes combinatorially and it becomes more difficult for the S-D assignment algorithm to solve the association problem. Therefore, it is not practical to apply the S-D assignment algorithm directly[9] when the number of sensors is large. We suggest the use of the sequential m-best assignment algorithm in applications with a large number of sensors.

For additional comparison, the performance of both the S-D assignment and the sequential m-best assignment algorithms for different p_d in a scenario with four targets and six sensors is listed in Tables V–VII.

D. EM-Based Algorithms

In Sections IV and V, we have considered the Poisson measurement model for each target, which leads to either the UGM or the PPP formulation. Both are solved using the EM algorithm. For the evaluation of these two EM-based algorithms in this section, the target measurements are generated according to this Poisson measurement model; specifically, if a sensor has a certain p_d, the number of measurements originated from a target is a Poisson random variable with parameter p_d. The clutter follows a Poisson model with parameter N_{fa}.

Note that the values of the probability of detection, p_d, and the expected number of false alarms, N_{fa}, are required to generate the target measurements. However, these EM-based algorithms do not need to know the values of N_{fa} and p_d. They adapt to these values by "learning them."

The EM-based algorithm starts with an initialization, which determines whether the objective function can converge to the global maximum or a local maximum.

TABLE VII
Comparison Between S-D Assignment and Sequential m-Best (SmB) 2-D Assignment Performance for $p_d = 0.7$ (Assumed Unknown)

	S-D			SmB($m = 4$)		
Assumed p_d	0.6	0.7	0.8	0.6	0.7	0.8
φ_{over}	0.5%	0.3%	0.2%	0	0	0
$\bar{M}_{over}$	1	1	1	N.A.	N.A.	N.A.
T_{over}^{RMSE} (m)	5.93	6.30	6.80	N.A.	N.A.	N.A.
θ_{over}^{RMSE} (°)	0.360	0.382	0.395	N.A.	N.A.	N.A.
φ_{under}	62.6%	57.4%	52.2%	27.5%	27.5%	27.5%
$\bar{M}_{under}$	1.45	1.46	1.39	1.14	1.14	1.14
T_{under}^{RMSE} (m)	35.39	35.24	34.48	6.93	6.97	6.59
θ_{under}^{RMSE} (°)	2.14	2.19	2.20	0.398	0.400	0.402
φ_{exact}	36.9%	42.3%	47.6%	72.5%	72.5%	72.5%
T_{exact}^{RMSE} (m)	14.78	14.90	14.92	6.96	6.96	6.96
θ_{exact}^{RMSE} (°)	0.941	0.911	0.910	0.860	0.860	0.860
T_{all}^{RMSE} (m)	27.41	26.29	24.99	6.95	6.96	6.88
θ_{all}^{RMSE} (°)	1.67	1.63	1.58	0.784	0.785	0.785
t (s)	0.318	0.318	0.322	0.131	0.127	0.131

[9]One possible practice is to use the S-D assignment algorithm on a subset of sensors followed by sequential processing as in [23].

TABLE VIII
UGM/EM and PPP/EM Performance Using Different Initialization ("I:") Approaches for Unknown $p_d = 0.7$

	I: Truth		I: Clustering		I: SmB($m = 1$)		I: SmB($m = 2$)	
	UGM/EM	PPP/EM	UGM/EM	PPP/EM	UGM/EM	PPP/EM	UGM/EM	PPP/EM
φ_{over}	0.1%	0	49.7%	50.5%	19.4%	19.3%	19.8%	19.7%
$\bar{M}_{over}$	1	N.A.	1.57	1.51	1.14	1.14	1.15	1.15
T^{RMSE}_{over} (m)	4.96	N.A.	36.69	28.51	3.46	3.47	3.34	3.35
θ^{RMSE}_{over} (°)	0.535	N.A.	2.23	2.37	0.493	0.494	0.449	0.451
φ_{under}	0.6%	0.7%	29.7%	30.5%	23.4%	23.4%	23%	22.9%
$\bar{M}_{under}$	1	1	1.52	1.48	1.10	1.10	1.09	1.08
T^{RMSE}_{under} (m)	3.06	2.57	34.21	31.8	8.28	8.30	2.88	3.04
θ^{RMSE}_{under} (°)	0.403	0.352	3.66	3.30	0.442	0.449	0.347	0.357
φ_{exact}	99.3%	99.3%	20.6%	19%	57.2%	57.3%	57.2%	57.4%
T^{RMSE}_{exact} (m)	3.05	3.01	70.95	52.96	5.56	5.54	5.53	5.55
θ^{RMSE}_{exact} (°)	0.351	0.347	5.38	5.27	0.615	0.615	0.615	0.620
T^{RMSE}_{all} (m)	3.05	3.01	46.37	35.69	5.81	5.81	4.74	4.78
θ^{RMSE}_{all} (°)	0.351	0.347	3.49	3.38	0.563	0.564	0.543	0.548
t (s)	2.52	2.40	4.72	4.32	1.23	1.21	1.31	1.31

Three initialization approaches are considered in this paper.

The first approach is to initialize the target locations and the emission times using their true values. This initialization approach works well as shown later; however, it critically depends on the truth, which is not available in the real world. Nevertheless, it provides a benchmark on how well the EM-based algorithms can perform. Since the number of targets N is unknown, one needs to evaluate a range of values for N and the algorithm selects the best N based on BIC. Such possible values for N can be selected based on the number of measurements obtained at each sensor; five values (2–6) are chosen for the four-target scenario considered here. When the evaluated number of targets is less than the true value, a subset of the true targets is used for initialization. When the evaluated number of targets is more than the true value, auxiliary targets in addition to the true ones are used for initialization.

The second approach is based on the k-means clustering algorithm. Any two bearing (or LOS) measurements from two different sensors can lead to a potential target. In the absence of measurement noise, the LOS measurements coming from the same target intersect at a single point. In the presence of measurement noise, the LOS measurements originating from the same target should intersect with each other in a close neighborhood. Therefore, the points of intersection from the LOS measurements of any two sensors are clustered and the centroids of each cluster are used to initialize the target locations. The emission times are initialized in the same way. As in the first approach, five values are evaluated for N.

The third approach is based on the SEQ[m(2-D)] assignment algorithm. The associations with more than two real measurements correspond to potential targets. Let N_{max} denote the number of such associations. These associations are ranked in terms of the association cost. A range of values from 1 to N_{max} will be evaluated for N, and the top N associations will be used to initialize the EM-based algorithm.

Tables VIII–X present the performance of the EM-based algorithm with both UGM and PPP formulations (UGM/EM and PPP/EM) using different initialization approaches at three levels[10] of p_d with a known false alarm rate ($N_{fa} = 1$) in a scenario where ten sensors are used to locate four targets.

Initialization at the truth enables the EM-based algorithm to estimate the number of targets, target locations, and target directions accurately and the estimation becomes more accurate as p_d increases. In this case, the global maximum is attained.

The clustering-based initialization is very prone to ghosting and therefore results in very large errors in terms of the number of targets, target locations, and target directions. It also takes a longer processing time with such a poor initialization. In this case, the algorithm terminates at a local maximum.

The assignment-based initialization overcomes the ghosting problem. With $m = 2$ in the SEQ[m(2-D)] assignment, the target direction errors are less than the standard deviation of the bearing measurement noise and the target location errors are close to those obtained using initialization at the truth. At a higher p_d, the number of overestimation cases increases. This is due to double counting of the same target by the assignment algorithm when two acoustic events occur at the same location. The assignment algorithm does not differentiate the acoustic events that occurred at the same location.

[10]The probability of detection is set to be the same for each target in the simulation studies only for simplicity; the EM-based algorithms can deal with the case that the probabilities of detection for different targets are distinct.

TABLE IX
UGM/EM and PPP/EM Performance Using Different Initialization ("I:") Approaches for Unknown $p_d = 0.8$

	I: Truth		I: Clustering		I: SmB($m = 1$)		I: SmB($m = 2$)	
	UGM/EM	PPP/EM	UGM/EM	PPP/EM	UGM/EM	PPP/EM	UGM/EM	PPP/EM
φ_{over}	0	0	51.5%	54.8%	26.9%	27.1%	27.5%	27.8%
$\bar{M}_{\text{over}}$	N.A.	N.A.	1.53	1.52	1.21	1.21	1.23	1.23
$T^{\text{RMSE}}_{\text{over}}$ (m)	N.A.	N.A.	30.97	40.07	2.84	2.87	2.85	2.88
$\theta^{\text{RMSE}}_{\text{over}}$ (°)	N.A.	N.A.	2.14	2.17	0.391	0.405	0.394	0.409
φ_{under}	0.1%	0.1%	29.6%	27.1%	14.2%	14.0%	14.1%	13.8%
$\bar{M}_{\text{under}}$	1	1	1.50	1.46	1.09	1.09	1.10	1.09
$T^{\text{RMSE}}_{\text{under}}$ (m)	0.69	0.74	33.60	29.94	2.58	2.57	2.57	2.57
$\theta^{\text{RMSE}}_{\text{under}}$ (°)	0.0886	0.0903	3.28	3.09	0.317	0.316	0.317	0.316
φ_{exact}	99.9%	99.9%	18.9%	18.1%	58.9%	58.9%	58.4%	58.4%
$T^{\text{RMSE}}_{\text{exact}}$ (m)	2.55	2.52	46.88	47.86	3.06	3.06	2.97	2.98
$\theta^{\text{RMSE}}_{\text{exact}}$ (°)	0.306	0.303	4.60	4.64	0.431	0.430	0.429	0.428
$T^{\text{RMSE}}_{\text{all}}$ (m)	2.55	2.52	35.44	40.07	2.95	2.96	2.89	2.91
$\theta^{\text{RMSE}}_{\text{all}}$ (°)	0.306	0.303	3.06	3.00	0.409	0.412	0.408	0.412
t (s)	2.17	2.16	4.51	4.62	1.69	1.71	1.85	1.84

TABLE X
UGM/EM and PPP/EM Performance Using Different Initialization ("I:") Approaches for Unknown $p_d = 0.9$

	I: Truth		I: Clustering		I: SmB($m = 1$)		I: SmB($m = 2$)	
	UGM/EM	PPP/EM	UGM/EM	PPP/EM	UGM/EM	PPP/EM	UGM/EM	PPP/EM
φ_{over}	0	0	52.1%	53.8%	33.6%	33.6%	34.7%	34.7%
$\bar{M}_{\text{over}}$	N.A.	N.A.	1.60	1.59	1.26	1.27	1.26	1.27
$T^{\text{RMSE}}_{\text{over}}$ (m)	N.A.	N.A.	37.92	29.36	2.44	2.46	2.49	2.50
$\theta^{\text{RMSE}}_{\text{over}}$ (°)	N.A.	N.A.	1.99	1.89	0.321	0.321	0.331	0.331
φ_{under}	0.3%	0.3%	28%	27.2%	9.9%	9.9%	8.9%	8.9%
$\bar{M}_{\text{under}}$	1	1	1.48	1.48	1.03	1.03	1.03	1.03
$T^{\text{RMSE}}_{\text{under}}$ (m)	1.41	1.40	27.82	25.97	2.55	2.58	2.29	2.36
$\theta^{\text{RMSE}}_{\text{under}}$ (°)	0.245	0.244	2.68	2.52	0.281	0.284	0.263	0.269
φ_{exact}	99.7%	99.7%	19.9%	19%	56.5%	56.5%	56.4%	56.4%
$T^{\text{RMSE}}_{\text{exact}}$ (m)	2.23	2.22	43.77	52.87	2.95	2.96	2.99	3.00
$\theta^{\text{RMSE}}_{\text{exact}}$ (°)	0.275	0.273	4.39	4.35	0.377	0.380	0.388	0.391
$T^{\text{RMSE}}_{\text{all}}$ (m)	2.23	2.21	37.59	35.14	2.76	2.77	2.77	2.79
$\theta^{\text{RMSE}}_{\text{all}}$ (°)	0.275	0.273	2.83	2.71	0.352	0.354	0.361	0.364
t (s)	2.22	2.22	4.88	4.92	1.97	1.95	2.21	2.19

Although it indicates that there are more targets than the truth, all real targets have actually been identified.

Fig. 8 compares the performance of the EM-based algorithm with both UGM and PPP formulations (UGM/EM and PPP/EM) with initialization at the truth for different known expected numbers of false alarms (or false alarm rate) in a scenario where ten sensors are used to locate four targets. The quality of the cardinality, location, and bearing estimates using both formulations is almost identical for the same p_d value, which demonstrates the effectiveness of the UGM formulation to incorporate the false alarm rate when it is known.

E. Assignment Algorithms and EM-Based Algorithms

In Sections VI-C and VI-D, the two types of algorithms—the assignment algorithms and the EM-based algorithms—were evaluated separately according to their assumed target-originated measurement models (Bernoulli and Poisson, respectively). Since the Bernoulli measurement model is the more realistic one, the target measurements are generated in the next evaluation according to this Bernoulli measurement model for comparing all the algorithms. Therefore, there is no measurement model mismatch for the assignment algorithms, but there is a measurement model mismatch for the EM-based algorithms.

Tables XI–XIII compare the assignment algorithms and EM-based algorithms with assignment-based initialization. In this case, one may consider the EM-based algorithms as postprocessing procedures. Such a processing increases the entire processing time and only leads to an insignificant improvement of the estimation accuracies. However, it reflects the capability of the EM-based

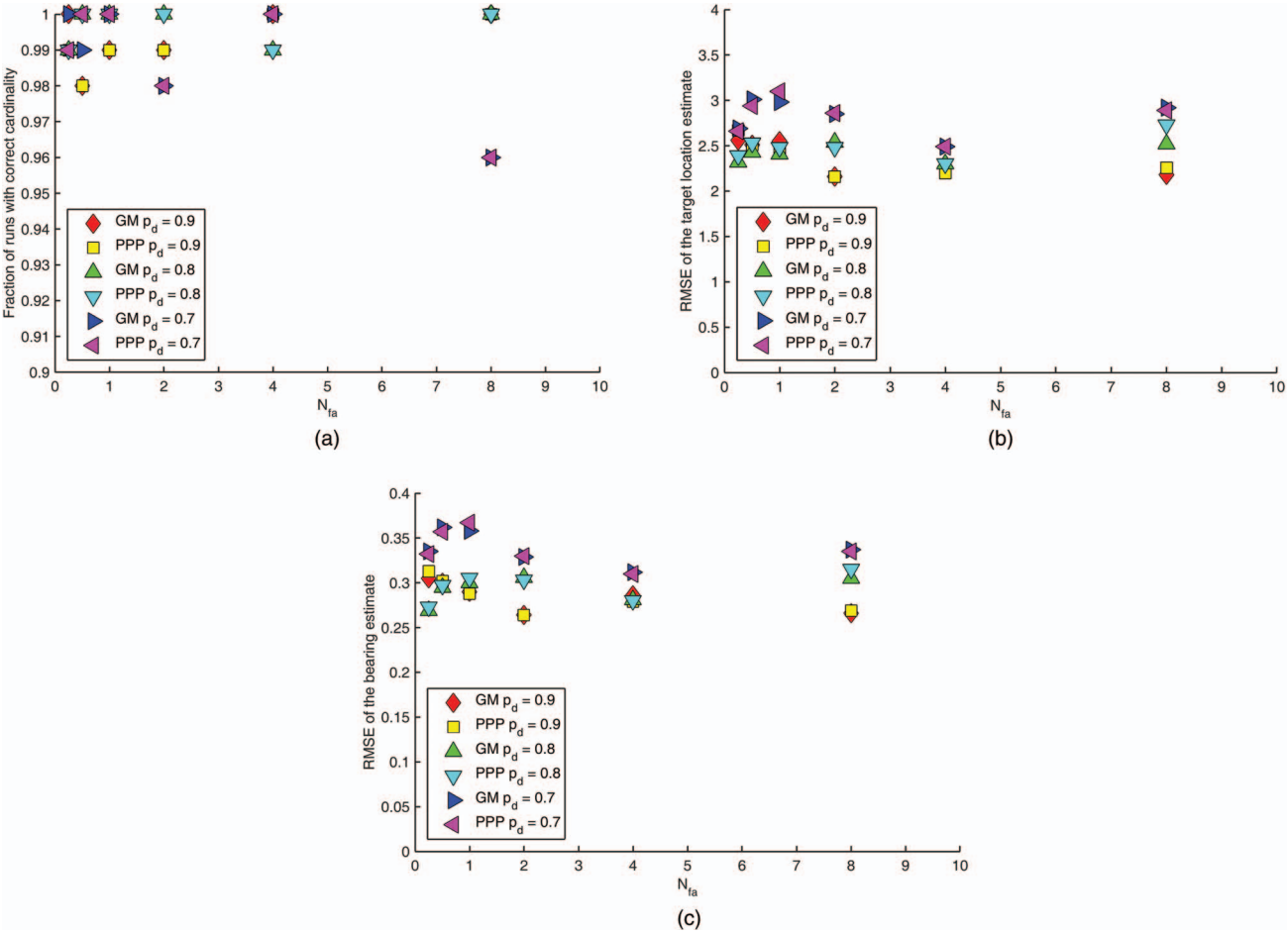


Fig. 8. The performance (in terms of φ_{exact}, $T_{\text{all}}^{\text{RMSE}}$, and $\theta_{\text{all}}^{\text{RMSE}}$) of UGM/EM and PPP/EM in scenarios with different known expected number of false alarms (0.25, 0.5, 1, 2, 4, and 8) for true p_{d} values at 0.7, 0.8, and 0.9.

TABLE XI
Performance Comparison Among S-D Assignment, Sequential m-Best (SmB) 2-D Assignment, and EM-Based Algorithms (With Different Initializations "I:") for $p_{\text{d}} = 0.9$ ($N_{\text{s}} = 6$)

	Assignment			I: SmB($m = 2$)		I: SmB($m = 4$)	
	S-D	SmB($m = 2$)	SmB($m = 4$)	UGM/EM	PPP/EM	UGM/EM	PPP/EM
φ_{over}	2.0%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%
$\bar{M}_{\text{over}}$	1	1	1	1	1	1	1
$T_{\text{over}}^{\text{RMSE}}$ (m)	14.24	3.55	3.55	3.54	3.54	3.54	3.54
$\theta_{\text{over}}^{\text{RMSE}}$ (°)	0.640	0.306	0.306	0.307	0.307	0.307	0.307
φ_{under}	1.9%	0.4%	0.3%	0.4%	0.4%	0.3%	0.3%
$\bar{M}_{\text{under}}$	1	1	1	1	1	1	1
$T_{\text{under}}^{\text{RMSE}}$ (m)	18.14	2.86	2.91	2.86	2.86	2.91	2.91
$\theta_{\text{under}}^{\text{RMSE}}$ (°)	0.792	0.279	0.286	0.279	0.279	0.286	0.286
φ_{exact}	96.1%	99.2%	99.3%	99.2%	99.2%	99.3%	99.3%
$T_{\text{exact}}^{\text{RMSE}}$ (m)	9.44	4.42	4.42	4.41	4.41	4.40	4.40
$\theta_{\text{exact}}^{\text{RMSE}}$ (°)	0.832	0.318	0.318	0.318	0.318	0.318	0.318
$T_{\text{all}}^{\text{RMSE}}$ (m)	9.74	4.41	4.42	4.40	4.40	4.40	4.40
$\theta_{\text{all}}^{\text{RMSE}}$ (°)	0.828	0.318	0.318	0.318	0.318	0.318	0.318
t (s)	0.568	0.096	0.162	0.181	0.190	0.249	0.247

TABLE XII
Performance Comparison Among S-D Assignment, Sequential m-Best (SmB) 2-D Assignment, and EM-Based Algorithms (With Different Initializations "I:") for $p_d = 0.8$ ($N_s = 6$)

	Assignment			I: SmB($m = 2$)		I: SmB($m = 4$)	
	S-D	SmB($m = 2$)	SmB($m = 4$)	UGM/EM	PPP/EM	UGM/EM	PPP/EM
φ_{over}	0.6%	0.5%	0.4%	0.4%	0.3%	0.3%	0.3%
$\bar{M}_{\text{over}}$	1	1	1	1	1	1	1
$T^{\text{RMSE}}_{\text{over}}$ (m)	4.21	6.75	6.74	5.69	6.27	6.26	6.26
$\theta^{\text{RMSE}}_{\text{over}}$ (°)	0.302	0.423	0.421	0.367	0.395	0.396	0.395
φ_{under}	25.4%	7.2%	7.5%	7.2%	7.2%	7.5%	7.5%
$\bar{M}_{\text{under}}$	1.23	1.03	1.03	1.03	1.03	1.03	1.03
$T^{\text{RMSE}}_{\text{under}}$ (m)	30.12	5.55	5.49	5.19	5.19	5.14	5.14
$\theta^{\text{RMSE}}_{\text{under}}$ (°)	1.58	0.369	0.367	0.359	0.359	0.358	0.358
φ_{exact}	74.0%	92.3%	92.1%	92.4%	92.5%	92.2%	92.2%
$T^{\text{RMSE}}_{\text{exact}}$ (m)	12.86	5.15	5.11	5.08	5.08	5.05	5.05
$\theta^{\text{RMSE}}_{\text{exact}}$ (°)	1.06	0.362	0.359	0.358	0.358	0.356	0.356
$T^{\text{RMSE}}_{\text{all}}$ (m)	17.49	5.18	5.14	5.09	5.09	5.06	5.06
$\theta^{\text{RMSE}}_{\text{all}}$ (°)	1.18	0.362	0.360	0.358	0.359	0.356	0.356
t (s)	0.420	0.070	0.141	0.151	0.148	0.231	0.237

TABLE XIII
Performance Comparison Among S-D Assignment, Sequential m-Best (SmB) 2-D Assignment, and EM-Based Algorithms (With Different Initializations "I:") for $p_d = 0.7$ ($N_s = 6$)

	Assignment			I: SmB($m = 2$)		I: SmB($m = 4$)	
	S-D	SmB($m = 2$)	SmB($m = 4$)	UGM/EM	PPP/EM	UGM/EM	PPP/EM
φ_{over}	0.3%	0	0	0	0	0	0
$\bar{M}_{\text{over}}$	1	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
$T^{\text{RMSE}}_{\text{over}}$ (m)	6.30	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
$\theta^{\text{RMSE}}_{\text{over}}$ (°)	0.382	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
φ_{under}	57.4%	27.6%	27.5%	27.6%	27.6%	27.5%	27.5%
$\bar{M}_{\text{under}}$	1.46	1.14	1.14	1.14	1.14	1.14	1.14
$T^{\text{RMSE}}_{\text{under}}$ (m)	35.24	7.10	6.96	5.69	5.68	5.65	5.65
$\theta^{\text{RMSE}}_{\text{under}}$ (°)	2.19	0.402	0.400	0.383	0.383	0.381	0.382
φ_{exact}	42.3%	72.4%	72.5%	72.4%	72.4%	72.5%	72.5%
$T^{\text{RMSE}}_{\text{exact}}$ (m)	14.90	7.44	6.96	7.27	7.27	6.88	6.86
$\theta^{\text{RMSE}}_{\text{exact}}$ (°)	0.911	0.872	0.860	0.864	0.864	0.851	0.851
$T^{\text{RMSE}}_{\text{all}}$ (m)	26.29	7.37	6.96	6.96	6.96	6.63	6.62
$\theta^{\text{RMSE}}_{\text{all}}$ (°)	1.63	0.795	0.785	0.786	0.786	0.775	0.775
t (s)	0.318	0.058	0.127	0.126	0.133	0.191	0.196

algorithm to solve the data association problem. Associating the measurements with a good degree of accuracy requires a good initialization, such as the assignment approach. In this case, although there is a mismatch in the measurement model, the EM-based algorithms estimate the number of targets, target locations, and target directions quite accurately due to the fact that the initialization by the SEQ[m(2-D)] assignment algorithm is close to the truth.

VII. CONCLUSION

This paper considers the problem of multiple transient emitter localization using a network of passive sensors. It is assumed that the number of targets as well as the association between measurements and targets is unknown and in the presence of missed detections and false alarms. Two different measurement models—the Bernoulli measurement model and the Poisson measurement model—are considered for each target and two types of algorithms—assignment- and EM-based—are presented, one for each measurement model. Simulation studies show that the SEQ[m(2-D)] assignment algorithm has very promising performance and can be employed in real-time applications. While the EM-based algorithms have the capability of solving the data association problem, simulation results suggest that they require the right initial estimates to provide reliable

localization results and the processing time could be longer than required. The fusion algorithms discussed in this paper assume that there is a fusion center to which each sensor can communicate. Fusion algorithms, which assume that no such fusion center exists and allow distributed processing and only single-hop communication, are developed in [10].

APPENDIX A PROOF OF PROPOSITION 1

For notational simplicity, let us denote

$$a_i = \sum_{\ell=1}^{N_s} \sum_{j=1}^{n_\ell} w_{\ell ji}^{(n-1)}, \quad i = 0, 1, \ldots, N \tag{100}$$

$$p_i = p_d(T_i), \quad i = 1, 2, \ldots, N \tag{101}$$

$$\mathbf{p} = [p_1, p_2, \ldots, p_N] \tag{102}$$

$$h_i(\mathbf{p}) = p_i - 1. \tag{103}$$

Substituting (25) into (45), the problem becomes

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & f(\mathbf{p}) = a_0 \ln N_{fa} + \sum_{i=1}^{N} a_i \ln p_i \\ & - \left(\sum_{i=0}^{N} a_i \right) \ln \left(\sum_{i=1}^{N} p_i + N_{fa} \right) \\ \text{subject to} \quad & h_i(\mathbf{p}) \le 0, \quad i = 1, \ldots, N. \end{aligned} \tag{104}$$

Let μ_i be a Lagrange multiplier corresponding to $p_i \le 1$ and $\boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_N]$. The Lagrangian is

$$L(\mathbf{p}, \boldsymbol{\mu}) = f(\mathbf{p}) + \sum_{i=1}^{N} \mu_i (0 - h_i(\mathbf{p})). \tag{105}$$

From the KKT conditions, the optimal values of $\mathbf{p}$ and $\boldsymbol{\mu}$ satisfy the following system of equations and inequalities:

$$\begin{cases} 0 = \dfrac{\partial f}{\partial p_i} - \mu_i \dfrac{\partial h_i}{p_i} = \dfrac{a_i}{p_i} - \dfrac{\sum_{i=0}^{N} a_i}{\sum_{i=1}^{N} p_i + N_{fa}} - \mu_i, \\ \quad i = 1, 2, \ldots, N \end{cases} \tag{106}$$

$$0 = \mu_i h_i(\mathbf{p}) = \mu_i (p_i - 1), \quad i = 1, 2, \ldots, N \tag{107}$$

$$0 \le \mu_i, \quad i = 1, 2, \ldots, N. \tag{108}$$

We need to break the analysis into cases according to (107).

Case 1: If

$$\mu_i = 0, \quad i = 1, 2, \ldots, N \tag{109}$$

then (106) is simplified to

$$p_i = \frac{a_i \left(\sum_{k=1}^{N} p_k + N_{fa} \right)}{\sum_{i=0}^{N} a_i}. \tag{110}$$

Summing over i from 1 to N, we have

$$\sum_{i=1}^{N} p_i = \sum_{k=1}^{N} p_k = \frac{\sum_{i=1}^{N} a_i \left(\sum_{k=1}^{N} p_k + N_{fa} \right)}{\sum_{i=0}^{N} a_i} \tag{111}$$

which can be simplified to

$$\sum_{k=1}^{N} p_k = \frac{\sum_{i=1}^{N} a_i N_{fa}}{a_0}. \tag{112}$$

Substituting (112) into (110), we have

$$p_i = \frac{a_i N_{fa}}{a_0}. \tag{113}$$

The feasibility of this solution depends on whether $a_i N_{fa}/a_0$ is greater than 1. Let

$$\mathcal{S} = \{ i \,|\, a_i N_{fa} > a_0 \}. \tag{114}$$

If the set $\mathcal{S}$ is empty, (113) will be the optimal solution for $\mathbf{p}$. If the set $\mathcal{S}$ is not empty, then we must have

$$\begin{cases} 0 = \mu_i h_i(\mathbf{p}) = \mu_i (p_i - 1), \quad i \notin \mathcal{S} & (115) \\ 0 \le \mu_i, \quad i \notin \mathcal{S} & (116) \\ 0 = p_i - 1, \quad i \in \mathcal{S} & (117) \\ 0 < \mu_i, \quad i \in \mathcal{S}. & (118) \end{cases}$$

Case 2: If

$$\begin{cases} 0 = \mu_i, \quad i \notin \mathcal{S} & (119) \\ 1 = p_i, \quad i \in \mathcal{S} & (120) \end{cases}$$

then (106) is simplified to

$$p_i = \frac{a_i \left(\sum_{k \notin \mathcal{S}} p_k + |\mathcal{S}| + N_{fa} \right)}{\sum_{i=0}^{N} a_i}, \quad i \notin \mathcal{S}. \tag{121}$$

Summing over i that is not in the set $\mathcal{S}$ and solving for $\sum_{i \notin \mathcal{S}} p_i$,

$$\sum_{i \notin \mathcal{S}} p_i = \frac{\sum_{i>0, i \notin \mathcal{S}} a_i (|\mathcal{S}| + N_{fa})}{\sum_{i>0, i \in \mathcal{S}} a_i + a_0}. \tag{122}$$

Substituting (122) into (121), we have

$$p_i = \frac{a_i (|\mathcal{S}| + N_{fa})}{\sum_{i>0, i \in \mathcal{S}} a_i + a_0}, \quad i \notin \mathcal{S}. \tag{123}$$

Since

$$a_i N_{fa} > a_0, \quad i \in \mathcal{S} \tag{124}$$

we have

$$\sum_{i>0, i \in \mathcal{S}} a_i N_{fa} > |\mathcal{S}| a_0 \tag{125}$$

$$\sum_{i>0, i \in \mathcal{S}} a_i N_{fa} + a_0 N_{fa} > |\mathcal{S}| a_0 + a_0 N_{fa} \tag{126}$$

$$\frac{N_{\text{fa}}}{a_0} > \frac{|\mathcal{S}| + N_{\text{fa}}}{\sum_{i>0, i\in\mathcal{S}} a_i + a_0}. \tag{127}$$

Since

$$a_i N_{\text{fa}} \le a_0, \quad i \notin \mathcal{S} \tag{128}$$

we have

$$p_i = \frac{a_i(|\mathcal{S}| + N_{\text{fa}})}{\sum_{i>0, i\in\mathcal{S}} a_i + a_0} < \frac{a_i N_{\text{fa}}}{a_0} \le 1, \quad i \notin \mathcal{S} \tag{129}$$

which verifies the feasibility of the solution consisting of (120) and (123). One can summarize the two cases as follows:

$$p_i = \begin{cases} 1, & \text{if } i \in \mathcal{S} \\ \dfrac{a_i(|\mathcal{S}| + N_{\text{fa}})}{\sum_{i>0, i\in\mathcal{S}} a_i + a_0}, & \text{if } i \notin \mathcal{S} \end{cases} \tag{130}$$

which is equivalent to (58) because of (100) and (101).

REFERENCES

[1] H. Akaike
"A new look at the statistical model identification,"
IEEE Trans. Autom. Control, vol. 19, no. 6, pp. 716–723, Dec. 1974.

[2] J. Areta, Y. Bar-Shalom, M. Levedahl, and K. Pattipati
"Hierarchical track association and fusion for a networked surveillance system,"
J. Adv. Inf. Fusion, vol. 1, no. 2, pp. 140–157, Dec. 2006.

[3] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan
Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software, Hoboken, NJ: Wiley, 2001.

[4] Y. Bar-Shalom, P. Willett, and X. Tian
Tracking and Data Fusion, Storrs, CT: YBS Publishing, 2011.

[5] C. M. Bishop
Pattern Recognition and Machine Learning, New York: Springer, 2006.

[6] S. Blackman
"Multiple hypothesis tracking for multiple target tracking,"
IEEE Aerosp. Electron. Syst. Mag., vol. 19, no. 1, pp. 5–18, Jan. 2004.

[7] D. Daley and D. Vere-Jones
An Introduction to the Theory of Point Processes: Volume I: Elementary Theory and Methods, New York: Springer, 2003.

[8] S. Deb, M. Yeddanapudi, K. Pattipati, and Y. Bar-Shalom
"A generalized $\mathcal{S}$-D assignment algorithm for multisensor–multitarget state estimation,"
IEEE Trans. Aerosp. Electron. Syst., vol. 33, no. 2, pp. 523–538, Apr. 1997.

[9] A. P. Dempster, N. M. Laird, and D. B. Rubin
"Maximum likelihood from incomplete data via the EM algorithm,"
J. R. Stat. Soc., Ser. B, vol. 39, no. 1, pp. 1–38, 1977.

[10] W. Dou, Y. Bar-Shalom, L. Kaplan, and J. George
"Distributed fusion algorithm for passive localization of multiple transient emitters,"
J. Adv. Inf. Fusion, vol. 13, no. 1, pp. 15–35, Jun. 2018.

[11] T. E. Fortmann, Y. Bar-Shalom, and M. Scheffe
"Sonar tracking of multiple targets using joint probabilistic data association,"
IEEE J. Ocean. Eng., vol. 8, no. 3, pp. 173–184, Jul. 1983.

[12] J. George and L. Kaplan
"Shooter localization using a wireless sensor network of soldier-worn gunfire detection systems,"
J. Adv. Inf. Fusion, vol. 8, no. 1, pp. 15–32, Jun. 2013.

[13] J. George, L. Kaplan, S. Deligeorges, and G. Cakiades
"Multi-shooter localization using finite point process,
Proc. 17th Int. Conf. Inf. Fusion, Jul. 2014.

[14] J. George and L. Kaplan
"A finite point process approach to multi-agent localization using transient measurements,"
J. Inf. Fusion, vol. 32, no. PA, pp. 62–74, Nov. 2016.

[15] H. W. Kuhn and A. W. Tucker
"Nonlinear programming,"
Proc. 2nd Berkeley Symp. Math. Stat. Probability, 1951, pp. 481–492.

[16] R. Mahler
"Multitarget Bayes filtering via first-order multitarget moments,"
IEEE Trans. Aerosp. Electron. Syst., vol. 39, no. 4, pp. 1152–1178, Oct. 2003.

[17] J. Moller and R. P. Waagepetersen
Statistical Inference and Simulation for Spatial Point Processes, London, UK: Chapman & Hall/CRC, 2003.

[18] K. G. Murty
"An algorithm for ranking all the assignments in order of increasing cost,"
Oper. Res., vol. 16, no. 3, pp. 682–687, 1968.

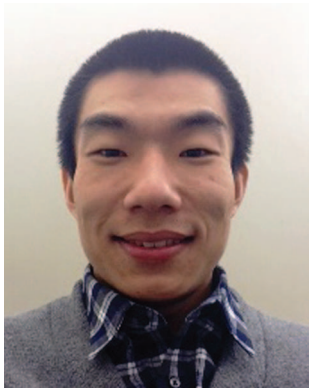
[19] R. W. Osborne, Y. Bar-Shalom, J. George, and L. Kaplan
"Data fusion from multiple passive sensors for multiple shooter localization via assignment,"
in *Proc. 17th Int. Conf. Inf. Fusion*, Jul. 2014.

[20] R. W. Osborne, Y. Bar-Shalom, J. George, and L. Kaplan
"Statistical efficiency of target localization from angle and shockwave measurements,"
J. Adv. Inf. Fusion, vol. 9, no. 2, pp. 75–89, Dec. 2014.

[21] K. R. Pattipati, S. Deb, Y. Bar-Shalom, and R. B. Washburn
"A new relaxation algorithm and passive sensor data association,"
IEEE Trans. Autom. Control, vol. 37, no. 2, pp. 198–213, Feb. 1992.

[22] G. Schwarz
"Estimating the dimension of a model,"
Ann. Stat., vol. 6, no. 2, pp. 461–464, 1978.

[23] S. Zhang and Y. Bar-Shalom
"Practical data association for passive sensors in 3D,"
J. Adv. Inf. Fusion, vol. 9, no. 1, pp. 38–46, Jun. 2014.



Wenbo Dou received the Ph.D. degree in electrical engineering from the University of Connecticut, Storrs, CT, USA, in 2017. His current research interests include bias estimation, multitarget localization and tracking, data fusion, and machine learning.

Jemin George received the M.S. and Ph.D. degrees in aerospace engineering from the State University of New York at Buffalo, Buffalo, NY, USA, in 2007 and 2010, respectively. In 2008, he was a Summer Research Scholar with the U.S. Air Force Research Laboratory's Space Vehicles Directorate, and in 2009, he was a National Aeronautics and Space Administration Langley Aerospace Research Summer Scholar. From 2009 to 2010, he was a Research Fellow with the Department of Mathematics, Technische Universität Darmstadt, Darmstadt, Germany. Since 2010, he has been with the U.S. Army Research Laboratory, Adelphi, MD, USA. His principal research interests include stochastic systems, control theory, nonlinear filtering, information fusion, adaptive networks, and distributed sensing and estimation.

Lance M. Kaplan received the B.S. degree with distinction from Duke University, Durham, NC, USA, in 1989, and the M.S. and Ph.D. degrees from the University of Southern California, Los Angeles, CA, USA, in 1991 and 1994, respectively, all in electrical engineering. From 1987 to 1990, he was a Technical Assistant with the Georgia Tech Research Institute. He held a National Science Foundation Graduate Fellowship and a USC Dean's Merit Fellowship from 1990 to 1993, and was a Research Assistant with the Signal and Image Processing Institute, University of Southern California, from 1993 to 1994. Then, he was on staff with the Reconnaissance Systems Department, Hughes Aircraft Company, from 1994 to 1996. From 1996 to 2004, he was a member of the faculty with the Department of Engineering and a senior investigator with the Center of Theoretical Studies of Physical Systems, Clark Atlanta University, Atlanta, GA, USA. He is currently a Researcher with the Networked Sensing and Fusion branch of the U.S. Army Research Laboratory (ARL), Adelphi, MD, USA. Dr. Kaplan is on the Board of Governors for the IEEE Aerospace and Electronic Systems Society (2008–2013, 2018–present) and a VP of Conferences for the International Society of Information Fusion (ISIF) (2014–present). Previously, he was Editor-in-Chief for the *IEEE Transactions on Aerospace and Electronic Systems* (2012–2017) and on the Board of Directors of ISIF (2012–2014). He is a three-time recipient of the Clark Atlanta University Electrical Engineering Instructional Excellence Award from 1999 to 2001. He is a Fellow of IEEE and of ARL. His current research interests include information/data fusion, reasoning under uncertainty, network science, resource management, and signal and image processing.

Richard W. Osborne, III, received the B.S., M.S., and Ph.D. degrees in electrical engineering from the University of Connecticut, Storrs, CT, USA, in 2004, 2007, and 2012, respectively. From 2012 to 2014, he was an Assistant Research Professor with the Electrical Engineering Department, University of Connecticut. From 2014 to 2015, he was a Senior Research Engineer with BAE Systems, Inc., Burlington, MA, USA, and since 2015, he has been a Senior Research Engineer with United Technologies Research Center, East Hartford, CT, USA. His academic interests include adaptive target tracking, information/sensor fusion, perception/computer vision, autonomous systems, and other aspects of estimation.

Yaakov Bar-Shalom received the B.S. and M.S. degrees from the Technion, Haifa, Israel, in 1963 and 1967, respectively, and the Ph.D. degree from Princeton University, Princeton, NJ, USA, in 1970, all in electrical engineering. From 1970 to 1976, he was with Systems Control, Inc., Palo Alto, CA, USA. He is currently a Board of Trustees Distinguished Professor with the Department of Electrical and Computer Engineering and Marianne E. Klewin Professor in Engineering with the University of Connecticut, Storrs, CT, USA. His current research interests include estimation theory, target tracking, and data fusion. He has authored or coauthored more than 550 papers and book chapters, and coauthored/edited 8 books, including *Tracking and Data Fusion* (YBS Publishing, 2011). He was elected Fellow of IEEE for contributions to the theory of stochastic systems and of multitarget tracking. He was an Associate Editor of the *IEEE Transactions on Automatic Control* and *Automatica*. He was General Chairman of the 1985 ACC, Chairman of the Conference Activities Board of the IEEE CSS and member of its Board of Governors, General Chairman of FUSION 2000, President of ISIF in 2000 and 2002, and Vice President for Publications during 2004–2013. In 1987, he was the recipient of the IEEE CSS Distinguished Member Award. Since 1995, he is a Distinguished Lecturer of the IEEE Aerospace and Electronic Systems Society. He was the corecipient of the M. Barry Carlton Award for the best paper in the *IEEE Transactions on Aerospace and Electronic Systems* in 1995 and 2000. In 2002, he was the recipient of the J. Mignona Data Fusion Award from the DoD JDL Data Fusion Group. He is a member of the Connecticut Academy of Science and Engineering. In 2008, he was the recipient of the IEEE Dennis J. Picard Medal for Radar Technologies and Applications, and in 2012 the Connecticut Medal of Technology. He has been listed by academic.research.microsoft (top authors in engineering) as #1 among the researchers in aerospace engineering based on the citations of his work. He was the recipient of the 2015 ISIF Award for a Lifetime of Excellence in Information Fusion. This award has been renamed in 2016 as the Yaakov Bar-Shalom Award for a Lifetime of Excellence in Information Fusion.